Macroscopic Tunneling of a Membrane in an Optomechanical Double-Well Potential

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Macroscopic tunneling of a membrane in an optomechanical double-well potential

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The macroscopic tunneling of an optomechanical membrane is considered. A cavity mode which couples quadratically to the membranes position can create highly tunable adiabatic double-well potentials, which together with the high Q-factors of such membranes render the observation of macroscopic tunneling possible. A suitable, pulsed measurement scheme using a linearly coupled mode of the cavity for the verification of the effect is studied.

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Optomechanical systems have seen a recent surge in experimental and theoretical interest, culminating in the cooling of micromechanical oscillators to within a fraction of a phonon of their quantum ground state [1–4] and the reaching of the strong coupling between cavity field and mechanical element [5, 6]. Despite these successes, few experimental demonstrations of non-classical behavior have been achieved so far. Notable exceptions include Ref. [1], which coupled the mechanical oscillator to a Josephson qubit to detect the presence of a single mechanical phonon, and Ref. [4], which demonstrated the asymmetry between up-converted and down-converted photons of a probe laser field, an unambiguous signature of the asymmetry between phonon absorption and emission. Additional proposals to generate and exploit non-classical effects in cavity optomechanics include schemes to squeeze a motional quadrature of the oscillator [7–12], perform quantum state tomography [1, 13, 14], or offer alternative ways to engineer non-classical mechanical states [15, 16] including most intriguingly perhaps the realization of Schrödinger cat states in truly macroscopic systems [17–19].

This paper extends these considerations by exploring the possibility to realize and verify the quantum tunneling of an optomechanical system operating deep in the quantum regime through a classically forbidden potential barrier. The observation of the tunneling of such a truly macroscopic object has not been achieved yet, although theoretical proposals have been made [20]. We find that this can be achieved in a "membrane-in-the-middle" (MiM) configuration [22–24] under conditions that are close to being realizable in current state-of-the-art experiments. We propose a detection scheme based on pulsed optomechanics ideas [14] that permits to monitor the tunneling dynamics through a series of weak measurements of the membrane position.

Our approach relies on adiabatically raising a potential barrier, whose parameters can be widely tuned, at the location of a mechanical harmonic oscillator. We show that the ground state of the resulting double-well potential can exhibit tunneling rates several orders of magnitude larger than the decoherence rate of the mechanical membrane, and that a weak optomechanical position measurement is enough to monitor the tunneling. Besides tunneling, the proposed scheme allows for the study of the quantum Zeno effect (QZE) [21] in a mechanical context and provides a comparatively simple scheme for the preparation and characterization of non-classical mechanical states of interest for quantum metrology and sensing.

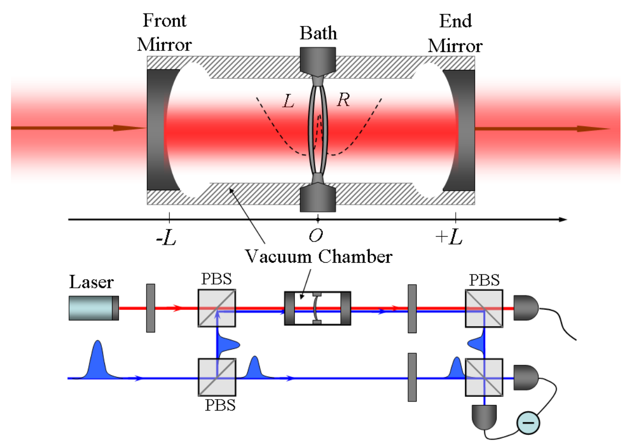

FIG. 1. (Color Online) Membrane-in-the-middle optomechanical system. The dashed lines illustrate the adiabatic double-well potential induced by the field mode. The membrane in shown in a superposition of left-well state and right-well state. The lower figure sketches a scheme for the detection of membrane tunneling.

We consider a MiM optomechanical system [22, 23, 25–27] consisting of two fixed mirrors and a partially reflecting membrane of center-of-mass oscillation frequency ω_M and effective mass M between them, see Fig. 1. A unique feature of this system is the option to realize either linear or quadratic optomechanical couplings, depending on the precise membrane's equilibrium position. Our proposed scheme involves one cavity mode that couples quadratically to the membrane to realize a double well potential, and a second mode that couples linearly and is excited by short optical pulses to first prepare and then monitor the membrane position.

Consider first the quadratic coupling mode. The corresponding system Hamiltonian is

$$\hat{H} = \hat{H}_{\rm M} + \hat{H}_{\rm opt} + \hat{H}_{\rm pump} + \hat{H}_{\kappa} + \hat{H}_{\gamma}, \tag{1}$$

where we have in harmonic oscillator units ($\omega_M = M = 1$)

$$\hat{H}_{\rm M} = \frac{\hat{p}^2}{2} + \frac{1}{2}\hat{x}^2, \tag{2}$$

$$\hat{H}_{\rm opt} = \hbar\left(\omega_c + \frac{1}{2}g_2\hat{x}^2\right)\hat{a}^\dagger\hat{a}, \tag{3}$$

$$\hat{H}_{\rm pump} = i\hbar\left(\eta e^{-i\omega_p t}\hat{a}^\dagger - \eta^* e^{i\omega_p t}\hat{a}\right). \tag{4}$$

Here $\hat{H}_{\rm M}$ is the energy of the mechanical resonator and $\hat{H}_{\rm opt}$ is the optomechanical interaction, with ω_c the resonance frequency of the cavity and g_2 the quadratic optomechanical coupling constant. The optical pumping at frequency ω_p with pumping rate η is described by $\hat{H}_{\rm pump}$. The dissipation terms $\hat{H}_\kappa$ and $\hat{H}_\gamma$ account for cavity and mechanical damping with rates κ and γ respectively.

The Heisenberg equations of motion for the membrane position can be cast as the second order equation

$$\frac{d^2\hat{x}}{dt^2} + \frac{\gamma}{2}\frac{d\hat{x}}{dt} = -\hat{x} - g_2\hat{a}^\dagger\hat{a}\hat{x} + \hat{\xi}\left(t\right), \tag{5}$$

where $\hat{\xi}\left(t\right)$ is the noise operator associated with thermal damping of the oscillator, and the evolution of the cavity field is governed by

$$\frac{d}{dt}\hat{a} = -i\left(\delta_c + \frac{1}{2}g_2\hat{x}^2\right)\hat{a} - \frac{\kappa}{2}\hat{a} + \eta + \sqrt{\kappa}\hat{a}_{\rm in}. \tag{6}$$

Given stable parameters, the cavity field approaches its steady state in a timescale κ^{-1}. For $\kappa \gg (\omega_M, \gamma)$ it therefore follows adiabatically the position of the membrane (note that this neglects the optomechanical membrane cooling), and the membrane dynamics are robust against fluctuations of the cavity field. Furthermore, a fast cavity decay rate destroys the quantum correlations between the light field and the membrane, in which case it is sufficient to treat the optical field classically, which we do in the following. The intracavity intensity as a function of the membrane position is

$$\bar{I}(\hat{x}, t) \approx \frac{\eta^2}{\left(\kappa/2\right)^2 + \Delta^2\left(\hat{x}, t\right)}, \tag{7}$$

where $\Delta\left(\hat{x}, t\right) \equiv \delta_c + g_2\langle\hat{x}^2\rangle\left(t\right)$, with $\delta_c = \omega_c - \omega_p$ the detuning of the pump from the cavity resonance. Inserting this expression into Eq. (5) and integrating the right hand side gives the effective potential acting on the membrane,

$$U(\hat{x}) = \frac{1}{2}\hat{x}^2 + \frac{4\eta^2}{\kappa}\arctan\left[\frac{\Delta\left(\hat{x}, t\right)}{\kappa/2}\right]. \tag{8}$$

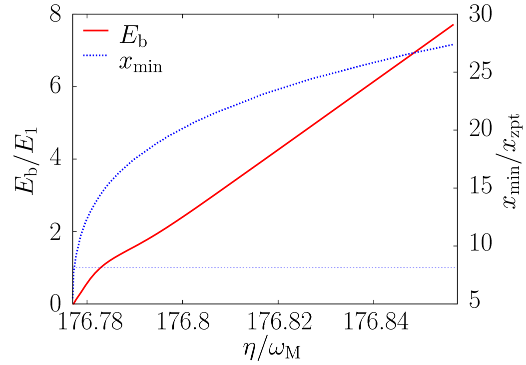

FIG. 2. (Color Online) Ratio of barrier height to ground state energy (solid line, left axis) and half minimum separation $x_{\rm min}$ in units of the zero-point width $x_{\rm zpt}$ (dotted line, right axis) as functions of the pumping rate η. The x-axis starts at the point where we have $D = 0$ (see main text). The coupling strength is $g_2/\omega_{\rm M} = -2 \times 10^{-4}$ and the detuning is zero.

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The light field adds an arctangent function to the harmonic potential, which for positive coupling g_2 leads to tighter confinement and for negative couplings, which is the case we are interested in, to a symmetric barrier whose parameters depend on η and δ_c. For $D \equiv -\kappa^2 - 16\eta^2 g_2 > 0$, the potential becomes a symmetric double well with two local minima at

$$\pm x_{\rm min} = \pm\sqrt{-\frac{2\delta_c + \sqrt{D}}{2g_2}}, \tag{9a}$$

separated by a barrier of height [28]

$$E_{\rm b} = -\frac{1}{2}x_{\rm min}^2 + \frac{4\eta^2}{\kappa}\left[\arctan\left(2\delta_c/\kappa\right) + \arctan\left(\sqrt{D}/\kappa\right)\right]. \tag{9b}$$

Typical parameters allow for a wide range of separations between minima and barrier heights, as illustrated in Fig. 2, which shows $x_{\rm min}$ and $E_{\rm b}$ as a function of the pumping rate η. The beginning of the x-axis is chosen at the pump rate which gives $D = 0$. The distances between minima can be substantial, and the traps should be accordingly shallow to give reasonable tunneling rates. For the numerical calculations, we assumed the quadratic single photon coupling $g_2/\omega_M = -2 \times 10^{-4}$, which for a typical membrane with $\omega_M/2\pi \approx 100$ kHz translates into a single photon optomechanical coupling of 20 Hz. Such relatively high coupling rates can be obtained by using avoided crossings between higher transverse modes [24]. In addition, a noteworthy aspect of this scheme is its versatility, as weak couplings and/or high decay rates can be compensated by adjusting the detuning or increasing the input power, see Eqs. (9), which for the presented calculations is of the order of 1 μW. We note that the shape of the potential is very sensitive to the input power. This constraint is weakened by the fact that the membrane is not sensitive to the *instantaneous* photon number in the cavity, but rather to its average value over the time of a mechanical oscillation. We therefore think that this sensitivity is not a fundamental issue. Generally speaking, low effective membrane mass and frequency are desirable to keep the tunneling rates high.

To estimate the tunneling rates in the double-well potential, we consider the membrane Hamiltonian $\mathcal{H} = \sum_{i=1}^{\infty} E_i \hat{c}_i^\dagger \hat{c}_i$, with eigenenergies E_i determined by the time independent Schrödinger equation $[\hat{p}^2/2+\hat{U}(\hat{x})]\psi = E\psi$. If E_1 and E_2 are smaller than the barrier height E_b, their values lie very close together. The corresponding eigenstates ψ_1 and ψ_2 are symmetric and antisymmetric, respectively, but exhibit similar squared amplitude. Thus the states

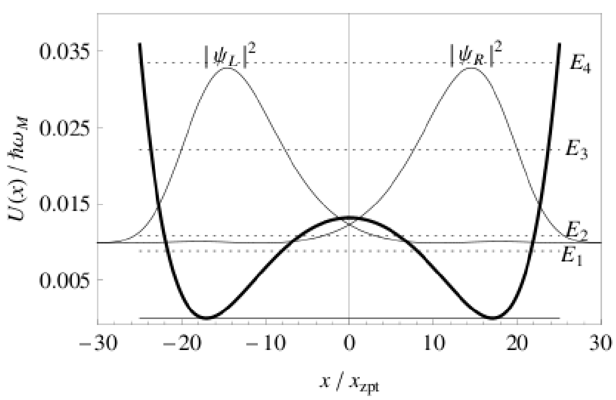

FIG. 3. Double well and resulting splitting of energy levels. Also shown the squared amplitudes of the two localized wavefunctions. Parameters used: $g_2/\omega_\mathrm{M} = -2\times10^{-4}$, $\eta/\omega_\mathrm{M} = 176.785$.

$$\psi_{R,L}(x) = 1/\sqrt{2}(\psi_1(x) \pm \psi_2(x)) \tag{10}$$

are located predominantly in one of the two potential wells. A typical situation is depicted in Fig. 3, which shows a double-well potential, its energy levels as well as the squared amplitudes of the located wave functions $\psi_{L/R}$.

Reexpressing the membrane Hamiltonian in terms of the localized modes

$$\hat{c}_{R,L}(x) = 1/\sqrt{2}(\hat{c}_1 \pm \hat{c}_2) \tag{11}$$

we obtain

$$\mathcal{H} = \sum_{j\in\{L,R\}} E_j \hat{c}_j^\dagger \hat{c}_j + \frac{J}{2}(\hat{c}_L^\dagger \hat{c}_R + \hat{c}_R^\dagger \hat{c}_L) + \sum_{i>2} E_i \hat{c}_i^\dagger \hat{c}_i, \tag{12}$$

with $E_L = E_R = (E_1+E_2)/2$ and the tunneling rate $J = E_2 - E_1$. Figure 4 shows the normalized tunneling rate J/ω_M as a function of the pumping rate η/ω_M, illustrating its exponential decrease for increasing η, i.e., increasing x_min and E_b, see Fig. 2. If more eigenvalues E_i lie below the barrier they can be split up analogously

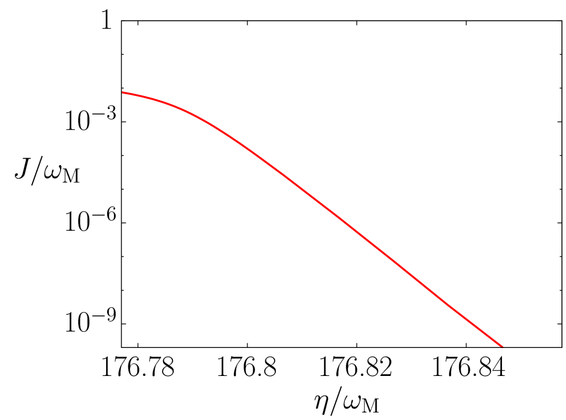

FIG. 4. (Color Online) Tunneling rate J of the two lowest lying states, normalized to ω_M, as a function of the scaled pumping rate η/ω_M for $g_2/\omega_\mathrm{M} = -2\times10^{-4}$, $\kappa/\omega_\mathrm{M} = 10$ and $\delta_c = 0$. Note the logarithmic y-axis.

but we find that ground state tunneling rates become unpractically small in such a situation. In the case of a thermal state, it could still be possible to verify tunneling of higher-lying states. The discussion of this situation is slightly more involved and beyond the scope of this letter.

In order for tunneling to survive decoherence effects the value of J must be large compared to $\bar{n}\omega_M/Q$, where $\bar{n}$ is the number of phonons in the thermal bath. Recent experiments with Si_3N_4 membranes demonstrated Q-factors in excess of 10^6 [29] at frequencies of 500kHz, with $\omega_\mathrm{M}/Q = 0.5\,\mathrm{sec}^{-1}$ and effective mass of $\sim$ 50ng. For such a membrane, we find tunneling rates in the range of $J \sim 100$Hz, indicating that the realization of macroscopic optomechanical tunneling should be within reach of a cryogenic experiment.

The detection of tunneling must be able to confirm its quantum nature – as opposed to classical motion or fluctuations driving the dynamics. The effect of continuous position measurements in a double-well setting has been previously studied in the context of single atom tunneling [30], and reveals two main effects. Firstly, continuous measurements can decrease the tunneling rate due to the QZE and secondly, back-action from the measurement heats up the membrane and thus eventually supplies enough energy to cross the barrier thermally. To avoid these difficulties, we consider instead a pulsed optomechanics approach adapted from the scheme recently proposed by Vanner *et al.* [14] in the context of ponderomotive squeezing, but adapted to the simpler task of monitoring the position of the membrane with a resolution better than the well width. Since the quadratically coupled resonator mode is insensitivite to the sign of x, the measurement sequence involves a sequence of optical pulses that drive a second resonator mode linearly coupled to the membrane.

In the specific experimental realization that we have in mind the center-of-mass motional mode of the membrane is initially prepared in its ground state, a step that is of course essential and that could be achieved either cryogenically or via an initial optomechanical cooling stage, depending on the specifics of the experimental setup. A symmetric potential barrier is then adiabatically raised by increasing the power of the quadratically coupled mode, such that the membrane finds itself in the symmetric ground state of the double well. The dynamics of the membrane is then monitored by performing a sequence of pulsed measurements [14].

Briefly, the pulsed interaction and measurement are described by non-unitary operators that determine the post-measurement mechanical state via

$$\rho_{\rm post} = \frac{\Upsilon \rho_{\rm pre} \Upsilon^\dagger}{{\rm Tr}\left(\Upsilon \rho_{\rm pre} \Upsilon^\dagger\right)}, \tag{13}$$

where the Kraus operator Υ is given explicitly by

$$\Upsilon = \exp\left[-\frac{(x_{\rm res} - \hat{x})^2}{2\sigma^2}\right], \tag{14}$$

where σ^2 is the uncertainty of the measurement, inversely proportional to the measurement strength [14]. It is determined by the coupling strength of the light mode to the membrane and the pulse intensity.

A "preparation" pulse that projects the membrane position with equal probabilities in the left or right well determines its initial position. That measurement must be strong enough to project the state of the membrane unambiguously into one of the two wells, but weak enough to keep its energy below the barrier. The membrane then starts to tunnel to the opposite well, which it will occupy with high probability after a time of the order of J^{-1}. A sequence of measurement pulses with a pulse separation short compared to that time is then applied to monitor the tunneling dynamics. Averaging trajectories starting in the same well, will unambiguously confirm quantum tunneling by revealing the slow, harmonic oscillation of the membranes position. In contrast, classical inter-well transitions that might result from too strong measurement pulses or an insufficient barrier height will result in a zero average displacement.

We demonstrated the validity of that measurement protocol by performing a numerical stochastic simulation for the parameter values of Fig. 3. Following that initial preparation that left the membrane in the left potential well, the density operator was propagated unitarily and a sequence of 20 pulsed measurements per tunneling cycle were simulated using Eq. (13). A typical trajectory is plotted in Fig. 5. The quantum nature of the jumps was confirmed by monitoring the membrane's energy and verifying that it stayed below the barrier height, and the signature of quantum tunneling is the emergence of harmonic oscillations of the position after averaging post-selected trajectories. The insert of the figure shows a typical histogram of measurement outcomes a short time after the initial preparation of the membrane.

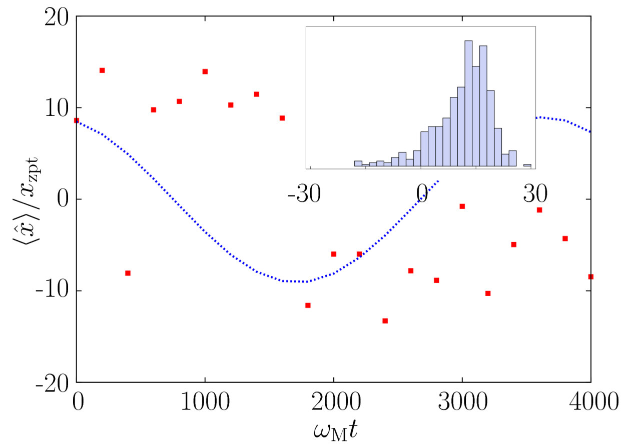

FIG. 5. (Color Online) Typical outcome of a sequence of 20 position measurements of a membrane initially prepared on the left potential well. The solid line gives the coherent evolution (i.e. without measurements) of $\langle \hat{x} \rangle$ for that particular trajectory. Inset: Distribution of 200 measurement outcomes at $\omega_{\rm M} t = 20$, where only trajectories that started in the left well were post-selected. Parameters as in Fig. 3,, measurement strength $1/\sigma = 0.02$.

In summary, we have investigated the possibility to observe the tunneling of a macroscopic optomechanical membrane in a cavity. The potential barrier is a consequence of the quadratic optomechanical coupling of the membrane to a lossy cavity mode. This set-up is a versatile tool to create wide, shallow double-well potentials. Using parameters from recent experiments, we find that achievable tunneling rates can exceed the decoherence rates of the membrane by several orders of magnitude. The effect could be verified in a pulsed optomechanics measurement scheme with a weak optical pulse train linearly coupled to the membrane. Besides the possibility of verifying quantum tunneling at unprecedented scales, this system also offer the potential to prepare and detect nonclassical "Schrödinger Cat" state. Further studies will include investigations of different ways to characterize such mechanical states nondestructively and the potential exploitation of their non-classical nature in applications such as high-precision quantum metrology, the study of decoherence mechanisms, and the quantum-classical transition.

ACKNOWLEDGEMENTS

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[1] A. D. O'Connell *et al.*, Nature **464**, 697 (2010).
[2] J. D. Teufel *et al.*, Nature **475**, 359 (2011).
[3] J. Chan *et al.* Nature **478**, 89 (2011).
[4] A. H. Safavi-Naeini *et al.*, arXiv:1108.4680v1 (2011).
[5] S. Gröblacher *et al.*, Nature **460**, 724 (2009).
[6] J. D. Teufel *et al.*, Nature **471**, 204 (20110.
[7] V. B. Braginsky, Y. I. Vorontsov, and K. P. Thorne, Science **209**, 5347 (1980).
[8] A. A. Clerk, F. Marquardt, and K. Jacobs, New J. Phys. **10**, 095010 (2008).
[9] A. Nunnenkamp *et al.*, Phys. Rev. A **82**, 021806(R) (2010).
[10] J. B. Hertzberg *et al.*, Nat. Phys. **6**, 213 (2010).
[11] M. J. Woolley *et al.*, Phys. Rev. A **78**, 062303 (2008).
[12] Junho Suh *et al.*, Nano Letters **10**, 3990 (2010).
[13] S. Singh and P. Meystre, Phys Rev A **81**, 041804(R) (2010).
[14] M. Vanner *et al.*, Proc. Nat. Acad. Sci. USA **108**, 16182 (2011).
[15] V. Peano and M. Thorwart, Phys. Rev. B **70**, 235401 (2004).
[16] L. Mazzola and M. Paternostro, Phys. Rev. A **83**, 062335 (2011).
[17] O. Romero-Isart *et al.*, Phys. Rev. Lett. **107**, 020405 (2010).
[18] O. Romero-Isart *et al.* New J. Phys. **12**, 033015 (2010).
[19] E. Chang *et al.*, Proc. Nat. Acad. Sci. **107**, 1005 (2010).
[20] M. A. Sillanpää *et al*, Phys. Rev. B **84**, 195433 (2011).
[21] E. C. G. Sudarshan and B. Misra, J. Math. Phys. **18**, 756 (1977).
[22] J. D. Thompson *et al.*, Nature **452**, 72 (2008).
[23] A. M. Jayich *et al*, New J. Phys. **10**, 095008 (2008).
[24] J. C. Sankey *et al.*, Nature Physics **6**, 707 (2010).
[25] M. Bhattacharya and P. Meystre, Phys. Rev. A **78**, 041801(R) (2008).
[26] M. Bhattacharya, H. Uys and P. Meystre, Phys. Rev. A **77**, 033819 (2008).
[27] J. Larson and M. Horsdal, Phys. Rev. A **84**, 021804(R) (2011).
[28] If $\delta_c > -\sqrt{D}/2$, $U(x=0)$ becomes a local minimum and the potential is a symmetric triple well. Although such a case is interesting in its own right, we focus on the situation of a double-well in this paper.
[29] P.-L. Yu, T. P. Purdy, C. A. Regal, arXiv:1111.1703v1 (2011).
[30] M. J. Gagen, H. M. Wiseman and G. J. Milburn, Phys. Rev. A **48**, 132–142 (1993); C. M. Caves and G. J. Milburn, Phys. Rev. A **36**, 5543 (1987).