Bridging the Gap by Squeezing Superfluid Matter

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Bridging the gap by squeezing superfluid matter

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Cooper pairing between fermions in dense matter leads to the formation of a gap in the fermionic excitation spectrum and typically exponentially suppresses transport properties. However, we show here that reactions involving conversion between different fermion species, such as Urca reactions in nuclear matter, become strongly enhanced and approach their ungapped level when the matter undergoes density oscillations of sufficiently large amplitude. We study both the neutrino emissivity and the bulk viscosity due to direct Urca processes in hadronic, hyperonic and quark matter and discuss different superfluid and superconducting pairing patterns.

Ultra-dense matter, such as might be found in compact stars, has certain transport properties, such as bulk viscosity and neutrino emissivity, that are dominated by flavor-changing beta (weak interaction) processes. However, the beta equilibration rate is exponentially suppressed when the matter cools below the critical temperature T_c for superconductivity/superfluidity, because Cooper pairing produces a gap in the fermion excitation spectrum [1, 2]. This suppression is relevant both in hadronic matter, where proton superconductivity and neutron superfluidity have $T_c = 10^8 - 10^{10}\,\mathrm{K}$ [3], and in exotic phases such as quark matter, where critical temperatures are as high as $T_c \simeq 3-5\times 10^{11}\,\mathrm{K}$ if flavor antisymmetric pairing channels are favored, and as low as $T_c \simeq 10^7\,\mathrm{K}$ for singlet pairing [4].

In this letter we show that compression oscillations can overcome the exponential suppression of flavor-changing processes if their amplitude is sufficiently high (but within the range for realistic oscillations in neutron stars). It is already known that beta-equilibration rates in unpaired matter are enhanced by "suprathermal" contributions at high amplitude. These arise from higher order terms in the chemical potential imbalance [5–7]. Here we report on a similar but more dramatic phenomenon in superfluid/superconducting phases of matter, arising from a threshold-like behavior with a rapid increase in available phase space when the typical energy in the equilibration processes approaches the gap.

The mechanism we discuss is generic and can be expected to operate in all situations where large perturbations drive the system out of beta-equilibrium. One possible application is to unstable oscillations of rotating compact stars, like f- or r-modes [8]. The amplitude of these modes grows exponentially due to gravitational wave emission until they are saturated by non-linear coupling to damped daughter modes [9] or damped by the suprathermal bulk viscosity [10]. Note, that even at amplitudes well below the level where bulk viscosity alone would saturate such modes, large amplitude effects can noticeably affect the thermal evolution of the star, see also [11], via viscous heating and enhanced neutrino emissivity [6]. Another class of scenarios stems from oscillations caused by singular events, like star quakes [12] or tidal forces preceding neutron star mergers [13]. Such events will inevitably cause large amplitude compressions locally and due to the enormous energies involved they may even trigger sizable global modes, which are only slowly damped by shear viscosity. The suprathermal bulk viscosity should then control the amplitude of the oscillation and correspondingly the emitted radiation.

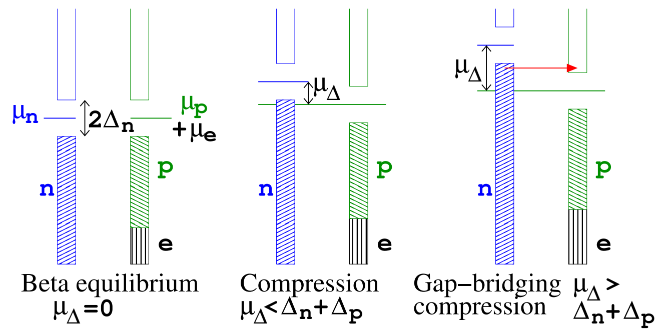

Figure 1: Schematic illustration of the opening of phase space for direct Urca interactions in gapped nuclear matter at $T=0$ when the deviation from beta equilibrium $\mu_\Delta = \mu_n - \mu_p - \mu_e$ is sufficiently large relative to the pairing gaps, $\mu_\Delta > \Delta_n + \Delta_p$.

Here we will study the effect of large amplitude oscillations for various *gapped* phases of dense matter. When, as in nuclear matter, Cooper pairs contain two particles of the same flavor, density oscillations cannot lead to pair breaking. Nevertheless, oscillations displace the gapped Fermi seas, and at sufficiently large amplitude can *bridge the gap*, opening phase space for beta equilibration processes. This is illustrated for the direct Urca process in nuclear matter in fig. 1, but we emphasize that gap-bridging is relevant to any reaction where nucleons or quarks change flavor. The mechanism is similar to electron tunneling between different superconductors separated by a large barrier when an electric potential is imposed [14]. As we will show, the effect begins at amplitudes far below the level where non-linear hydrodynamic effects, of higher order in the fluid velocity, would arise [5, 7, 15] and causes a huge increase in the corresponding rates, effectively bringing them up to the ungapped level.

Beta equilibration processes have a finite time scale, so a harmonic local density fluctuation with amplitude Δn around the equilibrium value $\bar{n}$ induces an oscillating displacement μ_Δ of the chemical potentials of the degenerate particles from their equilibrium. This oscillation leads to both heat generation (via dissipation due to bulk viscosity) and heat loss (from enhanced neutrino emission). Which of the two effects dominates depends among other factors on the amplitude of the oscillation [6]. In [7, 15] we found that over the entire range of physically reasonable density amplitudes the chemical potential oscillation is given by the linear, harmonic relation

$$\mu_\Delta(t) = \hat{\mu}_\Delta \sin(\omega t)\,, \quad \hat{\mu}_\Delta = C\frac{\Delta n}{\bar{n}}, \quad C \equiv \bar{n}\frac{\partial \mu_\Delta}{\partial n}\bigg|_x \tag{1}$$

where C characterizes the particular form of matter.

Gap-bridging is a generic phenomenon based on straightforward energetics, so, to illustrate the large enhancements that it produces, we will perform calculations for the simple case of direct Urca processes $d \to u+l+\bar{\nu}_l$ and $u+l \to d+\nu_l$. In reality, direct Urca occurs only in very dense or exotic forms of matter; elsewhere, modified Urca processes occur instead. We defer the more complex modified-Urca calculation to future work. However, the energetics that give rise to gap-bridging are the same in both cases, so it seems inevitable that gap-bridging effects will be as dramatic for modified Urca processes (see, for example, [1, 6]). To study all relevant cases within a unified framework we use a notation where d represents a down or strange quark or a hadron that contains it, u represents an up quark or a hadron that contains it; l represents a charged lepton (an electron, or in dense neutron matter a muon) and ν_l is the corresponding neutrino. We specialize to Fermi liquid theory, where the general dispersion relation of the matter fields is

$$(E_i-\mu_i)^2 = v_{Fi}^2\,(p_i-p_{Fi})^2 + \Delta_i^2 \tag{2}$$

where i labels the species and Δ_i represents a gap in the particle spectrum arising from superfluidity or (color)-superconductivity. Non-Fermi liquid effects should not play a role in a gapped system [16]. We neglect dependence of the gap on energy and momentum, but we include temperature and density dependence $\Delta_i = \Delta_i\,(T,\mu)$. The leptons can to a very good approximation be described by a free dispersion relation $E_l^2 = p_l^2 + m_l^2$, $E_\nu = p_\nu$. Urca processes enforce beta equilibrium, $\mu_\Delta \equiv \mu_d - \mu_u - \mu_l = 0$. Since in compact stars $T \ll \mu$ we can restrict the analysis to leading order in T/μ where we find the general results for the net rate and the emissivity

$$\Gamma_{dU}^{(\leftrightarrow)} \approx \frac{17G^2}{120\pi} D\Theta T^4 \mu_\Delta R_\Gamma\!\left(\frac{\mu_\Delta}{T},\frac{\Delta_d}{T},\frac{\Delta_u}{T}\right) \tag{3}$$

$$\epsilon_{dU} \approx \frac{457\pi G^2}{5040} D\Theta T^6 R_\varepsilon\!\left(\frac{\mu_\Delta}{T},\frac{\Delta_d}{T},\frac{\Delta_u}{T}\right) \tag{4}$$

In these expressions G is the effective coupling of the fields to the weak current, D is a function depending on the density and the equation of state

$$D = \frac{p_{Fd}p_{Fu}}{\mu_u v_{Fd} v_{Fu}}\left(\mu_d^2 - p_{Fd}^2 - \mu_u^2 + p_{Fu}^2 - m_l^2\right) \tag{5}$$

in which the parenthesis vanishes for a free massless dispersion relation. Any modification thereof due to interactions or masses opens phase space and yields a non-zero result. Eqs. (3) and (4) also contain a threshold function $\Theta \approx \theta(p_{Fu}+p_{Fl}-p_{Fd})$, a characteristic temperature and amplitude dependence, and a *modification function* R given for singlet gaps by the dimensionless integrals

$$\begin{aligned} R_i\!\left(\frac{\mu_\Delta}{T},\frac{\Delta_d}{T},\frac{\Delta_u}{T}\right) &= N_i\int_0^\infty dx_\nu x_\nu^{2+\lambda_i} \\ \cdot\left(\int_{-\infty}^{-\frac{\Delta_d}{T}} + \int_{\frac{\Delta_d}{T}}^{\infty}\right) &\frac{dx_d\,|x_d|}{\sqrt{x_d^2-\frac{\Delta_d^2}{T^2}}} \left(\int_{-\infty}^{-\frac{\Delta_u}{T}} + \int_{\frac{\Delta_u}{T}}^{\infty}\right) \frac{dx_u\,|x_u|}{\sqrt{x_u^2-\frac{\Delta_u^2}{T^2}}} \\ \cdot\,\tilde{n}(x_d)\,\tilde{n}(-x_u)&\left(\tilde{n}(X_-) - (-1)^{\lambda_i}\,\tilde{n}(X_+)\right) \end{aligned} \tag{6}$$

where $N_\Gamma = \frac{60}{17\pi^4}$, $N_\epsilon = \frac{2520}{457\pi^6}$, $\lambda_\Gamma = 0$ and $\lambda_\epsilon = 1$, $X_\pm \equiv x_\nu + x_u - x_d \pm \mu_\Delta/T$, and $\tilde{n}(x) \equiv 1/(\exp(x)+1)$. These functions reflect the phase space available to Urca reactions. They are normalized so that $R(0,0,0) = 1$ and are symmetric in the gap parameters $R(x,y,z) = R(x,z,y)$. In equilibrium they are pure reduction factors $R(0,\cdots) \le 1$ that can become extremely small [1]. However, for high-amplitude density oscillations ($\mu_\Delta > T$) the modification functions gain a *"suprathermal"* enhancement and can then greatly exceed one [5–7]. They depend on the equation of state only via dimensionless ratios and are the same for all direct Urca processes [1]. In ungapped matter they have an analytic polynomial form [6, 7]. The point of this paper is that they show an even more dramatic enhancement in matter with gapped fermions.

The oscillation period of a compact star is much smaller than its evolution time scale, so the relevant quantity is the averaged emissivity $\bar{\epsilon}_{dU} \equiv \frac{1}{2\pi}\int_0^{2\pi} d\varphi\,\epsilon_{dU}(\mu_\Delta(\varphi))$ which takes the same form eq. (4) with the averaged modification function $R_{\bar{\epsilon}}$ depending on $\hat{\mu}_\Delta$. The bulk viscosity of dense matter is [7]

$$\zeta = -\frac{C}{\pi\omega^2}\frac{\bar{n}_*}{\Delta n_*}\int_0^{2\pi} d\varphi\cos(\varphi)\int_0^{\varphi} d\varphi'\,\Gamma^{(\leftrightarrow)}(\mu_\Delta(\varphi')) \tag{7}$$

Inserting the rate eq. (3) and the chemical potential oscillation eq. (1) gives at low T valid for superfluid matter

$$\zeta_{dU} = \frac{17G^2}{120\pi} C^2 D\Theta \frac{T^4}{\omega^2} R_\zeta\!\left(\frac{\hat{\mu}_\Delta}{T},\frac{\Delta_d}{T},\frac{\Delta_u}{T}\right) \tag{8}$$

with an analogous modification function

$$R_\zeta\left(\frac{\hat{\mu}_\Delta}{T},\frac{\Delta_d}{T},\frac{\Delta_u}{T}\right)=-\frac{1}{\pi}\int_0^{2\pi}d\varphi\cos(\varphi)\int_0^{\varphi}d\varphi'\sin(\varphi')\cdot R_\Gamma\left(\frac{\hat{\mu}_\Delta}{T}\sin(\varphi')\,,\frac{\Delta_d}{T},\frac{\Delta_u}{T}\right)\qquad(9)$$

In hyperon and quark matter the viscosity is dominated by non-leptonic rather than Urca processes, but for those we also expect a high-amplitude enhancement [7].

To see the gap-bridging enhancement of beta-equilibration rates for oscillations of suprathermal amplitude, without specifying a particular form of matter, we show in fig. 2 the modification functions $R_{\bar{\epsilon}}$ and R_ζ. We vary the maximum gap Δ and the ratio of the two gaps. For strongly gapped particles $\Delta_i/T \gg 1$, the larger gap tends to dominate, e.g. $(\Delta_d,\Delta_u)=(2\Delta,0)$ is more suppressed than (Δ,Δ). Once the amplitude $\hat{\mu}_\Delta$ becomes comparable to Δ, the Urca reactions can bridge the gap(s) so that the modification functions rise steeply, and quickly reach their ungapped levels. Interestingly, for matter where the two gaps are the same (dotted lines) this rise starts already for $\hat{\mu}_\Delta \ll \Delta_i$ and eventually merges into the solid curve for a single gap $\Delta_d+\Delta_u$. In contrast, in the asymmetric case where one gap is much larger than the other, the effects of the smaller gap are only visible at large amplitudes and become negligible when it is more than an order of magnitude smaller. In the large amplitude limit $\hat{\mu}_\Delta \gg \Delta_i, T$ the modification functions approach $R_{\bar{\epsilon}} \approx \frac{105}{7312\pi^6}\frac{\hat{\mu}_\Delta^6}{T^6}$ and $R_\zeta \approx \frac{5}{136\pi^4}\frac{\hat{\mu}_\Delta^4}{T^4}$.

Fig. 2 shows that suprathermal enhancement in gapped matter is considerably bigger than in ungapped matter (the $\Delta/T=0$ line) [7]. This enhancement could be realistically achieved in compact star oscillations: $\hat{\mu}_\Delta$ is related to the density amplitude $\Delta n/\bar{n}$ by the factor C eq. (1). For hadronic matter with an APR equation of state [19], C increases from $C\,(n_0/4)\approx 20\,\mathrm{MeV}$ to $C\,(5n_0)\approx 150\,\mathrm{MeV}$ [2]; for quark matter it decreases from $C\,(n_0)\approx 30\,\mathrm{MeV}$ to $C\,(5n_0)\approx 20\,\mathrm{MeV}$. Thus for amplitudes $\Delta n/\bar{n}\sim 0.01$ the amplitude $\hat{\mu}_\Delta$ can indeed become large enough to bridge gaps $\Delta\sim 1\,\mathrm{MeV}$ [15].

To facilitate calculations of gap-bridging in various forms of matter, we show in tab. I the values of the relevant parameters for direct Urca processes in hadronic [2, 17, 20], hyperonic [21, 22] and quark matter [23, 24]. Inserted in the general results eqs. (4) and (8) they reproduce previous expressions in the subthermal case $\mu_\Delta \ll T$. In the ideal gas approximation our results simplify to

$$\bar{\epsilon}=\tilde{\epsilon}\left(\frac{\bar{n}}{n_0}\right)^{\nu_\epsilon}T_9^6R_{\bar{\epsilon}}\;,\;\zeta=\tilde{\zeta}\left(\frac{\bar{n}}{n_0}\right)^{\nu_\zeta}\left(\frac{1\mathrm{kHz}}{\omega}\right)^2T_9^4R_\zeta\qquad(10)$$

where T_9 is the temperature in units of $10^9\,\mathrm{K}$ and the values of $\tilde{\epsilon}$, $\tilde{\zeta}$, ν_ϵ and ν_ζ are given in tab. I.

Finally, to illustrate the likely magnitude and scope of gap-bridging effects in the neutron star context, we perform an illustrative calculation of the rate for direct Urca processes in a star, using a "toy model" of nuclear matter in which there is gapping of protons and neutrons, and direct Urca processes can occur at all densities. The model uses the APR equation of state [19] with the in-medium hadron masses given in [25]. For the density dependence of the $^1S_0$ proton gap we use a Gaussian fit to data from [3], with maximum $\Delta_{p0}=1\,\mathrm{MeV}$ centered at $n\approx 1.3\,n_0$. For illustrative purposes we assume the neutrons also have a $^1S_0$ gap (in reality it is expected to be $^3P_2$) with the density dependence proposed to explain the Cas A cooling data in [26], i.e. with maximum $\Delta_{n0}=0.12\,\mathrm{MeV}$ at $n\approx 3.7\,n_0$. The gaps have a BCS temperature dependence, but this has only a modest effect in small regions where $T\sim\Delta_p,\Delta_n$. Our results for neutrino emissivity and bulk viscosity at $T=10^8\,\mathrm{K}$ are shown in fig. 3. At infinitesimal amplitudes (thick $\Delta n/\bar{n}=0$ lines) there is enormous suppression by the proton gap, and moderate suppression by the smaller neutron gap. As the density oscillation amplitude increases, it first, at $\Delta n/\bar{n}\gtrsim 10^{-3}$, overcomes the suppression of emissivity and viscosity due to the neutron gap, increasing them at densities potentially relevant for direct Urca processes by many orders of magnitude. Then, at $\Delta n/\bar{n}\gtrsim 10^{-2}$, even the large proton gap at lower density is bridged, boosting the results by up to 10^{50}! Our calculation is for the direct Urca process, but the enhancement arises from pure energetics, so we expect that for the case of modified Urca reactions it will also provide enhancement that is capable of canceling the suppression due to Cooper pairing.

Fig. 3 shows that, if the star is cold enough that Urca processes are blocked by pairing gaps throughout the star, density oscillations could provide the dominant contribution to those processes, by bridging the pairing gaps, starting in the regions where the rate-controlling gap is the smallest. The smaller this minimum gap, the lower the amplitude required to bridge it. In hyperonic [21, 22] or quark matter [27, 28], as well as for a smaller neutron gap [29], there are processes which are only suppressed by $\Delta\lesssim 0.01$ MeV, that could be bridged entirely by oscillations with amplitude as small as $\Delta n/\bar{n}\lesssim 10^{-4}$.

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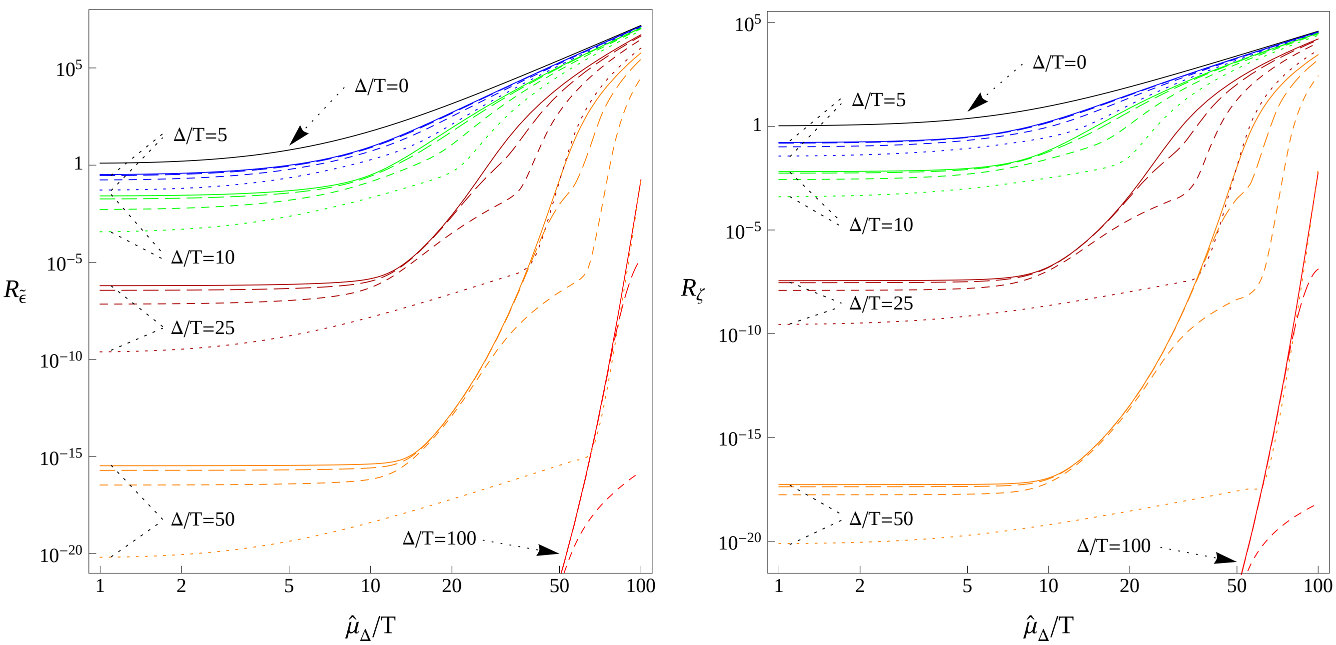

Figure 2: The modification functions R_i eqs. (4) & (8) as a function of oscillation amplitude $\hat{\mu}_\Delta$, for different pairing patterns with maximum gap Δ: symmetric case where only one particle is gapped, e.g. $\Delta_d = \Delta$, $\Delta_u = 0$ (solid lines); intermediate cases $\Delta_d = 5\Delta_u = \Delta$ (long dashed lines); $\Delta_d = 2\Delta_u = \Delta$ (short dashed lines); symmetric case $\Delta_d = \Delta_u = \Delta$ (dotted lines). Gap ranges from $\Delta/T = 0$ to $\Delta/T = 100$. *Left panel:* $R_{\tilde{\epsilon}}$ for the averaged neutrino emissivity. *Right panel:* R_ζ eq. (9) for the bulk viscosity.

process	G	f_V	g_A	D	μ_e	C	$\tilde{\epsilon}\left[\frac{\text{ergs}}{\text{cm}^3\text{s}}\right]$	$\tilde{\zeta}\left[\frac{\text{g}}{\text{cm s}}\right]$	ν_ϵ	ν_ζ
$n \to p l \bar{\nu}_l$	$\frac{1}{2}\sqrt{f_V^2+3g_A^2}\,\cos\theta_C G_F$	1	1.23	$2m_n^* m_p^* \mu_e$	$4\,(1-2x)\,S$	$4(1-2x)\left(n\frac{\partial S}{\partial n}-\frac{S}{3}\right)$	$6.82 \cdot 10^{26}$	$7.86 \cdot 10^{22}$	$\frac{2}{3}$	2
$\Lambda \to p l \bar{\nu}_l$	$\frac{1}{2}\sqrt{f_V^2+3g_A^2}\,\sin\theta_C G_F$	-1.23	0.89	$2m_\Lambda^* m_p^* \mu_e$	or $\frac{\left(3\pi^2 n\right)^{2/3}}{2m_n}$	or $\frac{\left(3\pi^2 n\right)^{2/3}}{6m_n}$	$3.05 \cdot 10^{25}$	$3.52 \cdot 10^{21}$		
$\Sigma^- \to n l \bar{\nu}_l$	$\frac{1}{2}\sqrt{f_V^2+3g_A^2}\,\sin\theta_C G_F$	-1	0.28	$2m_\Sigma^* m_n^* \mu_e$	for a free gas	for a free gas	$1.04 \cdot 10^{25}$	$1.20 \cdot 10^{21}$		
$d \to u e \bar{\nu}_e$	$\sqrt{3}\cos\theta_C G_F$	–	–	$\frac{8\alpha_s}{3\pi}\mu_q^2\mu_e$	$\frac{m_s^2}{4\mu_q}$	$-\frac{m_s^2}{3(1-c)\mu_q}$	$1.57 \cdot 10^{25}\alpha_s$	$5.09 \cdot 10^{21}\alpha_s$	$\frac{1}{3}$	$-\frac{1}{3}$
$s \to u e \bar{\nu}_e$	$\sqrt{3}\sin\theta_C G_F$	–	–	$m_s^2\mu_q$		free gas: $c = 0$	$3.98 \cdot 10^{24}$	$1.29 \cdot 10^{21}$		

Table I: Parameters for direct Urca processes in dense matter, with Fermi constant G_F, Cabbibo angle θ_C, vector and axial couplings f_V and g_A [1], proton fraction x, symmetry energy $S(\bar{n})$ [17], in-medium hadron masses $m^*(\bar{n})$ and quark interaction parameter c [7, 18]. The quark expression is to leading order in α_s and all results to leading order in T/μ, m_l/μ and μ_Δ/μ_e.

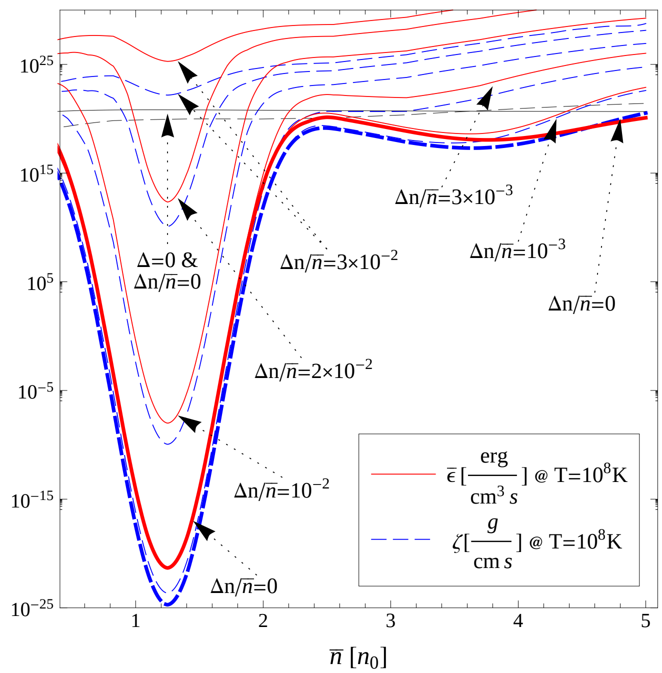

Figure 3: Illustration of the enhancement via suprathermal oscillations of the neutrino emissivity and the bulk viscosity of hadronic matter with a $^1S_0$ proton gap $\Delta_{p0} = 1\,\mathrm{MeV}$ at $n \approx 1.3\, n_0$ and a neutron gap $\Delta_{n0} \approx 0.1\,\mathrm{MeV}$ at $n \approx 4\, n_0$.

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