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Phys. Rev. Lett. **108**, 167603 — Published 19 April 2012

DOI: [10.1103/PhysRevLett.108.167603](https://doi.org/10.1103/PhysRevLett.108.167603)

1 **Direct observation of the proliferation of ferroelectric loop domains and**  
2 **vortex-antivortex pairs**

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12

13 **We discovered “stripe” patterns of trimerization-ferroelectric domains in hexagonal**  
14 **RE<sub>2</sub>MnO<sub>7</sub> (RE=Ho, ---, Lu) crystals (grown below ferroelectric transition temperatures ( $T_c$ ),**  
15 **reaching up to 1435 °C), in contrast with the vortex patterns in YMnO<sub>3</sub>. These stripe**  
16 **patterns roughen with the appearance of numerous loop domains through thermal**  
17 **annealing just below  $T_c$ , but the stripe domain patterns turn to vortex-antivortex domain**  
18 **patterns through a freezing process when crystals cross  $T_c$  even though the phase transition**  
19 **appears not to be Kosterlitz-Thouless-type. The experimental systematics are compared**  
20 **with the results of our six-state clock model simulation and also the Kibble-Zurek**  
21 **Mechanism for trapped topological defects.**

22

1           The subtle phase transitions straddling the boundary between long-range-order and  
2 disorder have attracted significant attention due to the fundamental science and the technological  
3 perspective [1-5]. For example, the ground state of two-dimensional (2D) spins with planar  
4 continuous degrees of freedom undergoes only quasi-long-range-order with spin-spin  
5 correlations falling off algebraically in space, and becomes a high-temperature disorder state  
6 with exponentially-decaying correlations through the so-called Kosterlitz-Thouless (KT)  
7 transition by the proliferation of unbound topological vortices [6-8]. Not only the KT transition  
8 in 2D planar spins but also various phenomena such as 2D melting and roughening transition at  
9 surface are associated with the emergence of a topological order, resulting from the binding of  
10 topological defects [9, 10]. Even though the ordering issue of 2D condensed matters is a time-  
11 honored topic, the topological ordering process in “large-scale real space” has little investigated  
12 experimentally. Furthermore, it is a profound question how the topological KT order is  
13 influenced by the 3<sup>rd</sup>-dimensional coupling and quenched disorder that exists often in real  
14 systems.

15           Layered hexagonal YMnO<sub>3</sub> is an improper ferroelectric where the size mismatch between  
16 Y and Mn-O layers induces a trimerization-type structural phase transition, and this structural  
17 transition leads to three antiphase domains ( $\alpha$ ,  $\beta$ ,  $\gamma$ ), each of which can support two directions  
18 (+,-) of ferroelectric polarization [11-13]. The antiphase and ferroelectric domains of YMnO<sub>3</sub>  
19 meet in cloverleaf arrangements that cycle through all six domain configurations. Occurring in  
20 pairs, the cloverleaves can be viewed as vortices and antivortices, in which the cycle of domain  
21 configurations is reversed [14]. Large-scale arrangements of topological vortices and antivortices  
22 in YMnO<sub>3</sub> reveal intriguing real space domain patterns with mathematical simplicity, which  
23 can be analyzed with graph theory [15]. The six possible characteristics of domains, combined  
24 with the layered structure of hexagonal YMnO<sub>3</sub>, suggest the analogy between the 2D six-state  
25 clock model and the physics of YMnO<sub>3</sub>. In the six-state clock model, it has been claimed the  
26 presence of three phases; long-range ordered (LRO) phase, intermediate KT phase, and high-  
27 temperature disordered phase [16]. However, vortex-antivortex domain patterns have been  
28 observed in YMnO<sub>3</sub> at room temperature far below structural transition temperature, which  
29 suggests the KT phase, rather than a LRO phase, as the ground state.

1           Herein, in order to unveil the origin of this inconsistency, we have studied the systematics  
2 of domain configurations in a series of hexagonal REMnO<sub>3</sub> (RE=rare earths) crystals grown with  
3 a flux method, and also the thermal evolution of the domain configurations. In order to reveal  
4 domain patterns, thin-plate-like crystals with optically flat surfaces were etched chemically in  
5 phosphoric acid at ~130 °C. The domain patterns of chemically-etched crystals were investigated  
6 using optical microscopy (OM), transmission electron microscopy (TEM) and atomic force  
7 microscopy (AFM). Note that preferential chemical etching of surface areas with upward  
8 polarization enables the observation of ferroelectric domain patterns on a crystal surface using  
9 OM or AFM. (see the detailed experimental method in the supplementary.)

10           Unlike vortex-antivortex domain patterns in YMnO<sub>3</sub>, stripe domain patterns with large  
11 downward (-) polarization domains are discovered in most of REMnO<sub>3</sub> crystals. The OM image  
12 of a stripe domain pattern on one entire surface of an ErMnO<sub>3</sub> crystal is shown in Fig. 1(a).  
13 These stripe lines in the OM image are identified as narrow trenches with the depth of ~500 nm  
14 as revealed in AFM scans (see the inset of Fig. 1(a)), and tend to be along the [110] direction (the  
15 hexagonal  $P6_3cm$  notation). These trenched lines correspond to narrow upward (+) polarization  
16 domains [15]. These stripe domain patterns are distinct from the topological vortex-antivortex  
17 domain pattern with small domains in YMnO<sub>3</sub>, shown in Fig. 1(b). In the vortex-antivortex  
18 domain pattern in YMnO<sub>3</sub>, a vortex consists of six trimerization antiphase ( $\alpha$ ,  $\beta$ ,  $\gamma$ ) and  
19 ferroelectric (+,-) domains merging at the center of the vortex, and is paired with an antivortex  
20 (or antivortices) with the opposite vorticity in terms of structural antiphase and ferroelectric  
21 relationship as shown in the inset of Fig. 1(b) [15]. On the other hand, the stripe domain pattern  
22 in Fig. 1(a) spans the entire crystal surface, and we have, in fact, observed only these stripe  
23 domain patterns without any hint of the presence of vortices in all REMnO<sub>3</sub> (RE=Ho, Er, Tm, Yb,  
24 Lu, but not Y) crystals. We also note that the stripe domain patterns with large downward-  
25 polarization domains appear to be consistent with an LRO phase, rather than the topological KT  
26 phase. As discussed below, this striking difference between stripe and vortex-antivortex domain  
27 patterns depends entirely on whether the crystal growth temperature is above the trimerization-  
28 structural transition temperature ( $T_c$ ) or not.

1 In order to explore the thermal evolution of these stripe domain patterns, we cooled down  
2 crystals from various annealing temperatures to room temperature and then etched them  
3 chemically. Figure 2(a) displays the AFM image of a chemically-etched  $\text{ErMnO}_3$  surface after  
4 cooling it down fast from  $1120^\circ\text{C}$ . Even though the equilibrating temperature is very high, the  
5 LRO stripe domain pattern changes little. However, when the temperature is raised by only  $20^\circ\text{C}$ ,  
6 the pattern changes significantly, and exhibits highly-curved lines with the appearance of many  
7 curved closed loops, as shown in Fig. 2(b). Dark stripe lines, conserved robustly up to  $1120^\circ\text{C}$ ,  
8 start to wiggle heavily at  $1140^\circ\text{C}$ , but never cross to each other, i.e., there is no hint of the  
9 presence of vortices in the entire crystal surface (see the supplementary section 2 and Fig. S2.).  
10 The more-or-less straight parts of dark stripe lines are indicated with white dashed lines in the  
11 upper region of Fig. 2(b). TEM image of closed loop domains is shown in Fig. 2(c), and the  
12 corresponding possible schematic is shown in Fig. 2(d). Therefore, when the system approaches  
13  $T_c$  from below, thermal fluctuations induce roughening of stripes domain walls and the  
14 appearance of a large number of loop domains.

15 When the annealing temperature is further raised by  $30^\circ\text{C}$  up to  $1170^\circ\text{C}$ , a complicated  
16 pattern was observed after chemical etching as shown in Fig. 2(e). This pattern, in fact, shows  
17 the crossing of lines and the presence of a large number of vortices. This vortex-antivortex  
18 domain pattern formation is more evident when the annealing temperature was raised to  $1200^\circ\text{C}$ ,  
19 as shown in Fig. 2(f). We have determined the characteristic temperatures for all  $\text{REMnO}_3$   
20 ( $\text{RE}=\text{Ho}, \text{Er}, \text{Tm}, \text{Yb}, \text{Lu}$ ) at which stripe domain patterns turn into vortex-antivortex domain  
21 patterns after the annealing/cooling experiments. The obtained characteristic temperatures are  
22 plotted in the inset of Fig. 2(f), and the reported  $T_c$  of  $\text{YMnO}_3$  is also plotted in the inset [17].  
23 The rough linear dependence in the inset strongly suggests that the characteristic temperatures,  
24 indeed, the trimerization-structural  $T_c$  of  $\text{REMnO}_3$ . Note that  $T_c$ 's of  $\text{REMnO}_3$  ( $\text{RE}=\text{Ho}, \text{Er}, \text{Tm},$   
25  $\text{Yb}, \text{Lu}$ ) have never been reliably determined because of the very high temperature nature, and  $T_c$   
26 drastically increases with decreasing RE size, which is consistent the notion that the structural  
27 transition is induced by the mismatch between small RE layers and large Mn-O layers in the  
28  $\text{REMnO}_3$  structure. Note that  $\text{REMnO}_3$  crystals were grown by slow cooling of the materials  
29 with  $\text{Bi}_2\text{O}_3$  flux in the temperature range of  $1200^\circ\text{C}$  and  $950^\circ\text{C}$ , but the real growth through  
30 nucleation occurs probably slightly above  $950^\circ\text{C}$ . Thus,  $\text{YMnO}_3$  crystals are likely grown above

1  $T_c$ , but other REMnO<sub>3</sub> crystals below  $T_c$ . Therefore, it appears that stripe domain patterns form  
2 when the crystal growth temperature is below  $T_c$  while vortex-antivortex domain patterns are  
3 realized when crystals are exposed to temperatures above  $T_c$ .

4 Interestingly, we found that once vortex-antivortex domain patterns, spanning the entire  
5 crystal surface (see the supplementary section 3 and Fig. S3), form by crossing  $T_c$ , they are  
6 conserved with various thermal treatments, but the domain size of vortex-antivortex domain  
7 patterns or the distance between vortices and antivortices can vary in a systematic manner. In  
8 order to find out the thermal evolution of vortex-antivortex domain patterns and the domain  
9 growth kinetics, the cooling rate near  $T_c$  was changed from 0.5 °C/h to 300 °C/h. In addition, we  
10 cooled one specimen from 1220 °C to 677 °C with cooling rate of 5 °C/h, followed by quenching  
11 to room temperature. Figures 3(a), (b), (c) and (d) show the AFM images of etched ErMnO<sub>3</sub>  
12 crystals with the above thermal treatments. With the large variation of cooling rate from 0.5 °C/h  
13 to 300 °C/h, vortex-antivortex domain patterns remain intact, but the domain size of vortex-  
14 antivortex domain patterns changes systematically.

15 We emphasize that the vortex-antivortex domain patterns in Figs. 3(b) and (c) are basically  
16 identical, indicating that the cooling rate below 677 °C does not influence the domain patterns.  
17 This is an important result for the origin of the mysterious second transition of REMnO<sub>3</sub> near  
18 600 °C reported in many early publications [12, 17-24]. This second transition at ~600 °C was  
19 identified as the ferroelectric transition from centrosymmetric  $P6_3/mmc$  to low-temperature polar  
20  $P6_3cm$  structures via an intermediate  $P6_3/mcm$  structure [18, 19] whereas other results argued for  
21 the nonexistence of intermediate  $P6_3/mcm$  state, but the presence of an isosymmetric phase  
22 transition with Y-O hybridization [12, 17, 20, 21, 23]. We, in fact, compared directly vortex-  
23 antivortex domain patterns at room temperature and 730 °C from TEM dark-field experiments  
24 using the  $1\bar{3}1$  diffraction spot as shown in Figs. 4(a) and (b). Basically there is little difference  
25 between two vortex-antivortex domain patterns, which, combined with no difference in vortex-  
26 antivortex domain patterns on cooling rate across the second transition temperature, are  
27 consistent with the possibility of an isosymmetric change at the second transition if it exists.

1        The analysis of the evolution of domain size of vortex-antivortex domain patterns with  
2        varying cooling rate demonstrates slow growth kinetics associated with the topological vortex-  
3        antivortex domain patterns. The average distance of vortex-antivortex pairs vs. inverse cooling  
4        rate,  $t$ , is plotted in Fig. 4(c). The cooling rate dependence of the average vortex-antivortex pair  
5        distance,  $D$ , can be described by  $D \propto t^n$  with  $n=0.23$  (See the supplementary section 4 and Fig.  
6        S4). This value of  $n \approx 1/4$  is rather different from the typical parabolic domain growth value of  
7         $n=1/2$  [25, 26]. This slow growth kinetics with vortex-antivortex domain patterns seem  
8        consistent with the Kibble-Zurek Mechanism for spontaneous trapping of topological defects in a  
9        system undergoing a continuous phase transition [27, 28]. For the average vortex-antivortex pair  
10       distance  $\propto t^n$  with  $n \approx 1/4$ , the density of vortex should vary like  $t^{1/2}$ , which is precisely the  
11       prediction of the Kibble-Zurek mechanism with mean field critical exponents [28]. We also note  
12       that chemical/structural-defects-induced pinning may play an important role for the variation of  
13       the vortex-antivortex distance with cooling rate. In fact, strong pinning tendency of domain  
14       patterns has been observed in many of our results. For example, the domain patterns in Fig. 2(a)  
15       and Fig. 4(b) change little with expected large thermal fluctuations at temperatures such as 1120  
16       °C and 730 °C. In addition, we have directly observed vortex-antivortex domain patterns pinned  
17       by large-scale surface defects, as shown in Fig. 4(d). Strong pinning, slow kinetics, and the  
18       different topology between vortex-antivortex and stripe domain patterns induce probably an  
19       “astronomical” time scale for the conversion of vortex-antivortex domain patterns to stripe  
20       domain patterns, even though the true ground state corresponds to stripe domain patterns.

21       The concept of topological KT order with binding vortices and antivortices was originally  
22       developed for the quasi-long-range-order in 2D XY systems. In the KT phase, algebraically  
23       decaying correlation can accompany topological defects, i.e., vortices. As the order parameter  
24       possesses a finite value,  $q$ , representing the number of evenly-spaced possible order orientations,  
25       there exists a long-range ordered phase at low temperatures when  $q$  is small. Convincing  
26       evidence has been amassed, based both on qualitative and quantitative considerations, to indicate  
27       that for large enough  $q$  (thought to be  $5 \leq q < 8$ ), there exists an intermediate phase similar to the  
28       KT phase [16]. The 2D  $q$ -state clock model with  $q \geq 8$  precisely reproduces the KT transition [29,  
29       30]. In the case of the 2D six-state clock model,  $q=6$ , the topological order exists only in an  
30       intermediate temperature range, and the ground state is a LRO phase (see the supplementary

1 section 5 and Fig. S5.). Any 3<sup>rd</sup> dimensional coupling will destabilize the intermediate KT phase  
2 in the six-state clock model. Therefore, one expects theoretically that a LRO phase will be the  
3 ground state of hexagonal REMnO<sub>3</sub> with six degrees of freedom, and the intermediate KT order  
4 may or may not exist, depending on the strength of 3<sup>rd</sup> directional coupling. Experimentally, our  
5 results indicate that the LRO ground state with stripe domain patterns can be observed, but only  
6 when crystals were grown below  $T_c$ . When crystals were grown from temperatures above  $T_c$  or  
7 crystals with stripe domain patterns were exposed to a temperature above  $T_c$ , domain patterns are  
8 always vortex-antivortex domain patterns. In addition, we have observed convincing evidence  
9 for strong pinning and also slow kinetics associated with domain patterns. Furthermore, our  
10 results do not show any indication for two step transitions from a long-range order to a KT order  
11 to disorder. Therefore, it is convincing that there exists only one LRO transition associated with  
12 six degrees of freedom. However, in any real situations without astronomical-time-scale  
13 annealing, slow kinetic creation of topological defects, which is well manifested in the Kibble-  
14 Zurek mechanism, and their strong pinning induce KT-type vortex-antivortex domain patterns  
15 when crystals undergoes the structural transition at  $T_c$ . In other words, a KT order is only implicit  
16 in hexagonal REMnO<sub>3</sub> and should not occur as the ground state, but it is arrested by slow kinetic  
17 and strong pinning. This “arresting” scenario is fully consistent with our simulation of the 2D  
18 six-state clock model with different cooling rates (see the supplementary section 5 and Fig. S5.).

19 In summary, the true ground state with stripe domain patterns can be realized when  
20 hexagonal REMnO<sub>3</sub> (RE=Ho, Er, Tm, Yb, Lu) crystals are grown below  $T_c$ , and is consistent  
21 with the long-range-ordered ground state of the six-state clock model with a significant 3<sup>rd</sup>  
22 dimensional coupling. When crystals cross  $T_c$ , domain patterns with topological vortices and  
23 antivortices are realized. These vortex domains can be enlarged with the time exponent of 1/4,  
24 which is much smaller than the typical domain growth exponent of 1/2. It is conceivable that the  
25 topological KT-type vortex-antivortex domain patterns eventually turn into long-range-ordered  
26 stripe domain patterns through astronomical-time-scale annealing. Therefore, the presence of  
27 stripe domain patterns in REMnO<sub>3</sub> crystals grown below  $T_c$  is an experimental marvel, enabling  
28 the observation of what happens in the true ground state of REMnO<sub>3</sub>.

29 Acknowledgement:



- 1 We thanks D. Vanderbilt for critical reading of the manuscript. This work was supported by
- 2 National Science Foundation DMR-1104484.

## 1 **References**

- 2 [1] C. H. Chen, J. M. Gibson, and R. M. Fleming, *Phys. Rev. Lett.* **47**, 723 (1981).
- 3 [2] A. Del Maestro, B. Rosenow, and S. Sachdev, *Phys. Rev. B* **74**, 024520 (2006).
- 4 [3] I. F. Herbut, and Z. Tesanović *Phys. Rev. Lett.* **73**, 484 (1994).
- 5 [4] O. Tchernyshyov, and G.-W. Chern, *Phys. Rev. Lett.* **95**, 197204 (2005).
- 6 [5] S. Larochelle, M. Ramazanoglu, and R. J. Birgeneau, *Phys. Rev. E* **73**, 060702 (2006).
- 7 [6] J. M. Kosterlitz, and D. J. Thouless, *J. Phys. C* **6**, 1181 (1973).
- 8 [7] D. R. Nelson, and J. M. Kosterlitz, *Phys. Rev. Lett.* **39**, 1201 (1977).
- 9 [8] V. Berezinskii, *Sov. Phys. JETP* **34**, 610 (1972).
- 10 [9] B. I. Halperin, and D. R. Nelson, *Phys. Rev. Lett.* **41**, 121 (1978).
- 11 [10] B. Kahng, and K. Park, *Phys. Rev. B* **47**, 5583 (1993).
- 12 [11] B. B. Van Aken, T. T. M. Palstra, A. Filippetti, and N. A. Spaldin, *Nature Mater.* **3**, 164
- 13 (2004).
- 14 [12] T. Katsufuji, S. Mori, M. Masaki, Y. Moritomo, N. Yamamoto, and H. Takagi, *Phys. Rev.*
- 15 *B* **64**, 104419 (2001).
- 16 [13] M. Fiebig, T. Lottermoser, D. Frohlich, A. V. Goltsev, and R. V. Pisarev, *Nature* **419**,
- 17 818 (2002).
- 18 [14] T. Choi, Y. Horibe, H. T. Yi, Y. J. Choi, W. Wu, and S. W. Cheong, *Nature Mater.* **9**, 253
- 19 (2010).
- 20 [15] S. C. Chae, Y. Horibe, D. Y. Jeong, S. Rodan, N. Lee, and S. W. Cheong, *Proc. Natl.*
- 21 *Acad. Sci. U. S. A.* **107**, 21366 (2010).
- 22 [16] M. B. Einhorn, R. Savit, and E. Rabinovici, *Nucl. Phys. B* **170**, 16 (1980).
- 23 [17] A. S. Gibbs, K. S. Knight, and P. Lightfoot, *Phys. Rev. B* **83**, 094111 (2011).

- 1 [18] G. Nénert, and et al., J. Phys.: Condens. Matter **19**, 466212 (2007).
- 2 [19] I. Ismailzade, and S. Kizhaev, Sov. Phys.-Solid State **7**, 236 (1965).
- 3 [20] K. Lukaszewicz, and J. Karut-Kalici ska, Ferroelectrics **7**, 81 (1974).
- 4 [21] T. Lonkai, D. G. Tomuta, U. Amann, J. Ihringer, R. W. A. Hendrikx, ouml, D. M. bbens,  
5 and J. A. Mydosh, Phys. Rev. B **69**, 134108 (2004).
- 6 [22] N. Fujimura, T. Ishida, T. Yoshimura, and T. Ito, Appl. Phys. Lett. **69**, 1011 (1996).
- 7 [23] C. J. Fennie, and K. M. Rabe, Phys. Rev. B **72**, 100103 (2005).
- 8 [24] G. A. Smolenskiĩ, and I. E. Chupis, Sov. Phys. Usp. **25**, 475 (1982).
- 9 [25] G. S. Grest, M. P. Anderson, and D. J. Srolovitz, Phys. Rev. B **38**, 4752 (1988).
- 10 [26] K. Kaski, M. Grant, and J. D. Gunton, Phys. Rev. B **31**, 3040 (1985).
- 11 [27] T. W. B. Kibble, J. Phys. A: Math. Gen. **9**, 1387 (1976).
- 12 [28] W. H. Zurek, Nature **317**, 505 (1985).
- 13 [29] C. M. Lapilli, P. Pfeifer, and C. Wexler, Phys. Rev. Lett. **96**, 140603 (2006).
- 14 [30] S. K. Baek, P. Minnhagen, and B. J. Kim, Phys. Rev. E **80**, 060101 (2009).

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## 1 **Figure legends**

2 **FIG 1.** (color online). Two distinct domain patterns of  $\text{RE}\text{MnO}_3$  crystals; stripe vs. vortex  
3 patterns. (a) optical microscope image of a chemically-etched  $\text{ErMnO}_3$  crystal surface. The inset  
4 shows the atomic force microscope (AFM) image of a stripe domain pattern after chemical  
5 etching. These domain patterns for a long-range ordered phase are also observed in  $\text{HoMnO}_3$ ,  
6  $\text{TmMnO}_3$ ,  $\text{YbMnO}_3$ ,  $\text{LuMnO}_3$  crystals, as shown in Fig. S1 in Supplementary. (b) Optical  
7 microscope image of a chemically-etched  $\text{YMnO}_3$  crystal surface. The inset shows the AFM  
8 image of an  $\text{YMnO}_3$  surface after chemical etching, showing a vortex-antivortex domain pattern.  
9 Vortex and antivortex are distinguished by the arrangement of trimerization antiphase and  
10 ferroelectric domains with the opposite sense of rotation around a core. Note that the entire  
11 surfaces of all as-grown crystals  $\text{RE}\text{MnO}_3$  ( $\text{RE}=\text{Ho}, \text{Er}, \text{Tm}, \text{Yb}, \text{Lu}$ ) exhibit no hint of the  
12 presence of any vortices or antivortices.

13 **FIG 2.** (color online). The thermal evolution of stripe domain patterns in  $\text{ErMnO}_3$ . (a), (b), (e)  
14 and (f), The AFM images of chemically-etched  $\text{ErMnO}_3$  crystals annealed at  $1120^\circ\text{C}$ ,  $1140^\circ\text{C}$ ,  
15  $1170^\circ\text{C}$ , and  $1200^\circ\text{C}$ , respectively. A stripe domain pattern with thermal roughening are evident  
16 in (b), but (e) and (f) display vortex-antivortex patterns. The inset displays  $T_c$ 's of  $\text{RE}\text{MnO}_3$   
17 estimated from the formation temperature of vortex-antivortex domain patterns.  $T_c$  of  $\text{YMnO}_3$  is  
18 from ref. [17]. (c) and (d), TEM image of loop domains of  $\text{ErMnO}_3$  quenched from  $1140^\circ\text{C}$  and  
19 a possible corresponding schematic, respectively. The loop domains observed in the TEM image  
20 are assigned with three antiphase domains ( $\alpha^+$ ,  $\beta^-$ ,  $\gamma^+$ ).

21 **FIG 3.** (color online). The evolution of vortex-antivortex domain patterns with varying cooling  
22 rate. The AFM images of chemically-etched  $\text{ErMnO}_3$  crystals; (a) cooled from  $1220^\circ\text{C}$  to  $890^\circ\text{C}$   
23 with rate of  $0.5^\circ\text{C/h}$ , followed by furnace cooling, (b) cooled from  $1200^\circ\text{C}$  to room temperature  
24 with rate of  $5^\circ\text{C/h}$ , (c) from  $1200^\circ\text{C}$  to  $677^\circ\text{C}$  with rate of  $5^\circ\text{C/h}$ , followed by quenching, and  
25 (d) from  $1200^\circ\text{C}$  to room temperature with rate of  $300^\circ\text{C/h}$ .

26 **FIG 4.** (color online). The pinning of vortex-antivortex domains. (a) and (b) TEM images of  
27 vortex-antivortex domain patterns of  $\text{ErMnO}_3$  at room temperature and  $730^\circ\text{C}$ , respectively. (c)  
28 The cooling rate dependence of the average distance of vortex-antivortex pairs, exhibiting a

1 power law dependence with the power of 0.23. (The open triangle is from Fig 3(c)). (d) The  
2 optical microscope and AFM (inset) images of a chemically-etched  $\text{ErMnO}_3$  crystal surface,  
3 suggesting strong pinning of vortex-antivortex domains by surface defects.

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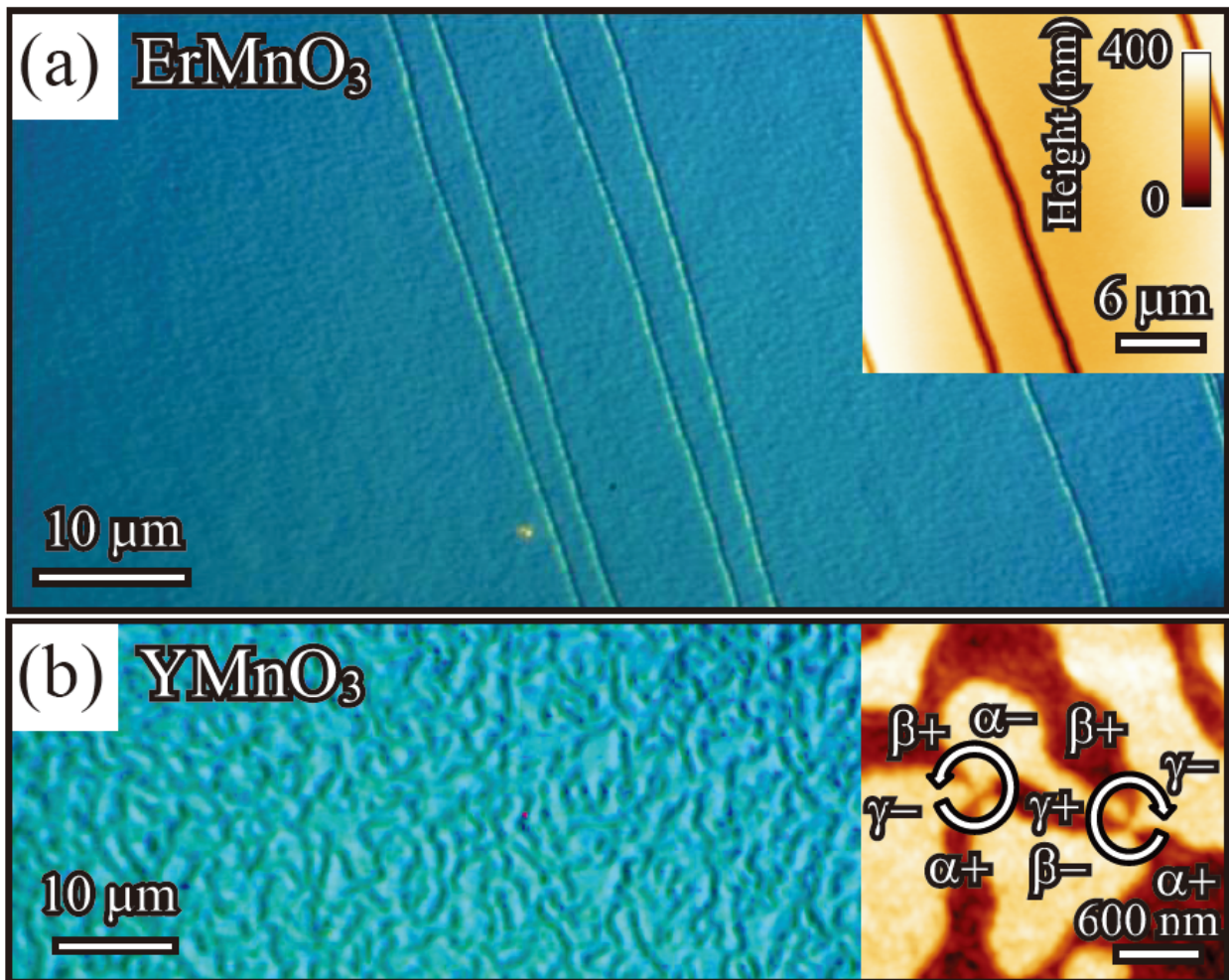


Figure 1

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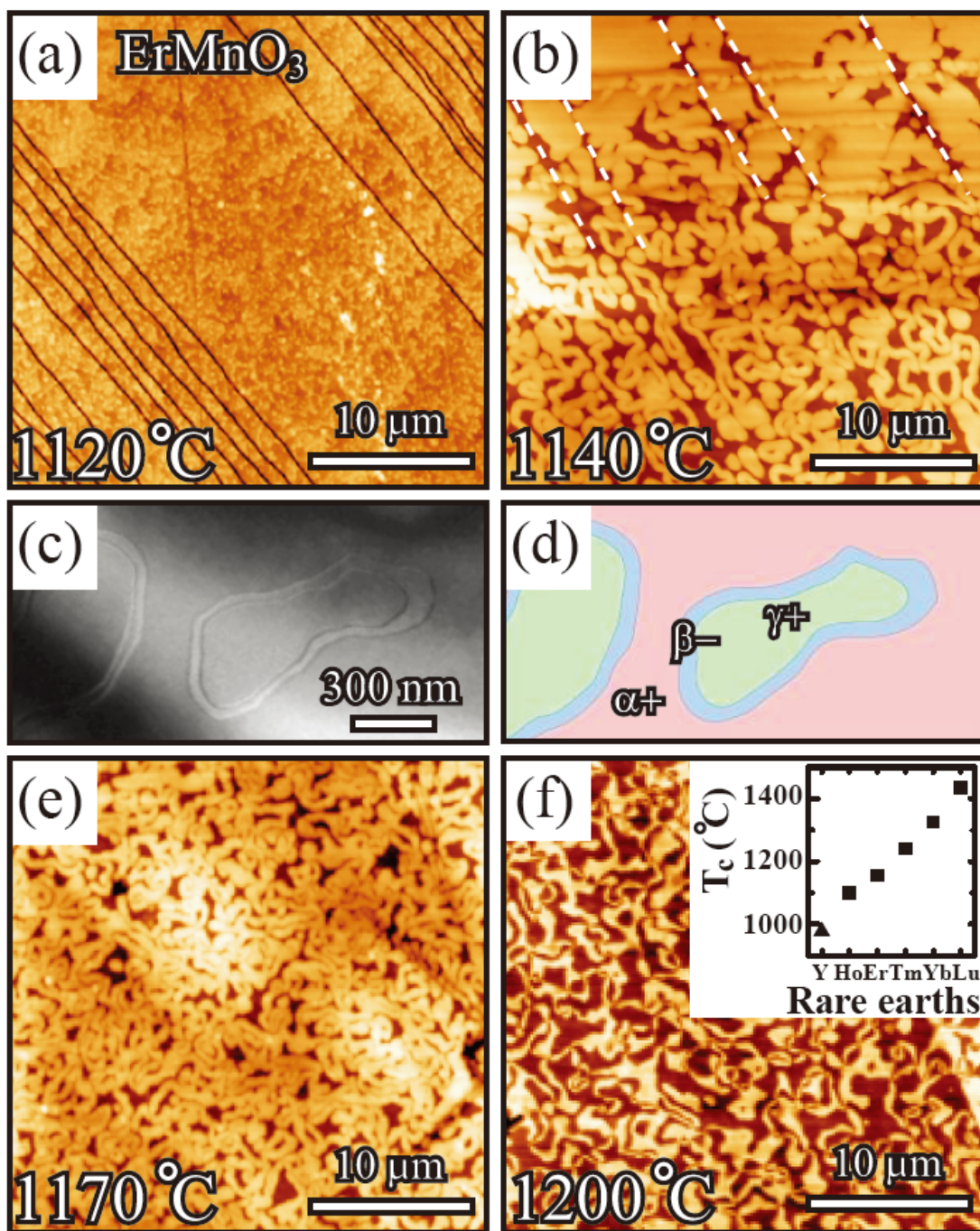


Figure 2

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2

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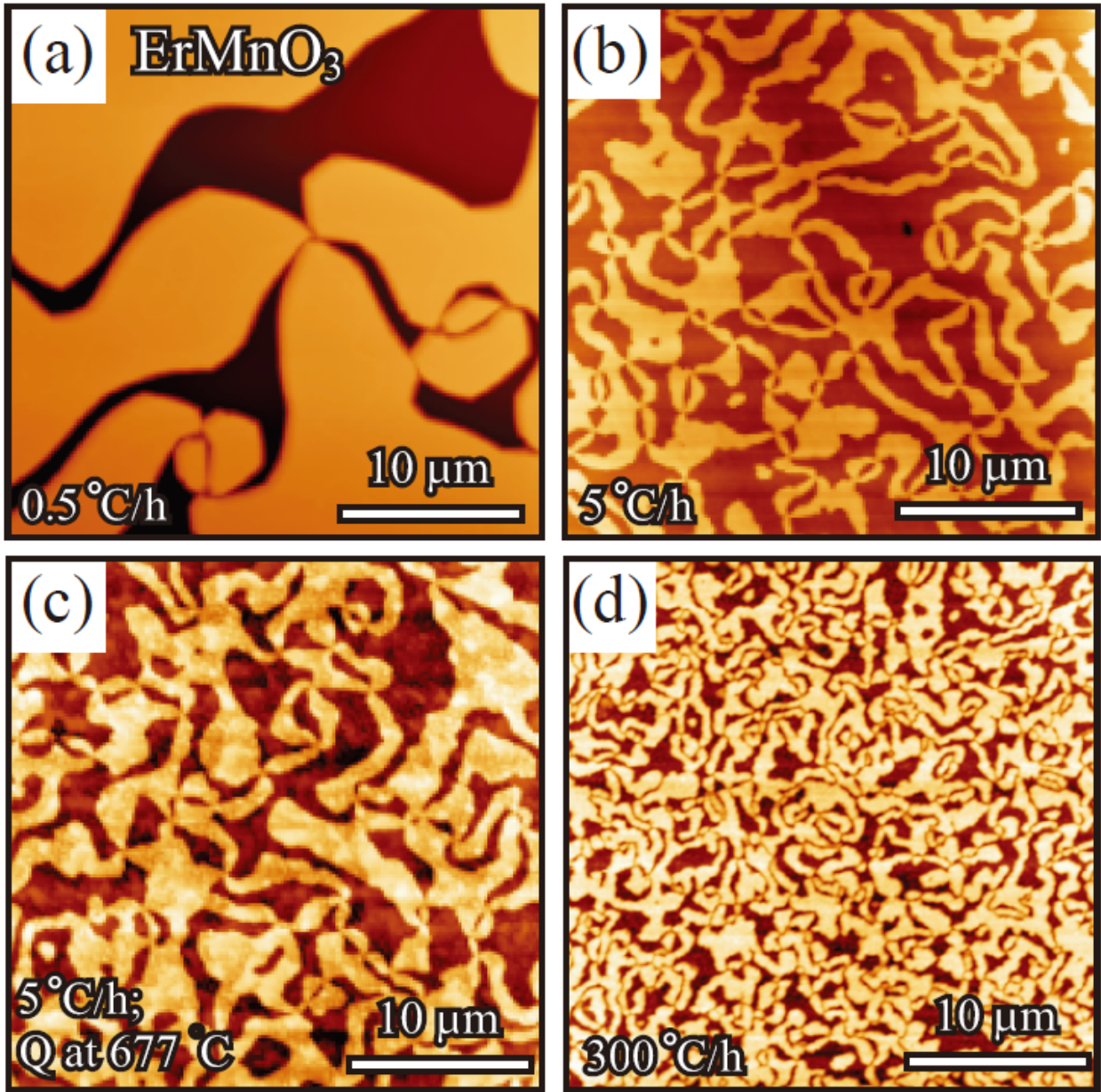


Figure 3

1

2

3



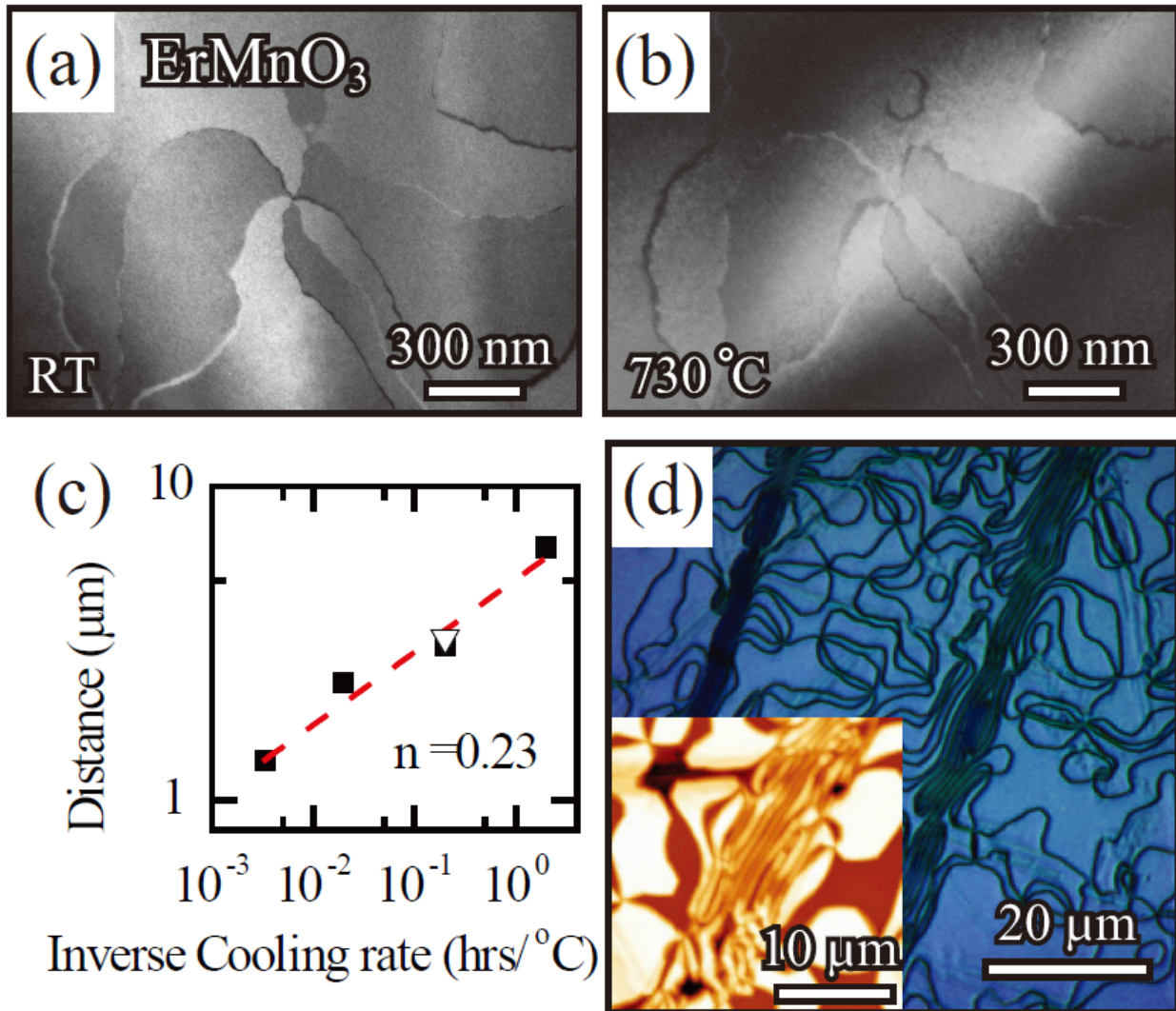


Figure 4

1

2