Parametric Amplification of Laser-Driven Electron Acceleration in Underdense Plasma

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Parametric amplification of laser-driven electron acceleration in underdense plasma

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A new mechanism is reported that increases electron energy gain from a laser beam of ultra-relativistic intensity in underdense plasma. The increase occurs when the laser produces an ion channel that confines accelerated electrons. The frequency of electron oscillations across the channel is strongly modulated by the laser beam, which causes parametric amplification of the oscillations and enhances the electron energy gain. This mechanism has a threshold determined by a product of beam intensity and ion density.

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Low-density plasmas have a notable ability to generate relativistic energetic electrons when irradiated by a high-intensity laser beam. This feature has been demonstrated experimentally and it can be utilized or be beneficial in a range of applications, including x-ray production [1], positron generation [2, 3], and ion acceleration [4]. The underlying electron acceleration mechanism depends significantly on laser pulse duration. For example, a laser-produced wakefield plays the dominant role when the beam is relatively short, with the duration shorter or comparable to the plasma wave period [5]. However, the wakefield mechanism becomes less efficient for longer laser pulses [6, 7]. Our interest in such pulses is motivated by experiments with solid-density targets on fast proton generation with 1-5 ps pulse duration [8]. A prepulse in these experiments can create a transparent preplasma, extending many wavelengths from the target surface along the beam path. The main pulse then interacts with a low-density plasma before reaching the target. It is important to understand if such interactions can generate hot electrons in addition to the ones produced at the critical surface [9].

In the case of a long laser beam, its ponderomotive pressure tends to expel plasma electrons from the beam in the transverse direction. The plasma mitigates this expulsion by generating a counteracting electric field via charge separation. As a result, the laser can create a positively charged channel with a quasi-static transverse electric field evolving on an ion time scale. An electron accelerated by the laser can become confined in this channel, oscillating across the channel while moving along with the laser beam. Considerable acceleration of electrons has been observed experimentally in such channels [1, 2, 7, 10–13]. Although the plasma in these experiments was significantly underdense, the measured electron energies were much greater than the energy expected in a vacuum for a laser beam of the same intensity.

Direct Laser Acceleration or DLA is the common term for this phenomenon. The DLA effect has been demonstrated in particle-in-cell simulations, with Ref. [14] being one of the first to report it. The prevailing explanation of the DLA mechanism is that "An energetic electron experiences transverse betatron oscillations in the static fields. These oscillations are along the polarization of the laser pulse electric field, and thus an efficient energy exchange is possible when the betatron frequency is close to the laser frequency, as witnessed by the relativistic electron" [14]. This explanation implies that the laser field acts as a driving force for betatron oscillations, which requires proper polarization of the laser beam. However, a strong laser field can also affect betatron oscillations via nonlinear modulations of their frequency, enabling parametric amplification of the oscillations. The parametric amplification is relevant to the problem and it is qualitatively different from the effect of the driving force. Nevertheless, it has so far been overlooked, and the goal of this letter is to fill the existing gap and describe the role of this mechanism.

We examine electron oscillations across the channel for an arbitrary polarization of the laser beam. We show that the oscillations become unstable regardless of the beam polarization and that the resulting amplification of the oscillations enhances electron acceleration in the direction of the laser beam. We find that the instability threshold at ultra-relativistic intensities is determined by the product of beam intensity and ion density. Consequently, the plasma density threshold decreases with laser intensity and the instability can develop even in a very underdense plasma.

In order to illustrate the mechanism responsible for the enhancement of electron acceleration, we consider the behavior of a single electron placed in a straight uniform ion channel and irradiated by a plane electro-magnetic wave. The ions are assumed to be immobile. We choose a two-dimensional spatial setup $(y; z)$, with the y-axis directed across the channel and the z-axis directed along the channel axis. The plane wave propagates in the positive direction along the z-axis. The electric and magnetic fields ($\mathbf{E}$ and $\mathbf{B}$) acting on the electron are thus given and the problem reduces to the analysis of the electron equations of motion,

$$\frac{d\mathbf{r}}{dt} = \frac{c}{\gamma}\mathbf{P}, \tag{1}$$

$$\frac{d\mathbf{P}}{dt} = -\frac{|e|\mathbf{E}}{m_e c} - \frac{|e|}{\gamma m_e c}\mathbf{P}\times\mathbf{B}, \tag{2}$$

where $\mathbf{r}$ is the electron position, t is the time in the laboratory (ion) frame of reference, e and m_e are the electron charge and mass, c is the speed of light, $\mathbf{P}$ is the dimensionless electron momentum normalized to $m_e c$, and $\gamma = \sqrt{1+P^2}$ is the relativistic factor.

The electric field, $\mathbf{E} = \mathbf{E}_{ion} + \mathbf{E}_{wave}$, is a sum of a static field of the ion space-charge, $\mathbf{E}_{ion}$, and an oscillating field of the wave, $\mathbf{E}_{wave}$. The static component in the uniform channel is $\mathbf{E}_{ion} = \mathbf{e}_y \omega_{p0}^2 m_e y/|e|$, where $\mathbf{e}_y$ is a unit vector, $\omega_{p0} \equiv \sqrt{4\pi n_0 e^2/m_e}$ is the plasma frequency, and n_0 is the density of the singly charged ions in the channel. The magnetic field, $\mathbf{B} = \mathbf{B}_{wave}$, is only due to the wave. It is convenient to express the wave field in terms of a dimensionless vector potential $\mathbf{a}$:

$$\mathbf{E}_{wave} = -\frac{m_e c}{|e|}\frac{\partial \mathbf{a}}{\partial t}, \quad \mathbf{B}_{wave} = \frac{m_e c^2}{|e|}\nabla\times\mathbf{a}. \tag{3}$$

The vector potential $\mathbf{a}$ depends on t and z only via a phase variable

$$\xi \equiv (ct - z)/\lambda, \tag{4}$$

where λ is the laser wavelength. We choose $\mathbf{a} = a(\xi)\left[\mathbf{e}_x \cos\theta + \mathbf{e}_y \sin\theta\right]$, where $\mathbf{e}_x$ and $\mathbf{e}_y$ are unit vectors, θ is a polarization angle, and $a(\xi)$ is a sinusoidal function with a slowly varying envelope. To be specific, we use the following expression for $a(\xi)$ in our subsequent numerical calculations: $a(\xi) = 0$ for $\xi \le 0$, $a(\xi) = 0.5\left[1 - \cos(\pi\lambda\xi/cT)\right] a_0 \sin(2\pi\xi + \psi)$ for $0 < \xi < cT/\lambda$, and $a(\xi) = a_0 \sin(2\pi\xi + \psi)$ for $\xi \ge cT/\lambda$, where a_0 is the maximum amplitude, ψ is an initial phase, and $T \gg \lambda/c$ is the pulse rise time.

The equations of motion simplify when dimensionless proper time τ (proper time normalized to λ/c) is used instead of t. The relation between the two is

$$dt/d\tau = \gamma\lambda/c. \tag{5}$$

We also introduce the dimensionless displacement u across the ion channel and a dimensionless parameter K, which is a ratio of the wavelength to the electron skin-depth:

$$u \equiv \omega_{p0} y/c, \quad K \equiv \omega_{p0}\lambda/c. \tag{6}$$

Note that $K \ll 2\pi$ in a significantly underdense plasma, which is the case of our primary interest here. Substituting the expressions for $\mathbf{E}$ and $\mathbf{B}$ into Eq. (2), we obtain the following closed set of equations:

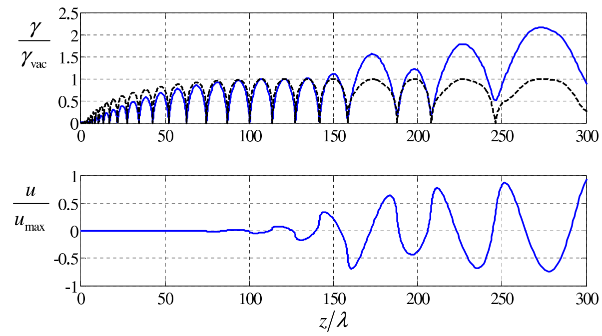

FIG. 1: Parametrically unstable oscillations in the s-polarized wave for $\kappa = 12$, with $a_0 = 10$, $K = 1.2$ and $T = 10\lambda/c$. The initial conditions are $u(0) = 0.01$, $P_y(0) = 0$, and $P_z(0) = 0$. The dashed (lower) curve in the upper panel shows a/a_0 as a function of the axial distance traveled by the electron.

$$\frac{d}{d\tau}\left[P_y - a\sin\theta\right] = -\gamma K u, \tag{7}$$

$$\frac{dP_z}{d\tau} = \left[a\cos^2\theta + P_y\sin\theta\right]\frac{da}{d\xi}, \tag{8}$$

$$\frac{d\xi}{d\tau} = \gamma - P_z, \tag{9}$$

$$\frac{du}{d\tau} = K P_y, \tag{10}$$

where $\gamma \equiv \sqrt{1 + P_y^2 + P_z^2 + a^2\cos^2\theta}$. Equation (9) is a derivative of Eq. (4) with respect to τ, with Eq. (5) taken into account. It follows from Eqs. (7) - (10) that there is an integral of motion for the electron:

$$\gamma - P_z + u^2/2 = C, \tag{11}$$

where C is a constant determined by initial conditions. This integral of motion sets an upper limit for the amplitude of electron oscillations across the ion channel, which is $u_{\max} = \sqrt{2 + u(0)^2}$ for an electron that is initially at rest ($C = 1$). The amplitude approaches $u_{\max}$ at very large values of P_z when $\gamma - P_z \to 0$.

Equations (7) - (10) reproduce a well-known result that, in the absence of ions, the maximum γ-factor for an electron that is initially at rest is

$$\gamma_{\rm vac} = 1 + a_0^2/2. \tag{12}$$

The electron moves axially slower than the wave and is therefore subject to dephasing, which limits electron energy gain from the wave. In the absence of ions, the dimensionless dephasing rate, $\gamma - P_z$, is constant and equal to unity. The ion channel can reduce this dephasing rate significantly by means of electron oscillations in the static field across the channel. Indeed, consider an electron that is initially at rest, but slightly displaced from the axis of the channel. The constant C in Eq. (11)

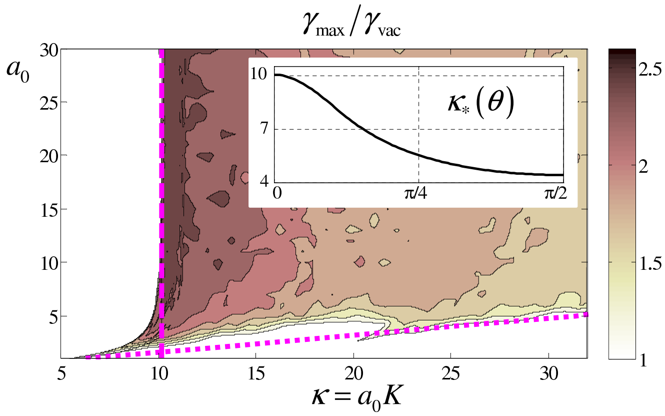

FIG. 2: Maximum γ-factor for $\theta = 0$ averaged over the initial wave phase. The vertical dashed line is the instability threshold $\kappa = \kappa_*(\theta = 0)$. The dotted line, $\kappa = 2\pi a_0$, is the boundary where the wave frequency is equal to ω_{p0}. The initial conditions are $u(0) = 0.01$, $P_y(0) = 0$, and $P_z(0) = 0$. The inset shows the threshold of the parametric instability κ_* as a function of the polarization angle θ.

is then very close to unity, so that the dephasing rate is $\gamma - P_z \approx 1 - u^2/2$. If the amplitude of the oscillations across the channel grows and approaches $u_{\max} \approx \sqrt{2}$, the dephasing rate becomes very small. This should allow the electron to stay in phase with the wave for a longer time and thereby gain more energy, with the resulting γ-factor exceeding γ_{vac}. The objective then is to find an amplification mechanism for the oscillations, as that would enable generation of electrons with $\gamma > \gamma_{\text{vac}}$.

In order to describe the electron motion across the channel, we combine Eqs. (7), (9), and (10) and use Eq. (11) in place of Eq. (8), which yields

$$u'' - \frac{uu'u'}{2\,(C - u^2/2)} + \frac{K^2 u}{2(C - u^2/2)^3} + \frac{K^2 u}{2\,(C - u^2/2)} + \frac{K^2 a^2 \cos^2\theta u}{2(C - u^2/2)^3} = \frac{\sin\theta}{C - u^2/2} K a', \tag{13}$$

where the prime denotes $d/d\xi$. This equation generalizes Eq. (11) of Ref. [14] to the case of arbitrary polarization. The two equations are identical when $\theta = \pi/2$, except for notations. We use phase ξ rather than time t as an independent variable, which helps to distinguish between resonant excitation of oscillations by an external force (betatron resonance) and parametric amplification of oscillations.

For small displacements, Eq. (13) reduces to a linear equation for a driven oscillator with a modulated eigenfrequency, $u'' + K^2(1 + \cos^2\theta a^2/2)u = \sin\theta K a'$. In the case of a non-relativistic wave ($a_0 \ll 1$), the oscillations across the channel are stable in an underdense plasma ($K \ll 1$). The reason is that the eigenfrequency, which is close to K for $a_0 \ll 1$, is significantly lower than the frequency of the modulations and the frequency of the driving force. However, the conditions for instability become much more favorable at ultra-relativistic intensities.

In the limit of low densities ($K \ll 1$) and ultra-relativistic intensities ($a_0 \gg 1$), the K^2-terms that do not involve a^2 can be neglected in Eq. (13). We also set $a(\xi) = a_0 \sin(2\pi\xi + \psi)$, implying that $\xi > cT/\lambda$, which simplifies Eq. (13) to an equation,

$$u'' - \frac{uu'u'}{2\,(C - u^2/2)} + \frac{\kappa^2 \cos^2\theta \sin^2(2\pi\xi + \psi)}{2(C - u^2/2)^3} u = \frac{2\pi\kappa \sin\theta}{C - u^2/2} \cos(2\pi\xi + \psi), \tag{14}$$

that depends only on a single parameter

$$\kappa \equiv a_0 K = \omega_{p0} a_0 \lambda / c. \tag{15}$$

In what follows, we set $C = 1$ in Eq. (14), which implies that $P_z(0) = 0$. The results for $C = 1$ can be rescaled to the case of an arbitrary C by replacing u with $C^{-1/2}u$ and κ with $C^{-3/2}\kappa$. This rescaling generalizes the results to the case of an arbitrary initial P_z via Eq. (11).

The case of small polarization angles offers further simplifications, because the driving term on the right hand side of Eq. (14) is small. It is instructive to consider a linearized version of Eq. (14),

$$u'' + \frac{1}{2}\kappa^2 \sin^2(2\pi\xi + \psi) u = 2\pi\theta\kappa \cos(2\pi\xi + \psi). \tag{16}$$

In contrast to the non-relativistic case, the eigenfrequency is now strongly modulated and, as a result, its maximum value increases with a_0 for a given ion density (fixed K). Solutions of Eq. (16) are stable for $\kappa \ll 1$, but they become unstable when κ is increased. The instability is caused by eigenfrequency modulations because it develops even at $\theta = 0$ when there is no driving force. We shall therefore use the term parametric to refer to this instability, with the changing parameter being the eigenfrequency. At $\theta = 0$, Eq. (16) becomes a Mathieu differential equation that has exponentially growing solutions when κ exceeds a certain threshold value κ_*. We find that $\kappa_* \approx 10.2$. The nonlinearity retained in Eq. (14) plays a destabilizing role and it formally allows the amplitude to reach $u_{\max}$, causing unlimited energy gain. This observation suggests that the energy gain in low-density plasma depends on K as well as on κ and that it needs to be determined from Eqs. (7) - (10).

The solution of Eqs. (7) - (10) for $\kappa = 12$ is shown in Fig. 1. The instability that develops in this case ($\kappa > \kappa_*$) causes the oscillations to grow significantly. The increase in their amplitude leads to increased axial acceleration, such that the maximum γ-factor ($\gamma_{\max}$) is greater than the maximum value achieved in the absence of ions [γ_{vac} of Eq. (12)]. Figure 2 shows $\gamma_{\max}$ as a function of a_0 and κ. At $a_0 > 10$, the stability boundary is indeed determined only by κ since the threshold coincides with the

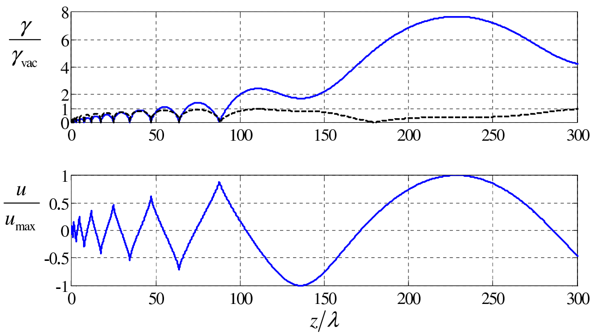

FIG. 3: Parametrically unstable oscillations in the p-polarized wave for $\kappa = 5$, with $a_0 = 10$, $K = 0.5$ and $T = 10\lambda/c$. The initial conditions are $u(0) = 0.0$, $P_y(0) = 0$, and $P_z(0) = 0$. The dashed (lower) curve in the upper panel shows a/a_0 as a function of the axial distance traveled by the electron.

dashed vertical line $\kappa = \kappa_*$. In Figure 2, $\gamma_{\max}/\gamma_{\rm vac}$ is averaged over the initial phase ψ. The value of $\gamma_{\max}/\gamma_{\rm vac}$ for specific ψ and T can deviate from the average value by order unity above the instability threshold, although $\gamma_{\max}/\gamma_{\rm vac}$ does not exhibit any sharp dependence on either ψ or T.

It is essential that electron oscillations across the ion channel become unstable for all polarization angles rather than just for $\theta \ll 1$. The θ-dependence of the instability threshold $\kappa_*(\theta)$ is shown in the inset of Fig. 2. The amplitude of the oscillations is small below the threshold only for $\theta \ll 1$. In general, this is not the case and the oscillations are nonlinear even below the threshold. In order to identify the cause of the instability in this regime, we linearize Eq. (14) for small deviations δu from the stable solution just below the threshold. The eigenfrequency is again strongly modulated. We deliberately neglect the force term in the linearized equation and we find that a slight variation of coefficients in the linearized equation (the coefficients are determined by the stable unperturbed solution) leads to exponential growth of δu. Such behavior is a signature of a parametric instability and it clearly distinguishes the parametric instability caused by modulations of the eigenfrequency from a resonance with the driving force. Figure 3 shows the unstable oscillations and the resulting maximum value of γ for $\theta = \pi/2$ and $\kappa = 5 > \kappa_*(\pi/2)$. The maximum value of γ is almost 8 times greater than $\gamma_{\rm vac}$ in this strongly nonlinear regime.

In summary, we have shown how parametric amplification of electron oscillations in the ion channel can enhance electron energy gain from a laser beam of ultra-relativistic intensity. The effect results from instability caused by a strong modulation of the electron oscillation frequency by the laser field. The origin of the modulation is the oscillations of the relativistic γ-factor caused by ultra-relativistic electron motion in the electromagnetic field of the wave. The instability develops when the laser intensity I exceeds a threshold value

$$I\left[\mathrm{W/cm^2}\right]\lambda^2[\mu\mathrm{m}] = 1.37\times10^{18}\left(\kappa_*/2\pi\right)^2 n_c/n_0, \quad (17)$$

where n_c is the critical density, n_0 is the ion density, and κ_* is the threshold parameter determined by the laser polarization (see the inset of Fig. 2). Our single particle model can also be applicable if the channel contains an underdense electron population in addition to the accelerated electron beam. Our analysis is not very restrictive with regard to the beam density because mutual repulsion of ultra-relativistic electrons can be much weaker than the focusing force from the ions even at substantial electron densities [15]. The effect of the electrons on the wave is negligible at sufficiently low electron densities. A self-consistent nonlinear calculation of the electron response to the laser field is needed to quantify the upper limit on electron density, since the electron motion in the channel is ultrarelativistic.

The presented mechanism could be employed in laser-target experiments to generate hot electrons, provided that appropriate applicability conditions are satisfied. The thickness of the preplasma layer needs to be comparable to the interaction length required for the enhanced acceleration, $l \approx \gamma_{\max}\lambda$. The width of the laser beam must exceed $\sqrt{a_0}c/\omega_{p0}$ to ensure that ultra-relativistic electrons with $\gamma \geq a_0$ are not expelled by the transverse gradient of the ponderomotive pressure. This condition also guarantees that the electron remains within the beam during its transverse oscillations. It is also plausible that this mechanism is involved in formation of energetic electron tails observed in Ref. [2, 7, 10, 12], because the laser intensity in the experiments was in the range of the threshold intensities defined by Eq. (17). A realistic modeling of the experiment is required for a more conclusive assessment. Finally, the amplification of electron oscillations via the presented mechanism may also be beneficial in those applications that employ transverse electron oscillations in a confining quasi-static field to generate x- and gamma-rays [12, 16], provided that the mechanism is robust with respect to spectral broadening of the laser beam [17].

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[1] S. Kneip *et al.*, SPIE Proc. **7359**, 73590T (2009).
[2] C. Gahn *et al.*, Phys. Plasmas **9**, 987 (2002).
[3] H. Chen *et al.*, Phys. Rev. Lett. **105**, 015003 (2010).
[4] M. Roth *et al.*, Phys. Rev. Lett. **86**, 436 (2001).
[5] E. Esarey *et al.*, Rev. Mod. Phys. **81**, 1229 (2009).
[6] V. Malka *et al.*, Phys. Plasmas **8**, 2605 (2001).
[7] S.P.D. Mangles *et al.*, Phys. Rev. Lett. **94**, 245001 (2005).
[8] K. Flippo *et al.*, Journal of Physics: Conference Series **244**, 022033 (2010); J. Fuchs *et al.*, Nat. Phys. **2** 48 (2006).
[9] S. C. Wilks *et al.*, Phys. Rev. Lett. **69**, 1383 (1992).
[10] C. Gahn *et al.*, Phys. Rev. Lett. **83**, 4772 (1999).
[11] Y. Kitagawa *et al.*, Phys. Rev. Lett. **92**, 205002 (2004).
[12] S. Kneip *et al.*, Phys. Rev. Lett. **100**, 105006 (2008).
[13] B. R. Walton *et al.*, Phys. Plasmas **13**, 113103 (2006).
[14] A. Pukhov *et al.*, Phys. Plasmas **6**, 2847 (1999).
[15] G.I. Budker, Atomic Energy **1**, 673 (1956).
[16] S. Cipiccia *et al.*, Nature Phys. **7**, 867 (2011).
[17] S.Y. Kalmykov *et al.*, Phys. Plasmas **18**, 056704 (2011).