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Taming the Pion Cloud of the Nucleon

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We present a light-front determination of the pionic contribution to the nucleon self-energy, Σ_π, to second-order in pion-baryon coupling constants that allows the pion-nucleon vertex function to be treated in a model-independent manner constrained by experiment. The pion mass μ dependence of Σ_π is consistent with chiral perturbation theory results for small values of μ and is also linearly dependent on μ for larger values, in accord with the results of lattice QCD calculations. The derivative of Σ_π with respect to μ^2 yields the dominant contribution to the pion content, which is consistent with the $\bar{d}-\bar{u}$ difference observed experimentally in the violation of the Gottfried sum rule.

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Understanding the pion and its interaction with and amongst nucleons is a necessary step in learning how QCD describes the interaction and existence of atomic nuclei. As a nearly massless excitation of the QCD vacuum with pseudoscalar quantum numbers, the pion plays a central role in particle and nuclear physics as a harbinger of spontaneous symmetry breaking. The pion is associated with large distance structure of the nucleon [1, 2] and the longest ranged component of the nucleon-nucleon force [3]. In lattice QCD calculations the nucleon mass depends on an input value of the quark mass, which generates a pion mass μ, and extrapolation formulae depending on μ are typically used [4–7] (see the review [8].) In addition, the pion cloud plays an important role in deep inelastic scattering on the nucleon, especially in understanding the violation of the Gottfried sum rule [9, 10].

Phenomenological calculations of pion-nucleon interactions are beset with uncertainties related to the dependence of the vertex function on momentum transfer and on the possible dependence upon the virtuality (difference between the square of the four-momentum and mass squared) of any intermediate nucleon or baryon. Moreover, modern treatments of spin 3/2 baryons such as the Δ (baryon excitation of lowest mass) within the Rarita-Schwinger (RS) [11] formalism have been problematic as discussed in [12]. The pathologies of the $\pi N\Delta$ coupling have long been known [13–17]. The aim of the present letter is to develop and apply a method that is free of those ambiguities.

As a specific example, consider the role of the pion cloud in deep inelastic scattering. This is related to the pion contribution to the nucleon self-energy of Fig. 1a. One needs to include the term in which the virtual photon interacts with the pion [18], Fig. 1b, but one also needs to include the effects of the virtual photon hitting the nucleon, Fig. 1c. Conservation of momentum and charge would seem to require that the argument of the vertex function depends on the square of the invariant mass of the intermediate pion-baryon system (s) [19]. Taking the form factor to have the standard form of depending on the square of the four-momentum transfer, between the initial nucleon and intermediate baryon (t), while natural, popular and effective [20],[2] seemingly disagrees with charge and momentum conservation according to [19].

But chiral symmetry (limit of vanishing pion mass) provides strong guidance. It is known that the πN vertex function $G_{\pi N}(t)$ and the nucleon axial form factor are related by the generalized Goldberger-Treiman relation [21]):

$$MG_A(t) = f_\pi G_{\pi N}(t), \tag{1}$$

where t is the square of the four-momentum transfered to the nucleons, $G_A(t)$ is the axial vector form factor and f_π is the pion decay constant. The result Eq. (1), obtained from a matrix element of the axial vector current between two on-mass-shell nucleons, follows from PCAC and the pion pole dominance of the pseudoscalar current. Using Eq. (1) has obvious practical value because it relates an essentially unmeasurable quantity

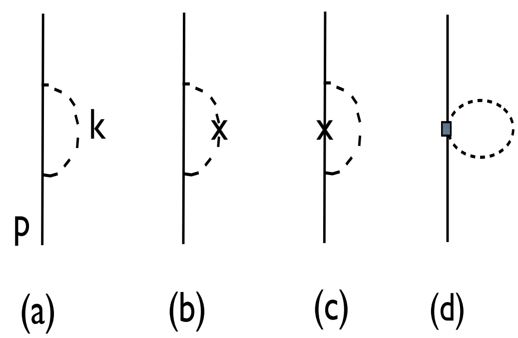

FIG. 1: (a) Pionic (dashed line) contribution to the nucleon (solid line) self-energy. (b) External interaction, x, with the pion (c) External interaction, x, with the intermediate nucleon. (d) Effect of 2π-nucleon interaction.

$G_{\pi N}$ with one G_A that is constrained by experiments. However the t dependence inherent in Eq. (1) would seem to violate the purported consequence of momentum conservation. Similarly the pionic coupling between nucleons and Δ particles has an off-diagonal Goldberger-Treiman relation [22–24], obtained using similar logic:

$$2MC_5^A(t) = f_\pi G_{\pi N\Delta}(t), \tag{2}$$

where C_5^A is the Adler form factor [25, 26], accessible in neutrino-nucleon interactions.

The present manuscript develops a method that satisfies momentum conservation, utilizes Eq. (1) and involves only on-mass-shell nucleons. The key to removing ambiguities lies in evaluating the relevant Feynman diagrams by carrying out the integration over the four-momentum k by first integrating over k^- (the light front energy) in such a way that the intermediate baryon is projected onto its mass shell. This allows the use of the on-mass shell form factors Eqs. (1,2) and is manifestly consistent with charge and momentum conservation.

Consider the contribution to the nucleon self-energy $\Sigma_\pi(N)$, involving an intermediate nucleon, Fig. (1a), given by Feynman rules as

$$\Sigma_\pi(N) = -i3g_{\pi N}^2\bar{u}(P)\int\frac{d^4k}{(2\pi)^4}\frac{\gamma^5(\not p - \not k + M)\gamma^5}{(k^2-\mu^2+i\epsilon)((p-k)^2-M^2+i\epsilon)}u(P)F^2(k^2), \tag{3}$$

where M,μ are the nucleon and pion masses. The quantity P represents the nucleon momentum and spin, (p,s), evaluated in the proton rest frame. We use the notation: $G_{\pi N}(t) \equiv g_{\pi N}F(t) = \frac{M}{f_\pi}G_A(t)$, with $G_A(0) = 1.267 \pm 0.04$, $M = 0.939$ GeV, $f_\pi = 92.6$ MeV, $g_{\pi N} \equiv G_{\pi N}(0) = 13.2$ with $F(0) = 1$. The term $F(k^2)$ represents the pion nucleon form factor. Its dependence on a single variable is justified only if the pionic vertex function appears between two on-mass-shell nucleons. In that case, one may use a dispersion relation:

$$F(k^2) = \frac{1}{\pi}\int_{(3m_\pi)^2}^{\infty} dt'\; Im[F(t')]/(k^2 - t'). \tag{4}$$

Performing the spin average of Eq. (3) leads to the result

$$\Sigma_\pi(N) = \frac{3g^2_{\pi N}}{M}\int \frac{d^4k\; F^2(k^2)}{i(2\pi)^4}\frac{k\cdot p}{(k^2-\mu^2+i\epsilon)((p-k)^2-M^2+i\epsilon)}. \tag{5}$$

We evaluate $\Sigma_\pi(N)$ using light-front coordinates: $k^\pm \equiv k^0 \pm k^3,\; k^2 = k^+k^- - k^2_\perp$. Thus $\Sigma_\pi(N) = \frac{3g^2_{\pi N}}{M}\int dk^+ d^2k_\perp J$, with

$$J \;= \frac{1}{i(2\pi)^4}\frac{1}{2}\int dk^- F^2(k^2)\frac{k\cdot p}{k^+(p-k)^+(k^- - \frac{k^2_\perp+\mu^2-i\epsilon}{k^+})((p-k)^- - \frac{k^2_\perp+M^2-i\epsilon}{p^+-k^+})}. \tag{6}$$

The expression Eq. (4) for $F(k^2)$ is not written explicitly here because the analytic structure is the same as that of $1/(k^2-\mu^2+i\epsilon)$. If $0 < k^+ < p^+$, the first pole in k^- is in the lower half k^- plane (LHP) (as are the ones arising from $F(k^2)$) and the intermediate nucleon pole is in the upper half plane (UHP). We integrate over the UHP, so that the *only* pole we need to consider is the one in which the intermediate nucleon is on its mass shell and the momentum k is space-like. For $k^+ < 0$ and $k^+ > p^+$ all of the poles are on the same side of the real axis, and one obtains 0. We take the residue of the integral for which the nucleon is on shell so that $k^- = p^- - \frac{M^2+k^2_\perp}{p^+-k^+}$. Using the residue theorem and integrating over k^+ leads to the result

$$\Sigma_\pi(N) = -3g^2_{\pi N}\frac{\pi}{8M(2\pi)^3}\int_0^\infty dt\;\frac{t\,F^2(-t)}{(t+\mu^2)}\left(-\frac{t}{M^2}+\sqrt{\frac{t^2}{M^4}+\frac{4t}{M^2}}\right). \tag{7}$$

This result is obtained by using the pseuodscalar form of πN coupling in Eq. (3), but the use of pseudovector coupling would give the same result because the intermediate nucleon is on its mass shell.

To proceed we use a specific form of the form factor F, the commonly used dipole parametrization

$$F(Q^2) = 1/(1+(Q^2/M_A^2))^2, \tag{8}$$

with M_A as the so-called axial mass. The values of M_A are given by $M_A = 1.03 \pm 0.04$ GeV as reviewed in [21]. This range is consistent with the one reported in a later review [27]. A somewhat lower value (0.85 Gev) is obtained [28] if one restricts the extraction region to very low values of Q^2, but we need higher values to evaluate Eq. (7). Using this dipole parameterized form factor F gives

$$\begin{aligned}\Sigma_\pi(N) &= -3Mg^2_{\pi N}\frac{\pi}{4(2\pi)^3}\frac{1}{6\left(\frac{4}{b}-1\right)^{5/2}(a-b)^4}\times\\ &[\sqrt{(4-b)b}\left((a-b)^2(a(b-10)+2(b-1)b) - 3a^2(b-4)^2 b\log\left(\frac{b}{a}\right)\right)\\ &+6\left(4a^3 + a^2(b-6)b((b-4)b+6) - 2ab^2((b-10)b+18) - 2(b-2)b^3\right)\tan^{-1}\left(\sqrt{\frac{4}{b}-1}\right)\\ &+6ab(b-4)^2\sqrt{(a-4)a(b-4)b}\tan^{-1}\left(\sqrt{\frac{4}{a}-1}\right)],\; a\equiv \mu^2/M^2,\; b\equiv M_A^2/M^2.\end{aligned} \tag{9}$$

To relate to chiral perturbation theory we expand in powers of a up to order μ^4 and b around unity to obtain a very accurate representation of the exact expression for $0 \le a \le 0.04,\; 0.6 \le b \le 1.6$. We find

$$\begin{aligned}\widetilde{\Sigma}_\pi(N) &= -3Mg^2_{\pi N}\frac{\pi}{4(2\pi)^3}[\frac{2\pi}{27\sqrt{3}} + \left(-\frac{1}{6} - \frac{10\pi}{27\sqrt{3}}\right)a + \pi a^{3/2} +\\ &\left(\left(\frac{2}{3}+\frac{104\pi}{81\sqrt{3}}\right)a^2 - \frac{16\pi a}{81\sqrt{3}} + \frac{8\pi}{81\sqrt{3}}\right)(b-1) + a^2\left(\frac{\log(a)}{2} - \frac{67\pi}{27\sqrt{3}} - \frac{4}{3}\right)],\end{aligned} \tag{10}$$

where the tilde indicates that a chiral expansion has been made. The term independent of the pion mass provides a -0.222 M correction to the bare nucleon mass, in contrast with an early approach (not using the heavy baryon expansion) which gives a contribution of formal order $M(M/4\pi f_\pi)^2$ [29]. The term of order μ^3 reproduces the standard expression: $-3g_A^2/(32\pi f_\pi^2)\mu^3$ [30].

The next step is to include terms with an intermediate Δ, the baryon excited state of lowest mass, which couples strongly to the πN system. The effects of other intermediate baryons are not included in this first evaluation, but our technique can be applied to those states. We use the isospin-invariant interaction Lagrangian of the form $\mathcal{L}_{\pi N\Delta} = \frac{g_{\pi N\Delta}}{2M}\bar{\Delta}^i_\mu(\mathbf{p}')g^{\mu\nu}u(\mathbf{p})\partial_\nu\pi^i$ +H.c. [22, 23] which yields the same result as the gauge invariant coupling of [12] for an on-shell intermediate Δ. We note that Δ^i is a vector spinor in both spin and isospin space and $g_{\pi N\Delta} = \sqrt{6}/2G_{\pi N\Delta}(0)$, a notational relation between re-normalized coupling constants [22]. The contribution of the intermediate Δ to the nucleon self-energy is given by

$$\Sigma_\pi(\Delta) = i2(\frac{g_{\pi N\Delta}}{2M})^2\bar{u}(P)\int\frac{d^4k}{(2\pi)^4}\frac{(\not p - \not k + M_\Delta)}{(k^2-\mu^2+i\epsilon)((p-k)^2-M_\Delta^2+i\epsilon)}\frac{(p-k)^2}{M_\Delta^2}$$
$$\times P^{(3/2)}_{\mu\nu}(p-k)k^\mu k^\nu u(P)F_\Delta^2(k^2), \tag{11}$$

where the factor of 2 arises from the isospin matrix element, M_Δ is the mass of the Δ and our notation for the projection operator $P^{(3/2)}_{\mu\nu}$ is given in [12]. We take the ratio of coupling constants to be $(\frac{g_{\pi N\Delta}}{g_{\pi N}})^2 = 72/25$, which is the $SU(6)$ quark model result. The form factor F_Δ is defined via $G_{\pi N\Delta}(t) \equiv g_{\pi N\Delta}F_\Delta(t) = \frac{2M}{f_\pi}C_5^A(t)$. Performing the spin average leads to the result

$$\Sigma_\pi(\Delta) = 2(\frac{g_{\pi N\Delta}}{2M})^2\frac{1}{M}\int\frac{d^4k\ F_\Delta^2(k^2)}{i(2\pi)^4}(M^2 - p\cdot k + MM_\Delta)\frac{(p-k)^2}{M_\Delta^2}$$
$$\times\frac{2}{3}[k^2 - \frac{(k\cdot(p-k))^2}{(p-k)^2}]\frac{1}{(k^2-\mu^2+i\epsilon)((p-k)^2-M_\Delta^2+i\epsilon)}. \tag{12}$$

We evaluate $\Sigma_\pi(\Delta)$ using light-front coordinates in a procedure analogous to that used for $\Sigma_\pi(N)$. The integral over k^- is done in the upper half k^- plane (UHP), so that the only pole is the one in which the intermediate Δ is on its mass shell and the momentum k is space-like. The result is

$$\Sigma_\pi(\Delta) = -2(\frac{g_{\pi N\Delta}}{2M})^2\frac{\pi}{M(2\pi)^3}\frac{1}{3}\int_0^\infty dt\ \frac{F_\Delta^2(-t)}{(t+\mu^2)}\left(t + \frac{1}{4M_\Delta^2}(M^2 - M_\Delta^2 + t)^2\right)$$
$$\times\frac{1}{2}\left((M+M_\Delta)^2 + t\right)\left(\frac{-t + M^2 - M_\Delta^2}{2M^2} + \frac{1}{2M^2}\sqrt{(M_\Delta^2 - M^2 + t)^2 + 4tM^2}\right). \tag{13}$$

We turn to numerical evaluations. Lattice calculations [24] indicate that the ratio $G_{\pi N\Delta}(t)/G_{\pi N}(t)$ is constant as a function of the space-like values of t, thus here we use $F_\Delta(t) = F(t)$. The integration of Eq. (13) yields a lengthy closed form expression. To gain insight, and compare with the general form of the chiral expansion of baryon masses in QCD *e.g.* [31–33] we take $M_A = M, b = 1$ and expand in μ/M, $(\xi - 0.72)$, $\xi \equiv \frac{M_\Delta^2 - M^2}{M^2}$ to find

$$\widetilde{\Sigma}_\pi(\Delta) = -2M(\frac{g_{\pi N\Delta}}{2})^2\frac{\pi}{(2\pi)^3}\frac{1}{3}[f_1(a) + (\xi - 0.72)f_2(a)] \tag{14}$$

$$f_1(a) \equiv -0.888a^2 + 1.01a^2\log(a) - 1.55a^2(\log(a) + 1.20)$$
$$-0.402a^2(\log(a) + 1.24) - 0.00369a + 0.280a\log(a) + 0.310 \tag{15}$$

$$f_2(a) \equiv (5.48a^2 + 1.46a^2\log(a) + 2.39a^2(\log(a) + 1.20)$$
$$+0.128a^2(\log(a) + 1.24) + 1.02a + 0.318a\log(a) - 0.0196), \tag{16}$$

where the tilde indicates that a chiral expansion has been made. The terms of order $\mu^4\log\mu^2$ emphasized by [5],[34] are included, but the expression also contains previously noted [33] dominating non-analytic terms of the form $\mu^2\log\mu^2$.

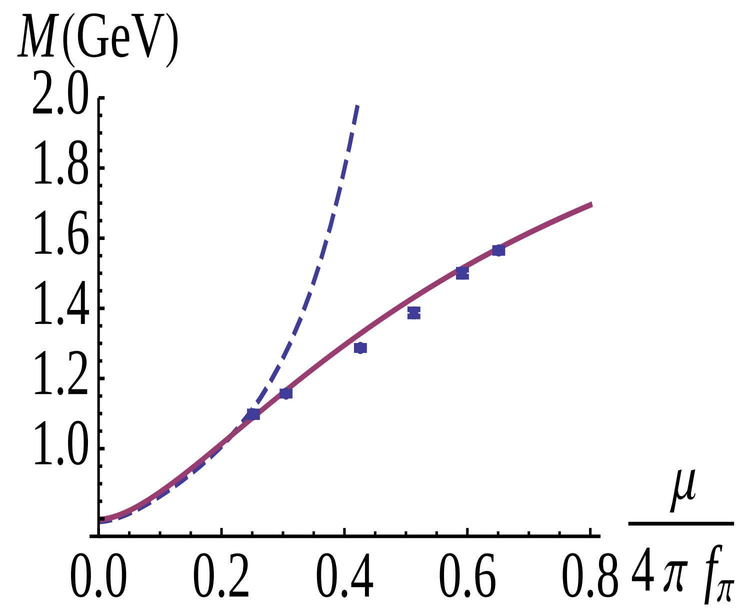

FIG. 2: Nucleon mass as a function of μ. Square blocks: LHP lattice data [35]. Solid: Σ_π of Eq. (17), Eq. (9) and Eq. (13). Dashed: chiral approximation $\widetilde{\Sigma}_\pi = \widetilde{\Sigma}_\pi(N) + \widetilde{\Sigma}_\pi(\Delta)$, Eq. (10) and Eq. (14).

The total pionic contribution to the nucleon mass Σ_π is given by

$$\Sigma_\pi \equiv \Sigma_\pi(N) + \Sigma_\pi(\Delta), \tag{17}$$

and the chiral approximation $\widetilde{\Sigma}_\pi$ is given by $\widetilde{\Sigma}_\pi \equiv \widetilde{\Sigma}_\pi(N) + \widetilde{\Sigma}_\pi(\Delta)$. These are shown in Fig. 2 as a function of the varying pion mass μ, the only parameter that is varied. Bare masses, $M_0 = 2.42$ GeV, $\widetilde{M}_0 = 2.06$ GeV have been added to $\Sigma_\pi, \widetilde{\Sigma}_\pi$ so as to reproduce the lattice data point at $\mu/4\pi f_\pi = 0.252(\mu = 293)$ MeV. We use $M_A = 1.03$ GeV. The use of the exact expression gives an approximately linear dependence on the pion mass, in agreement with the "surprisingly linear" results of lattice QCD simulations [8, 35], found for values of μ greater than about 290 MeV. The LHP lattice data [35] are shown, and these are consistent with other lattice calculations as reviewed Varying the value of M_A within the stated range changes the value of Σ_π only for $\mu > 0.5$ GeV, and by 5 % or less. The low-order chiral approximation of Eq. (10) and Eq. (14) fails badly, showing that the chiral logarithms do not dominate for the relatively large values of μ used in many previous lattice QCD calculations. One could carry out the expansions of Eq. (10) and Eq. (14) to higher order in μ, but convergence requires many terms. One achieves a satisfactory description of $\Sigma(N)$ up to $\mu = 0.65$ GeV by keeping terms up to order μ^{24}, and of $\Sigma(\Delta)$ up to $\mu = M_\Delta - M$ GeV by keeping terms up to order μ^{20}.

It is worthwhile to compare our procedure with that of some others. For example, if one uses the heavy baryon limit to simplify Eq. (3), evaluates the integral by taking the pion to be on its mass shell and regularizes the divergent integral over momentum using a cutoff at a maximum momentum, one obtains results that correspond to the terms used in [5]. The relativistic procedure of [36] avoids the use of the heavy baryon limit by treating the nucleon recoil terms using an expansion procedure and uses dimensional regularization. We include all of the recoil terms and employ a cut-off procedure that is constrained by experimental data. In chiral perturbation theory, our procedure corresponds to keeping a specific set of higher-order terms with a fixed relation between them, a relation fixed by experimental data.

Our results do not include contributions of order higher than $1/f_\pi^2$. These may be considered as keeping the lowest order pion cloud corrections using an expansion in powers of $\varepsilon \equiv 1/(4\pi f_\pi R)^2$, where R is a confinement radius [1, 37, 38]. Here $R \sim \sqrt{12}/M_A$, so $\varepsilon \approx 1/12$. Thus we expect our results for the terms computed here to be accurate within about 10%. This argument was mainly applied to terms involving combinations of couplings of the nucleon to a single pion, but also holds for the n-pion-nucleon vertex *e.g.* as appearing in Fig. 1d. These terms enter at higher orders in μ in chiral perturbation theory [39]. The coupling constant $g_{\pi N}$ and the confinement sizes of the pion and the nucleon, although not explicit in chiral perturbation theory, enter into the calculation of the diagram in terms of quarks and gluons and via the implicit dependence of f_π and $g_{\pi N}$ on the underlying strong coupling constant, α_S. Therefore we expect that the terms of the chiral Lagrangian will be consistent with the expansion in ε.

To test our treatment of the nucleon self-energy, we consider the contribution to lepton-nucleon deep inelastic scattering DIS arising from virtual pions. This is related to the term $\mathcal{M}_\pi$, obtained from Feynman rules for the diagram of Fig 1b, as

$$\mathcal{M}_\pi = 2M \frac{\partial \Sigma_\pi}{\partial \mu^2}. \tag{18}$$

This expression does not involve a "probability", because the square of a nucleon light-front wave function does not appear. Note that charge and momentum are explicitly conserved: production of a pion of momentum k is accompanied by an intermediate nucleon of momentum $p-k$. The integrations over $k^-, k_\perp$ are carried out explicitly, and with the definition $y = k^+/p^+$ one finds

$$\begin{aligned}
&\mathcal{M}_\pi = \int_0^1 dy f_\pi(y),\ f_\pi(y) \equiv f_\pi^N(y) + f_\pi^\Delta(y),\\
&f_\pi^N(y) \equiv 3 g_{\pi N}^2 \frac{\pi}{2(2\pi)^3} \int_{y^2M^2/(1-y)}^\infty dt\ \frac{t\ F^2(-t)}{(t+\mu^2)^2},\\
&f_\pi^\Delta(y) \equiv 2(\frac{g_{\pi N\Delta}}{2M})^2 \frac{\pi}{(2\pi)^3} \frac{2}{3} \int_{(y^2M^2+y(M_\Delta^2-M^2))/(1-y)}^\infty dt\ \frac{F^2(-t)}{(t+\mu^2)^2}\\
&\times \left(t + \frac{1}{4M_\Delta^2}(M^2 - M_\Delta^2 + t)^2\right) \frac{1}{2}((M+M_\Delta)^2 + t).
\end{aligned} \tag{19}$$

The functions $f_\pi^N(y), f_\pi^\Delta(y)$ are shown in Fig. 3 where one observes that these functions are of roughly equal importance.

The change in the quark distribution functions of the nucleon, $\delta q_i(x)$, from this effect is given by the convolution formula as $\delta q_i(x) = \int_x^1 dy f_\pi(y) q_i^\pi(x/y)$, with q_i^π the distribution functions for quarks of flavor i in the pion. The related contribution to the nucleon structure function $\delta F_2(x)$ is

$$\delta F_2(x) = \int_x^1 y f_\pi(y) F_2^\pi(x/y) dy, \tag{20}$$

where F_2^π is the pion structure function [9, 40].

An integral involving the difference between the proton and neutron structure functions is particularly interesting:

$$\int_0^1 \frac{dx}{x}(F_2^p(x) - F_2^n(x)) = 1/3 - \frac{2}{3}\int_0^1 dx(\bar{d}(x) - \bar{u}(x)), \tag{21}$$

where the first term, obtained if the bare nucleon has a symmetric sea, i.e. $\bar{d} = \bar{u}$, represents the Gottfried sum rule [41]. Experiment has clearly established violation of the Gottfried sum rule, and the most precise determination of the sea asymmetry [42] is

$$D \equiv \int_0^1 (\bar{d}(x) - \bar{u}(x))dx = 0.118 \pm 0.012. \tag{22}$$

Henley & Miller [10] showed that the pion cloud provides a natural explanation of the measured asymmetry. For Fig. 1b, the pion cloud of a proton will include $\pi^+(u\bar{d})$ and the π^0, which has equal numbers of $\bar{d}$ and $\bar{u}$.

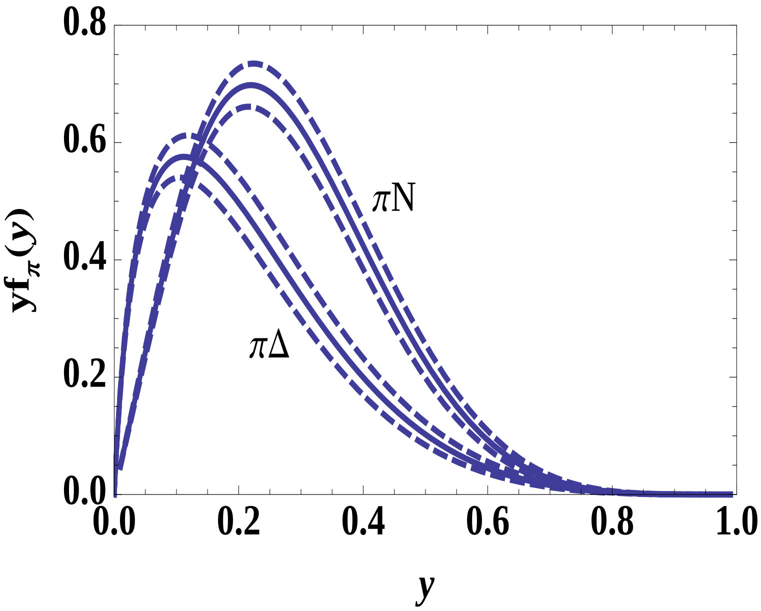

FIG. 3: $yf_\pi(y)$ for the intermediate πN and $\pi\Delta$ states for M_A = 0.99,1.03,1.07 GeV

Only valence quarks of the pions are considered; the pion sea distributions are assumed to be symmetric. The probability for a $\pi^+ n$ intermediate state is 2/3, and that for a $\pi^0 p$ state is 1/3. Including also the effects of an intermediate Δ leads to

$$D_\pi = \int_0^1 dyy(\frac{2}{3}f_\pi^N(y) - \frac{1}{3}f_\pi^\Delta(y)), \tag{23}$$

with the probability of $\pi^-\Delta^{++} = 1/2$ and that for $\pi^+\Delta^0$ =1/6. Since a bare baryon is assumed to have a symmetric sea, possible contributions from Fig. 1c do not enter. Using M_A = 1.03 GeV, the nucleonic contribution is 0.173, and the Δ contribution is -0.064, so that the total is 0.109, within the experimental range of Eq. (22).

To summarize: our light front treatment of the relevant Feynman diagrams reveals that the pion-baryon vertex function appears only between on-mass-shell baryons. This allows the vertex function to be expressed in terms of one variable, the invariant momentum transfer t, and to be constrained by experimental data. All ambiguities regarding the theoretical input needed to evaluate effects of the pion cloud to second-order in the coupling constants for the effects of intermediate N, Δ are resolved. The uncertainty due to the neglect of higher-order terms is estimated to be about 10%. Our procedure reproduces the observed linear dependence of the nucleon mass on the pion mass found in lattice QCD calculations and the flavor asymmetry of the nucleon sea. This work has implications for nucleon-nucleon scattering because one is instructed to use the coupling implied by Eq. (1), and also for computing pion cloud effects on the elastic electromagnetic form factors of nucleons [43].

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