Using Interference for High Fidelity Quantum State Transfer in Optomechanics

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Using interference for high fidelity quantum state transfer in optomechanics

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We revisit the problem of using a mechanical resonator to perform the transfer of a quantum state between two electromagnetic cavities (e.g. optical and microwave). We show that this system possesses an effective mechanical dark mode which is immune to mechanical dissipation; utilizing this feature allows highly efficient transfer of intra-cavity states, as well as of itinerant photon states. We provide simple analytic expressions for the fidelity for transferring both Gaussian and non-Gaussian states.

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Introduction– The field of quantum optomechanics, where a mechanical resonator is coupled to photons in a cavity, has seen remarkable recent progress. Milestones include using the backaction of photons to cool a mechanical resonator to near its ground state [1, 2], and the observation of strong coupling effects [3–5]. The ability of a mechanical resonator to couple to diverse electromagnetic cavities naturally leads to what is perhaps the most promising application of this field: the possibility of efficiently transferring a quantum state between photons with vastly differing wavelengths [6–9]. Such state transfer would have direct utility in quantum information processing (e.g. the transfer of quantum information from a superconducting qubit in a microwave circuit QED setup to an optical photon, or highly non-classical microwave states as prepared in Ref. [10] to optical photons).

Previous investigations of this problem have largely considered schemes based on two successive "swap" operations in a two-cavity optomechanical system (Fig. 1a). One pulses the optomechanical interactions to first exchange the states of the first cavity and the mechanical resonator; this is then repeated to exchange the mechanical and the second cavity states [6, 7, 11]). While intuitively simple, achieving high-fidelity with this protocol is only possible with low levels of cavity and mechanical dissipation; *we quantify this below.* In particular, one requires extremely low mechanical bath temperatures. This is true even if the mechanics is initially prepared in its ground state [6, 7], as heating during the transfer nonetheless degrades the state. Aspects of this swap-scheme were recently implemented experimentally [14].

Given the above, it would be highly advantageous to find new state transfer schemes less sensitive to mechanical dissipation. This is the goal of this paper. We show that the two-cavity optomechanical system possesses a mode which is delocalized between the two cavities while simultaneously being decoupled from the mechanical dissipation; we term this decoupled mode a "mechanically-dark" mode, as it is analogous to an atomic state which is protected against optical excitation by destructive interference [15]. We show that by using this dark mode, one can perform high-fidelity quantum state transfer of intra-cavity states at levels of mechanical dissipation where the conventional double-swap scheme is essentially unusable.

We also show that this dark mode can be used for efficient transfer of *itinerant* photons (e.g. transferring the state of photons incident on a microwave cavity to the state of photons leaving an optical cavity). This approach is particularly attractive, as it does not require any time-dependent variation of optomechanical couplings. Further, if one is willing to only consider the transfer of small-bandwidth states, the scheme can also be used without requiring optomechanical strong coupling. We quantify analytically the fidelity of this scheme for Gaussian states (in a way that allows easy comparison against the intra-cavity transfer schemes mentioned above), as well as non-classical states; we also consider limitations on the bandwidth of the states that can be transferred. These analytic expressions yield a simple intuitive picture of the factors limiting fidelity. In the limit of weak coupling, this itinerant-photon transfer scheme is equivalent to that described by Safavi-Naeni et al. [8] (though that work did not discuss fidelities, strong coupling, or the role of the dark mode).

Model– We consider an optomechanical system where a single mechanical resonator is simultaneously coupled to both an optical cavity and a microwave cavity via dispersive couplings (see Fig. 1a); particular experimental realizations are discussed in Ref. [7, 8]. We also focus on the standard situation where a weak bare optomechanical coupling $g \ll \kappa, \omega_M$ is enhanced by strongly driving each cavity, resulting in effective linear couplings (see, e.g., [16, 17]). We work in an interaction picture with respect to the two cavity drives, and in a displacement picture with respect to the average (classical) field in each cavity. The Hamiltonian is:

$$\hat{H} = \omega_M \hat{a}^\dagger \hat{a} - \sum_{i=1,2} \left[\Delta_i \hat{d}_i^\dagger \hat{d}_i - G_i \left(\hat{a}^\dagger \hat{d}_i + \hat{d}_i^\dagger \hat{a} \right) \right] + \hat{H}_{\text{diss}} \tag{1}$$

Here, ω_M ($\hat{a}$) is the mechanical frequency (annihilation operator), $\hat{d}_i$ is the annihilation operator of cavity i ($i = 1, 2$) in the displaced frame, and Δ_i is the detuning of the drive applied to cavity i. The driven optomechanical coupling between the mechanical resonator and cavity i is denoted as G_i; note that these are proportional to the

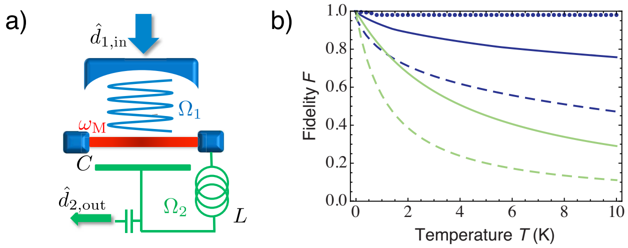

FIG. 1: (Color online) (a) The two-cavity optomechanical system. (b) Fidelity of the double swap (DS) protocol (light green) and the adiabatic transfer (AT) protocol (blue) to transfer a coherent state $|\alpha = 1\rangle$, where $G/2\pi = 2$ MHz (solid), $G/2\pi = 0.5$ MHz (dashed), and $\gamma = 2\pi \times 1$ KHz, $\omega_M/2\pi = 10$ MHz. Cavity 1 (2) is a microwave (optical) cavity: $\Omega_1/2\pi = 10$ GHz ($\Omega_2/2\pi = 100$ THz), $\kappa_1 = \kappa_2 = 2\pi \times 50$ KHz. For DS, $G_1 = G_2 = G$ and the total transfer time is $t^{\mathrm{s}} = \pi/G$. For AT, we used an optimal modulation with $G_1^2(t) + G_2^2(t) = G^2$ constant, and an optimal transfer time at each temperature, which represents a trade-off between heating (via non-adiabatic transitions) and amplitude decay [30]. The blue dotted line corresponds to AT of $|\alpha = 0.1\rangle$ with $G/2\pi = 2$ MHz; as the amplitude decay effect is negligible, a long transfer time can be used to suppress heating caused by non-adiabatic transitions. For DS, the fidelity is mainly limited by heating, hence F versus T for $|\alpha = 0.1\rangle$ is almost indistinguishable from $|\alpha = 1\rangle$.

drive amplitude applied to cavity i, and thus can be controlled in time. H_{diss} describes the damping and driving of the two cavities and mechanical resonator by independent Ohmic baths. We let γ (κ_i) denote the damping rate of the mechanical resonator (cavity i), and let N_M (N_i) denote the bath temperature (expressed as a number of thermal quanta). We also assumed the optimal situation where each cavity is far into the resolved-sideband regime $\omega_M \gg \kappa_i$, and where each cavity is driven near the red-detuned mechanical sideband (i.e. $\Delta_i \sim -\omega_M$). This permits us to make a rotating wave approximation in writing the optomechanical interactions, resulting in a "beam-splitter" form which is optimal for state transfer [11]. We assume below negligible phase noise in the cavity drives. The effect of such noise on coherent transfer was studied in Ref. [12, 13]; the resulting requirements are similar to those needed for cavity cooling [13], and are thus within reach of experiment.

Double-swap protocol– The optomechanical interactions in Eq. (1) can be used to swap states between the three modes of the system [6, 7, 11]. The swap protocol involves first turning on the interaction G_1 for a time $t_1^{\mathrm{s}} = \pi/(2G_1)$ (while $G_2 = 0$), which if $\gamma = \kappa_1 = \kappa_2 = 0$ would swap the states of cavity 1 and the mechanical resonator (i.e. $\hat{a}(t_i^{\mathrm{s}}) = -i\hat{d}_i(0)$ and $\hat{d}_i(t_i^{\mathrm{s}}) = -i\hat{a}(0)$). One then shuts off G_1 and turns on G_2 for a time t_2^{s} to swap the mechanical state to cavity 2.

The presence of mechanical and cavity dissipation degrades this protocol's fidelity. To quantify this, we consider the simple case of transferring a Gaussian state, and calculate the Uhlmann fidelity F [18] between initial and final states. Letting ρ_1 (ρ_2) denote the density matrix of cavity 1(2) at the start (end) of the transfer, we find [28]:

$$F \equiv \left(\mathrm{Tr}[(\sqrt{\rho_1}\rho_2\sqrt{\rho_1})^{1/2}]\right)^2 = \frac{1}{1+\bar{n}_{\mathrm{h}}} \exp\left(-\frac{\lambda^2}{1+\bar{n}_{\mathrm{h}}}\right). \tag{2}$$

Note that we will optimize the fidelity over simple rotations in phase space (so that if ρ_2 is a rotated version of ρ_1, $F = 1$). F depends on just two parameters: $\bar{n}_{\mathrm{h}}$ represents the heating of the state during the protocol by noise emanating from cavity and mechanical dissipative baths, while λ characterizes the decay of the mean value of $\hat{d}$ due to cavity and mechanical damping. Efficient transfer requires minimizing both these effects. In the double-swap protocol, the amplitude-decay will completely suppress F unless one is in the strong coupling limit $G_i > \kappa_i$. In this relevant limit, and for the case where the state to be transferred is a coherent state $|\alpha\rangle$, we find the simple result [29]:

$$\bar{n}_{\mathrm{h}} = \sum_i \frac{\gamma N_M + \kappa_i N_i}{2} t_i^{\mathrm{s}},\ \lambda = |\alpha| \sum_i \frac{\kappa_i + \gamma}{4} t_i^{\mathrm{s}}, \tag{3}$$

where $(\gamma N_M + \kappa N_i)/2$ is the average heating rate and $(\kappa_i+\gamma)/4$ is the average amplitude decay rate during each time interval. We have assumed the optimal situation where the mechanical resonator is initially in its ground state ([6, 7]). Despite this pre-cooling, the mechanical contribution to $\bar{n}_{\mathrm{h}}$ can still be large. One thus requires an extremely low mechanical bath temperature to ensure good fidelity using swap scheme (see Fig. 1b).

Effective mechanically-dark mode– From Eq. (3), we see that the heating $\bar{n}_{\mathrm{h}}$ due to mechanical noise in the double-swap scheme is simply the heating rate times transfer time, and hence scales as $1/G$. We now show that transfer protocols exist where the mechanical heating effect is even more greatly suppressed with increasing G. This is possible by making use of a mode of the two-cavity optomechanical system which is simultaneously delocalized between both cavities, but at the same time is largely immune to to mechanical dissipation.

Focusing as before on the case where each cavity is driven at the red-detuned sideband, we note that the coherent part of the Hamiltonian $\hat{H}_0 = \hat{H} - \hat{H}_{\mathrm{diss}} = \sum_j \hbar\omega_j \hat{c}_j^\dagger \hat{c}_j$ with $j = \pm, \mathrm{dk}$. $\hat{c}_\pm \equiv (\hat{c}_{\mathrm{br}} \pm \hat{a})/\sqrt{2}$ describe hybridized modes of the bright cavity mode $\hat{c}_{\mathrm{br}} \equiv \left(G_1^2 + G_2^2)\right)^{-1/2}\left(G_1\hat{d}_1 + G_2\hat{d}_2\right)$ and the mechanical mode $\hat{a}$, with eigenfrequencies $\omega_\pm = \omega_M \pm \sqrt{G_1^2+G_2^2}$. Whereas

$$\hat{c}_{\mathrm{dk}} \equiv \left(G_1^2 + G_2^2\right)^{-1/2}\left(-G_2\hat{d}_1 + G_1\hat{d}_2\right) \tag{4}$$

describes a delocalized cavity mode which is decoupled from the mechanics. We thus refer to $\hat{c}_{\mathrm{dk}}$ as a "mechanically dark" mode; its frequency is $\omega_{\mathrm{dk}} = \omega_M$, independent of coupling. As we now demonstrate, utilizing this mode allows the efficient transfer of both intra-cavity and itinerant photon states.

Adiabatic transfer– Consider first the same problem addressed by the double-swap scheme, the transfer of an intra-cavity state initially in cavity 1 to cavity 2. This can be accomplished by using an adiabatic passage approach, similar to the well-known STIRAP scheme [19]. One modulates $G_1(t)$ and $G_2(t)$ so that the dark mode adiabatically evolves from being $-\hat{d}_1$ at $t=0$ to $\hat{d}_2$ at the end of the protocol at a time $t=t_f$. The cavity state is thus transferred from cavity 1 to cavity 2 using the the coupling to the mechanics, but without actually populating the mechanics; the result is a greatly enhanced protection against mechanical sources of dissipation.

Fig. 1b shows how such an adiabatic transfer protocol improves the state transfer fidelity over the double-swap scheme when the mechanical heating effect is non-negligible. When transferring a Gaussian state, F again takes the general form described by Eq. (2). The adiabatic "dark mode" transfer protocol dramatically suppresses $\bar{n}_{\rm h}$ compared to the swap scheme. However, to remain adiabatic, the transfer must ideally occur over a time long compared with $1/G$. Thus, similar to the swap scheme, one needs strong coupling (i.e. $\kappa_i \ll G_i$) to avoid the amplitude-decay suppression of F described by λ in Eq. (2). Nonetheless, the greater resilience against mechanical noise presents a strong advantage over the double-swap scheme. A somewhat related scheme for transferring atomic motional states was discussed in Ref. [11]; the uni-directional "cascaded" coupling used there is fundamentally different from that considered here.

Itinerant state transfer– While the previously discussed transfer schemes require a strong optomechanical coupling (i.e. $G_i \gg \kappa_i$), mechanically-mediated transfer is also possible in the opposite regime if the goal is to transfer a narrow-bandwidth state of photons incident on cavity 1 to the state of photons leaving cavity 2 [8, 20]. We now show that the mechanically-dark mode discussed above plays an important role in this itinerant-photon state transfer, and even allows it to be highly effective in regimes of strong optomechanical coupling. We begin by writing the Heisenberg-Langevin equations [21, 22]:

$$\begin{aligned}\dot{\hat{a}} &= -i\omega_M \hat{a} - \gamma\hat{a} - i\sum G_i \hat{d}_i - \sqrt{2\gamma}\hat{a}_{\rm in} \\ \dot{\hat{d}}_i &= -i\Delta_i \hat{d}_i - \kappa_i \hat{d}_i - iG_i\hat{a} - \sqrt{2\kappa_i}\hat{d}_{i,\rm in}\end{aligned} \tag{5}$$

with $\hat{a}_{\rm in}$ and $\hat{d}_{i,\rm in}$ representing both input noise (taken to be white) and signals driving each resonator. Solving Eq. (5) and using standard input-output relations [21] yield the relation between input and output fields $\hat{\mathbf{A}}_{\rm out}[\omega] = \mathbf{s}[\omega]\hat{\mathbf{A}}_{\rm in}[\omega]$ with $\hat{\mathbf{A}}[\omega] = \{\hat{d}_1[\omega], \hat{d}_2[\omega], \hat{a}[\omega]\}$ and $\mathbf{s}[\omega]$ is the scattering matrix (see [30] for details).

High fidelity transfer from $\hat{d}_{1,\rm in}$ to $\hat{d}_{2,\rm out}$ requires that over the input signal bandwidth, the transmission coefficient $|s_{21}[\omega]|^2 \sim 1$, as well as that $|s_{23}[\omega]|^2 \sim 0$ (i.e. negligible transmission of mechanical noise). To quantify this, we consider a Gaussian input state in a temporal mode defined by $\hat{u}_1 = (2\pi)^{-1/2}\int d\omega f[\omega]\,\hat{d}_{1,\rm in}[\omega]$ (see, e.g., Ref. [22]). $f[\omega]$ describes a wavepacket incident on cavity 1 which is localized in both frequency and time; $\int d\omega |f[\omega]|^2 = 1$ to ensure that $\hat{u}_1$ is a canonical bosonic annihilation operator. The fidelity of transferring this itinerant Gaussian state again takes the general form of Eq. (2), and the parameters $\bar{n}_{\rm h}$ and λ can be calculated analytically [23]. For a coherent state input $|\psi_{\rm in}\rangle \propto \exp\left(\alpha\hat{u}_1^\dagger\right)|0\rangle$:

$$\bar{n}_{\rm h} = \sum_{i=1,2,M} \int d\omega \left|f[\omega]\, s_{2i}[\omega]\right|^2 N_i \tag{6}$$

$$\lambda = |\alpha| \max_\tau \left(1 - \left|\int d\omega e^{-i\omega\tau} s_{21}[\omega]\,|f[\omega]|^2\right|\right) \tag{7}$$

We have optimized the final state ρ_2 in Eq. (2) over a time-translation τ, so that if the output pulse is simply a time-delayed copy of the input pulse, $F=1$.

To have protection against mechanical dissipation, one would ideally like the input state incident on cavity 1 to only excite the dark mode. Without dissipation, the dark mode is energetically separated from the coupled modes $\hat{c}_\pm$, and hence protection is achieved by using an input signal with mean frequency ω_M in the displaced frame. Including dissipation (and the consequent lifetime broadening), the input signal incident on cavity 1 will also excite the bright cavity mode $\hat{c}_{\rm br}$ as well as the mechanical mode $\hat{a}$. This unwanted excitation is irrelevant as long as the cooperativity of each cavity $C_i = G_i^2/\gamma\kappa_i \gg 1$. In this limit, the bright mode amplitude $\langle\hat{c}_{\rm br}\rangle$ (average over $|\psi_{\rm in}\rangle$) is a factor $\sim 1/C$ smaller than the dark mode amplitude, due to a destructive interference akin to the optomechanical analogue of electromagnetic-induced transparency [24–26]. The mechanical mode amplitude may be large in the case of weak coupling, but only results in a small flux to the mechanical bath (and hence a small loss) due to the smallness of γ (i.e. $s_{31} \sim 1/\sqrt{C}$). The transfer of the input signal thus occurs almost entirely via the dark mode in this limit (see [30] for more details).

Good fidelity also requires that the dark mode, once excited by the input state, only leaks out via cavity 2, ensuring $|s_{21}[\omega_M]| \sim 1$. This requires a destructive interference between the promptly reflected input signal and the wave leaving the dark mode via cavity 1. For $C_i \gg 1$, this interference cancelation results in the simple impedance matching condition $C_i = C \equiv C$, i.e.:

$$G_1^2/\kappa_1 = G_2^2/\kappa_2 \tag{8}$$

Taking our input mode $|f[\omega]|^2$ to have mean frequency ω_M and a Gaussian profile with variance $\Delta\omega^2$, and as-

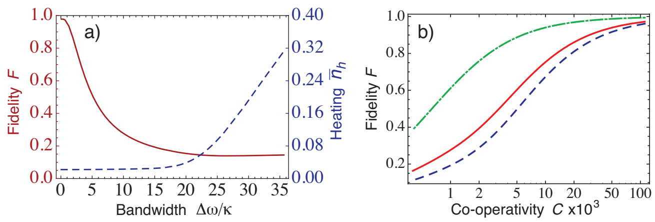

FIG. 2: Fidelity for transferring itinerant photon states at $T = 2$ K. (a) Fidelity (red, solid) and heating (blue, dashed) versus input bandwidth for transferring $|\alpha = \sqrt{3}\rangle$ coherent state. (b) Fidelity versus cooperativity $C = G^2/\kappa\gamma$ in the narrow-bandwidth limit. The input states are $|\alpha = \sqrt{3}\rangle$ (green, dash-dot), $|n = 3\rangle$ Fock state (blue, dashed), and $(|1\rangle + |3\rangle)/\sqrt{2}$ (red, solid). Unless specified here, the system parameters are the same as the dashed line in Fig. 1b.

suming $C_1 = C_2 \gg 1$, we find to leading order in $\Delta\omega$:

$$\bar{n}_{\mathrm{h}} \approx \frac{N_M}{4C}\left(1+\left(\frac{\Delta\omega}{G}\right)^2\left(1-\frac{\kappa^2}{16G^2}\right)\right) \tag{9}$$

$$\lambda \approx |\alpha|\left[\frac{1}{8C}+\left(\frac{2\Delta\omega}{\kappa}\right)^2\left(1+\left(\frac{\kappa^2}{8G^2}\right)^2\right)\right] \tag{10}$$

Good fidelity requires a high cooperativity $C \gg |\alpha|, N_M$. In the weak-coupling regime $G < \kappa$, one also needs $\sqrt{|\alpha|}\Delta\omega \ll (G^2/\kappa)$, which reflects the width of the $s_{21}[\omega]$ transmission resonance. In the opposite regime $G \gg \kappa$, one needs $\Delta\omega \leq \kappa/\sqrt{|\alpha|}$ as shown in Fig. 2a. Further, we see that in comparison against the double-swap scheme, the mechanical-heating effect described by $\bar{n}_{\mathrm{h}}$ is reduced by a factor κ/G. The expression of $\bar{n}_{\mathrm{h}}$ is the usual *weak-coupling* expression for the mechanical temperature cavity cooling [16, 17]; unlike cavity-cooling, it describes $\bar{n}_{\mathrm{h}}$ in both weak and strong coupling regimes.

Transfer of non-classical itinerant states– Given the advantages of the itinerant transfer scheme, it is also interesting to consider how well it is able to transfer non-classical states. While in general it is difficult to obtain analytic expressions for the evolution of non-Gaussian states, we show that here, one can obtain useful and reliable analytic approximations.

We again consider an input mode in a given temporal mode $\hat{u}_1$; we take this mode to be centered on ω_M, and for simplicity, to have a vanishingly small bandwidth $\Delta\omega$. Suppose now the input state incident on cavity 1 is a Fock state of this mode $|n\rangle \propto \left(\hat{u}_1^\dagger\right)^n|0\rangle$. We also take the noise driving both cavities to be zero-temperature ($N_1 = N_2 = 0$), but allow the mechanical resonator to be driven by thermal noise. Letting $p_{\mathrm{th}}(q, N_m)$ be the probability of having q thermal quanta incident on the mechanical resonator, the fidelity can be decomposed as

$$F = \sum_{r=0}^{\infty} P(r,n) = \sum_{r=0}^{\infty}\sum_{q=0}^{\infty} p_{\mathrm{th}}(q,N_m)\left|f_q^{(r,n)}\right|^2 \tag{11}$$

where $P(r, n)$ is the probability of having r outgoing photons leaving cavity 1 and n photons leaving cavity 2, and the expression of $f_q^{(r,n)}$ can be found in Supplemental Materials.

Note that in the regime of optimal state transfer $C_1 = C_2 \equiv C \gg 1$, the probability of having photons leave cavity 1 is small: the dark state effectively prevents mechanical photons from contributing, and Eq. (8) ensures minimal reflection of signal photons. One can thus get a good approximation by simply retaining the $r = 0$ and $r = 1$ term in Eq. (11): F is approximately the probability of obtaining n photons in the cavity 2 output mode and at most one photon leaving cavity 1. This is a rigorous lower bound on the exact fidelity, and is exact to order $1/C$.

In the limit $C \gg 1$, one finds that to leading order in $1/C$ the fidelity for transferring the n-photon itinerant Fock state is $F \simeq 1 - [N_M(3+2n)+n]/4C$. For $N_M \gg 1$, the condition for a near-unity fidelity is thus $C \gg N_M n$; for a large-n Fock state, this is more stringent than the condition for having a large fidelity transfer of a coherent state with $|\alpha| \sim \sqrt{n}$ (c.f. Eqs. (9),(10)).

Finally, we note that the same approach can be used to compute the fidelity of transferring an arbitrary pure input state of the form $|\Psi_1\rangle = \sum_m c_m |m\rangle$; the full expression is provided [30]. The transfer fidelity of different non-Gaussian states together with a coherent state (for realistic parameters) are shown in Fig. 2b.

Conclusions– In this paper, we have proposed using a mechanically dark delocalized mode in a two-cavity optomechanical system for quantum state transfer. We have demonstrated that both intra-cavity states and itinerant photon states can be transferred with high fidelity, using parameters within reach of current experiments.

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[28] An expression for the fidelity of Gaussian state transfer was given in Ref. [27]; however, this work neglected the effects of cavity and mechanical damping, and hence their expression lacks the exponential factor in our Eq. (2).

[29] The full expression for F for an arbitrary Gaussian state is rather unwieldy, and will be presented elsewhere [23].

[30] See Supplemental Material for details of the adiabatic pulse shape, the scattering matrix,and the transfer fidelity of superposition Fock states.