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# Spin Liquid Phases for Spin-1 systems on the Triangular lattice

Kenke Xu,<sup>1</sup> Fa Wang,<sup>2</sup> Yang Qi,<sup>3</sup> Leon Balents,<sup>1,4</sup> and Matthew P. A. Fisher<sup>1</sup>

<sup>1</sup>*Department of Physics, University of California, Santa Barbara, CA 93106*

<sup>2</sup>*Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139*

<sup>3</sup>*Institute for Advanced Study, Tsinghua University, Beijing 100084, China*

<sup>4</sup>*Kavli Institute for Theoretical Physics, University of California, Santa Barbara, CA, 93106*

Motivated by recent experiments on material  $\text{Ba}_3\text{NiSb}_2\text{O}_9$ , we propose two novel spin liquid phases ( $A$  and  $B$ ) for spin-1 systems on a triangular lattice. At the mean field level, both spin liquid phases have gapless fermionic spinon excitations with quadratic band touching, thus in both phases the spin susceptibility and  $\gamma = C_v/T$  saturate to a constant at zero temperature, which are consistent with the experimental results on  $\text{Ba}_3\text{NiSb}_2\text{O}_9$ . On the lattice scale, these spin liquid phases have  $\text{Sp}(4) \sim \text{SO}(5)$  gauge fluctuation; while in the long wavelength limit this  $\text{Sp}(4)$  gauge symmetry is broken down to  $\text{U}(1) \times Z_2$  in type  $A$  spin liquid phase, and broken down to  $Z_4$  in type  $B$  phase. We also demonstrate that the  $A$  phase is the parent state of the ferro-quadrupole state, nematic state, and the noncollinear spin density wave state.

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A quantum spin liquid (QSL) is a ground state of an insulating magnet with vanishing static local moments and exotic emergent excitations.[1] Within spin wave theory for the simplest Heisenberg Hamiltonians, quantum fluctuations rapidly decrease with increasing spin quantum number  $S$ , so it is often believed that QSLs may occur only in the extreme case of  $S = 1/2$  spins. Indeed, the most promising empirical QSL materials are comprised of spin-1/2 moments[2–7]. However, when the Hamiltonian deviates from the Heisenberg form, quantum effects can be enhanced also for higher spin, leading to ground states beyond the usual magnetically ordered ones. Theoretically, biquadratic and other higher order exchange terms have been argued to favor multipolar ordered and QSL states in particular materials, such as the triangular lattice spin-1 magnet  $\text{NiGa}_2\text{S}_4$  [8–12] and certain ordered double perovskites[13]. Quite unexpectedly, recent experiments have evidenced QSL behavior in the spin-1 magnet  $\text{Ba}_3\text{NiSb}_2\text{O}_9$ , with spins residing on triangular lattices with AB stacking.[14] Although the Curie-Weiss temperature of this material is  $\theta_{CW} \sim -75\text{K}$ , no magnetic ordering or phase transition was observed down to a temperature of  $0.35\text{K}$ , approximately 200 times lower than  $|\theta_{CW}|$ . The low temperature thermodynamics of this material is strikingly similar to that of the geometrically similar spin-1/2 organic triangular lattice QSLs [5, 15–17]. In particular, the spin susceptibility  $\chi$  and linear coefficient of specific heat  $\gamma = c_v/T$  in  $\text{Ba}_3\text{NiSb}_2\text{O}_9$  both saturate to constants at low temperature [14].

Most theoretical approaches to QSLs rely on slave particle methods, and/or wave functions which correspond to slave particles. While these approaches have been extensively developed for  $S = 1/2$  systems, there has been little theoretical work on them for the  $S = 1$  case. We consider this here. To sharpen the discussion, we assume the presence of  $\text{SU}(2)$  spin symmetry, and seek QSL states in this framework which match the basic phenomenology

so far observed in the low temperature thermodynamics.

One way of studying spin-1 system is by introducing three flavors of fermionic spinon  $f_\alpha$  ( $\alpha = 1-3$ ) as follows [18, 19]:  $\hat{S}^a = f_\alpha^\dagger S_{\alpha\beta}^a f_\beta$ , and  $S^a$  are three spin-1 matrices. In order to guarantee the equivalence of the spin Hilbert space and the spinon Hilbert space, one must impose the gauge constraint  $\sum_\alpha f_{i,\alpha}^\dagger f_{i,\alpha} = 1$ , fixing the spinon density locally to 1/3-filling. At the mean field level, the spinon  $f_\alpha$  forms a Fermi surface whose area is 1/3 of the Brillouin zone. A spinon Fermi surface seems to be consistent with constant  $\chi$  and  $\gamma$  observed experimentally. However, beyond the mean field theory, due to the single occupancy constraint, the spinon fermi surface is coupled to a dynamical  $\text{U}(1)$  gauge field. This  $\text{U}(1)$  gauge field has a “dressed” over-damped  $z = 3$  dynamics due to its coupling to the Fermi surface, which leads to a  $\gamma = C_v/T \sim T^{-1/3}$  at low temperature [20, 21], inconsistent with experiment. One solution of this problem is to introduce pairing of the spinons in the mean field state. This has its own difficulties: either a gap is induced and impurities must be invoked to restore the proper thermodynamics,[22] or spin-rotational symmetry must be strongly broken.[19]

*General Formalism* We start instead by representing the spin-1 operators in the following way:

$$\hat{S}_i^\mu = \frac{1}{2} \sum_{\alpha,\beta=\uparrow,\downarrow} \sum_{a=1,2} f_{\alpha,a,i}^\dagger \sigma_{\alpha\beta}^\mu f_{\beta,a,i}. \quad (1)$$

Here  $\sigma^\mu$  are three spin-1/2 Pauli matrices. Each spinon  $f_{\alpha,a}$  has two indices:  $\alpha = \uparrow, \downarrow$  denotes spin and  $a = 1, 2$  is an “orbital” quantum number. Thus we can consider not only the usual spin  $\text{SU}(2)$  rotations in the  $\alpha - \beta$  space, but also orbital  $\text{SU}(2)$  transformations in the  $a - b$  space. Matching with the spin Hilbert space requires not only constraining the total fermion number to *half-filling* (two fermions per site), but also requiring each site to be an orbital  $\text{SU}(2)$  singlet, which guarantees that the spin

space is a symmetric spin-1 representation:

$$\begin{aligned}\hat{n}_i &= \sum_{a=1,2} \sum_{\alpha=\uparrow,\downarrow} f_{\alpha,a,i}^\dagger f_{\alpha,a,i} = 2, \\ \hat{\tau}^\mu &= \sum_{\alpha,a,b} f_{\alpha,a,i}^\dagger \tau_{ab}^\mu f_{\alpha,b,i} = 0.\end{aligned}\quad (2)$$

Here  $\tau_{ab}^\mu$  are three Pauli matrices that operate on the orbital indices. A similar slave fermion formalism with orbital indices was introduced in Ref. [23], and it was applied to two-orbital  $SU(N)$  magnets that can be realized in Alkaline earth cold atoms [24–26].

Due to these two independent constraints in Eq. 2, the spinon  $f_{\alpha,a}$  appears to have the following  $U(1) \times SU(2)$  gauge symmetries:

$$\begin{aligned}U(1)_c &: f_{\alpha,a,i} \rightarrow e^{i\theta_i} f_{\alpha,a,i}; \\ SU(2)_o &: f_{\alpha,a,i} \rightarrow [e^{i\vec{\theta}_i \cdot \vec{\tau}/2}]_{ab} f_{\alpha,b,i}.\end{aligned}\quad (3)$$

By rewriting  $f_{\alpha,a,i}$  in terms of Majorana fermions  $\eta$  as follows, however, a larger gauge symmetry is exposed:

$$f_{\alpha,a,i} = \frac{1}{2}(\eta_{\alpha,a,1,i} + i\eta_{\alpha,a,2,i}).\quad (4)$$

On every site,  $\eta_i$  has in total three two-component spaces, making the maximal possible transformation on  $\eta_i$   $SO(8)$ . Within this  $SO(8)$ , the spin  $SU(2)$  transformations are generated by the three operators  $(\sigma^x \lambda^y, \sigma^y, \sigma^z \lambda^y)$ , where the Pauli matrices  $\lambda^a$  operate on the two-component space  $(\text{Re}[f], \text{Im}[f])$ . The total gauge symmetry on  $\eta$  is the maximal subgroup of  $SO(8)$  that commutes with the spin- $SU(2)$  operators. This is  $Sp(4) \sim SO(5)$  generated by the ten matrices  $\Gamma_{ab} = \frac{1}{2i}[\Gamma_a, \Gamma_b]$ , where

$$\begin{aligned}\Gamma_1 &= \sigma^y \tau^y \lambda^z, \quad \Gamma_2 = \sigma^y \tau^y \lambda^x, \quad \Gamma_3 = \tau^y \lambda^y, \\ \Gamma_4 &= \tau^z, \quad \Gamma_5 = \tau^x.\end{aligned}\quad (5)$$

These  $\Gamma^a$  with  $a = 1 \cdots 5$  define five gamma matrices that satisfy the Clifford algebra  $\{\Gamma_a, \Gamma_b\} = 2\delta_{ab}$ .  $\Gamma_{ab}$  and  $\Gamma_a$  are all  $8 \times 8$  hermitian matrices.  $\Gamma_{ab}$  are all antisymmetric and imaginary, while  $\Gamma_a$  are symmetric.

We consider a spin-1 Heisenberg model on the triangular lattice with both nearest neighbor and 2nd neighbor antiferromagnetic couplings. Based on the above spinon representation of spin-1 operators, the Heisenberg model can be rewritten as follows:

$$\begin{aligned}\sum_{i,j,\mu} J_{ij} \hat{S}_i^\mu \hat{S}_j^\mu &\sim \sum_{i,j,\mu} J_{ij} f_{\alpha,a,i}^\dagger \sigma_{\alpha\beta}^\mu f_{\beta,a,i} f_{\gamma,b,j}^\dagger \sigma_{\gamma\rho}^\mu f_{\rho,b,j} \\ &\sim -2J_{ij} \hat{\Delta}_{ab}^* \hat{\Delta}_{ba,ji} + \text{Const}, \\ \hat{\Delta}_{ab,ji} &= \varepsilon_{\alpha\beta} f_{\alpha,a,j} f_{\beta,b,i}.\end{aligned}\quad (6)$$

Decoupling through a hopping term is also possible, but we do not pursue this here. To analyze Eq. (6), we adopt

a mean field ansatz with nonzero pairing  $\langle \hat{\Delta}_{ab,ji} \rangle$ , so that the spinon  $f_{\alpha,a}$  fills a mean field band structure. To improve beyond mean field, a variational spin wave function may be obtained by projecting the mean field ground state to satisfy Eq. (2):

$$|G_{\text{spin}}\rangle = \prod_i P(\hat{n}_i = 2) \otimes P(\hat{\tau}_i^\mu = 0) |f_{\alpha,a}\rangle.\quad (7)$$

The general formalism discussed above can describe many novel spin liquid states, with various different gauge fluctuations that are subgroups of  $Sp(4)$ . Here we focus on simple states which satisfy the phenomenology of  $\text{Ba}_3\text{NiSb}_2\text{O}_9$  [14], and in particular demand linear specific heat and constant susceptibility. We consider the following ansatz, which is a  $d+id$  pairing state of spinons:

$$\langle \hat{\Delta}_{ab,(i,i+\hat{e})} \rangle = \left( \delta_{ab} \Delta_1^{(m)} + \tau_{ab}^z \Delta_2^{(m)} \right) (e_x + ie_y)^2, \quad (8)$$

where  $\hat{e}$  is any of the nearest-neighbor or 2nd neighbor unit vectors, and  $\Delta^{(m)}$  with  $m = 1, 2$  denotes the pairing amplitude on the nearest and 2nd neighbor links respectively. This is a spin singlet but orbital triplet. Because the pair wave function vanishes when two spinons are on the same site, such states may be particularly insensitive to the projection in Eq. 7.

*Continuum theory:* In the majority of the paper, we consider the case with  $\Delta_2^{(m)} = 0$ . Then, expanded at  $\vec{k} = 0$ , the low energy mean field Hamiltonian reads

$$\begin{aligned}H &\sim \eta^\dagger \{ (\partial_x^2 - \partial_y^2) \Gamma_{13} + 2\partial_x \partial_y \Gamma_{23} \} \eta, \\ \Gamma_{13} &= -\sigma^y \lambda^x, \quad \Gamma_{23} = \sigma^y \lambda^z.\end{aligned}\quad (9)$$

This mean field Hamiltonian has quadratic band-touching at  $\vec{k} = 0$ . Using the same method as introduced in Ref. [27], one can verify that this mean field Hamiltonian breaks the  $Sp(4)$  gauge symmetry down to a  $U(1) \times Z_2$  gauge symmetry:

$$\eta_i \rightarrow e^{i\theta_i \Gamma_{45}} \eta_i, \quad \eta_i \rightarrow Q_i \eta_i, \quad Q_i \in \{\mathbf{1}, \Gamma_4\}.\quad (10)$$

Notice that the  $U(1)$  and  $Z_2$  gauge transformations do *not* commute with each other.

In addition to the quadratic band touching at  $\vec{k} = 0$ , depending on  $\Delta^{(m)}$ , there are multiple Dirac points in the Brillouin zone. For instance, when  $\Delta^{(2)} < \Delta^{(1)}$ , there are Dirac points at the Brillouin zone corners  $\vec{Q} = \pm(4\pi/3, 0)$ . A complex Dirac fermion field  $\psi$  at momentum  $\vec{Q} = (4\pi/3, 0)$  can be defined as

$$\eta_{\vec{r}} = \psi_{\vec{r}} e^{i\vec{Q} \cdot \vec{r}} + \psi_{\vec{r}}^\dagger e^{-i\vec{Q} \cdot \vec{r}}.\quad (11)$$

The low energy Hamiltonian for  $\psi$  reads  $H_\psi \sim \psi^\dagger (i\Gamma_{13} \partial_x - i\Gamma_{23} \partial_y) \psi$ . However, the Dirac fermion has vanishing density of states at zero energy, and thus contributes sub-dominantly to the  $\eta$  spinon in many physical properties.

The spinon carries a projective representation of physical symmetry. Under discrete symmetry transformations, the low energy spinon fields  $\eta$  and  $\psi$  transform as:

$$\begin{aligned}
T_x &: x \rightarrow x + 1, & \eta &\rightarrow \eta, & \psi &\rightarrow e^{i4\pi/3}\psi; \\
T &: t \rightarrow -t, & \eta &\rightarrow i\Gamma_{12}\eta, & \psi &\rightarrow i\Gamma_{12}\psi^\dagger; \\
\mathcal{I} &: \vec{r} \rightarrow -\vec{r}, & \eta &\rightarrow \eta, & \psi &\rightarrow \psi^\dagger; \\
P_y &: x \rightarrow -x, & \eta &\rightarrow i\Gamma_{13}\eta, & \psi &\rightarrow i\Gamma_{13}\psi^\dagger; \\
R_{\pi/3} &: (x + iy) \rightarrow e^{i\pi/3}(x + iy), \\
&& \eta &\rightarrow e^{i\frac{\pi}{3}\Gamma_{12}}\eta, & \psi &\rightarrow e^{i\frac{\pi}{3}\Gamma_{12}}\psi^\dagger.
\end{aligned} \tag{12}$$

These transformations guarantee that there is no relevant fermion bilinear perturbation that does not break physical symmetry. For instance, the fermion bilinear  $f_i^\dagger f_i \sim \eta^t \Gamma_{12} \eta$  breaks the time-reversal symmetry; thus it is forbidden in the Hamiltonian. In Ref. [28], a stable quadratic band touching model was discussed, and it was generally argued that the  $Z_4$  symmetry and  $Z_6$  symmetry of the lattice is crucial to the stability of the quadratic band touching.

*Effect of gauge fluctuations:* The spinons are coupled to a U(1) gauge field  $a_\mu \Gamma_{45}$ . The gauge field Lagrangian is renormalized by the fermion loop, which generates a mass gap for  $a_0$ , thus  $a_0$  can be ignored hereafter. The same fermion loop will also renormalize the dynamics of the transverse component of gauge field  $a^T$  to be:

$$\begin{aligned}
\mathcal{L}_1 &= \sum_{\omega, \vec{q}} \left( c|\omega| + \frac{q^2}{e_{\omega, \vec{q}}^2} + c_1 \sqrt{\omega^2 + v^2 q^2} \right) |a_{\omega, \vec{q}}^T|^2, \\
e_{\omega, \vec{q}}^2 &\sim \frac{e^2}{1 + c_2 e^2 \log \left( \frac{\Lambda^2}{4\omega^2 + q^4} \right)}.
\end{aligned} \tag{13}$$

In Lagrangian  $\mathcal{L}_1$ , the first two terms come from the screening of spinons at the quadratic band touching, while the third term comes from the Dirac points. At low energy, the gauge field therefore obeys  $z = 1$  scaling with  $\omega \sim q$ , so that the  $q^2/e^2$  term is negligible in Eq. (13). For this reason, the gauge field decouples from  $\eta$  (which has  $z = 2$  scaling) at low energy, but remains strongly coupled to the  $z = 1$  Dirac fermion  $\psi$ .

*Thermodynamic and transport properties:* The finite density of states of the  $\eta$  spinon leads to a constant  $\gamma = C_v/T$  at zero temperature. In terms of  $\eta$ , the spin density  $S^z$  is represented as

$$\sum_i S_i^z = \sum_{\alpha, a, i} \eta_i^\dagger \sigma^z \lambda^y \eta_i. \tag{14}$$

Since the spin density commutes with the mean field Hamiltonian Eq. 9, turning on an external magnetic field creates a Fermi surface of  $\eta$  and  $\psi$ , and since the density of states is finite at the quadratic band touching, the

spin susceptibility saturates to a constant at zero temperature. Thus this spin liquid phase is consistent with the scaling of specific heat and spin susceptibility observed experimentally.

We also find a unique scaling of thermal conductivity. The thermal conductivity is proportional to the specific heat and the velocity of entropy carriers of the system:  $\kappa \sim cvl$ , where  $l$  is the mean free path. Since the spinon has quadratic band touching, at low temperature the average velocity scales as  $v \sim T^{1/2}$ , thus the thermal conductivity scales as  $\kappa \sim T^{3/2}$ . Note that the thermal conductivity contribution from the Dirac fermion and gauge field scales as  $T^2$ , and is thus subdominant.

*Fluctuating orders:* The gauge invariant fermion bilinear operators can be viewed as physical order parameters with power-law correlations. They can be classified according to their transformations under symmetry and gauge symmetry. Some of the fermion bilinears are summarized as follows:

- (1) Spin density wave
 
$$\begin{aligned}
\vec{S}_{\vec{r}} &= \cos(\vec{Q} \cdot \vec{r}) \vec{n}_1 + \sin(\vec{Q} \cdot \vec{r}) \vec{n}_2, \\
\vec{n}_1 + i\vec{n}_2 &\sim a_1 \psi^\dagger \vec{S} \psi^* + b_1 \eta^t \vec{S} \psi,
\end{aligned}$$
- (2) Spin – Nematic :
 
$$\vec{d} \sim a_2 \eta^t \vec{S} \Gamma_3 \eta + b_2 \psi^\dagger \vec{S} \Gamma_3 \psi,$$
- (3) Nematic :
 
$$\begin{aligned}
N &= \sum_{\vec{\epsilon}} \vec{S}_i \cdot \vec{S}_{i+\vec{\epsilon}} (e_x + i e_y)^2 \\
N &= N_1 + i N_2 \\
&\sim a_3 (\eta^t \Gamma_{13} \eta + i \eta^t \Gamma_{23} \eta) + b_3 (\psi^\dagger \Gamma_{13} \psi + i \psi^\dagger \Gamma_{23} \psi);
\end{aligned}$$
- (4) Spin – chirality :
 
$$\begin{aligned}
C &= \sum_{ijk \in \Delta} \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k) + \dots \\
C &\sim a_4 \eta^t \Gamma_{12} \eta + b_4 \psi^\dagger \Gamma_{12} \psi,
\end{aligned} \tag{15}$$

where  $\vec{S} = (\sigma^x \lambda^y, \sigma^y, \sigma^z \lambda^y)$  are the spin matrices.

Several of these orders are germane to spin-one triangular antiferromagnets. The spin density wave order parameter is precisely that which describes the classical  $120^\circ$  planar spin state, with  $\vec{Q} = (4\pi/3, 0)$  coinciding with the Brillouin zone corner. As a consequence the spin structure factor of this state is singular at this momentum. Spin nematic order occurs naturally when biquadratic interactions are present in spin one systems.[11] In fact,  $\vec{d}$  changes sign under the  $Z_2$  gauge transformation  $\eta \rightarrow \Gamma_4 \eta$ , so it is a headless nematic director. The physical order parameter is actually a bilinear

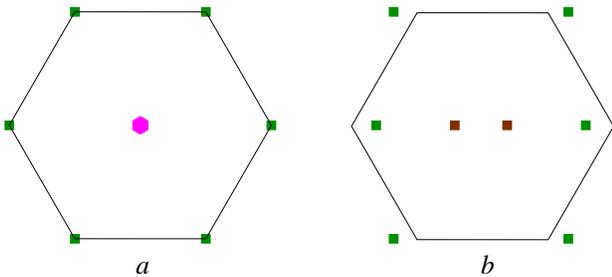


FIG. 1: *a*, The spin liquid we are considering contains a quadratic band touching at  $\vec{k} = 0$  (hexagon), and Dirac points (squares) at the corners of the Brillouin zone. *b*, with a nonzero and small nematic order  $N_1 > 0$ , the quadratic band touching is split into two Dirac points, and the locations of the other Dirac points are shifted.

of  $\vec{d}$ , which corresponds to the ferro-quadrupole tensor

$$Q^{\mu\nu} = \frac{1}{2} \langle \hat{S}_i^\mu \hat{S}_i^\nu + \hat{S}_i^\nu \hat{S}_i^\mu \rangle - \frac{2}{3} \delta^{\mu\nu} = d^\mu d^\nu - \frac{|\vec{d}|^2}{3} \delta^{\mu\nu}. \quad (16)$$

Spatial nematic order, in which lattice rotation symmetry is broken but time-reversal and spin symmetry are preserved, is described by  $N_1$  and  $N_2$ . Order of this type was suggested for  $S=1$  triangular antiferromagnets in Ref.[9], but also can be realized by spontaneous formation of Haldane chains. The spin-chirality order parameter  $C$  is less obvious from a microscopic perspective, but is a fluctuating order for this QSL state.

At the mean field level, the equal time correlation of spin chirality, nematic, and spin density wave order parameters all fall off as  $1/r^4$ ; the correlation of spin quadrupole order parameter falls off as  $1/r^8$ . The  $U(1)$  gauge fluctuation will modify the scaling dimension of the order parameters, and its correction can be calculated systematically using a  $1/N$  expansion. We will leave this calculation to future studies.

*Potential instabilities:* One potential instability of this spin liquid state is instanton proliferation of the compact  $U(1)$  gauge field[29]. However, due to screening by the gapless fermions, the instantons are greatly suppressed. By analogy with the theory of the algebraic spin liquid [30] (in which the  $z = 1$  gauge field is similarly strongly coupled to Dirac fermions), we expect the spin liquid phase here to be similarly stable in principle.

Furthermore, the mean field Hamiltonian Eq. 9 is subject to perturbations such as four-fermion interactions, which are marginal perturbations at the quadratic band touching. These four-fermion interactions can modify the correlation functions of the order parameters discussed above. The renormalization group may lead to weak run-away flow of these four-fermion interactions, which eventually can break the symmetry of the system, and develop one of the orders in Eq. 15.

If one of these orders develops, it can completely or

partially gap the fermions and introduce interesting effects. Nonzero spin nematic order,  $\vec{d} \neq 0$ , gaps out the quadratic band touching and Dirac fermion  $\psi$ . Depending on the sign of  $a_2$  and  $b_2$ , a nonzero  $\vec{d}$  drives the mean field band structure of spinon into either a quantum spin Hall type of topological insulator or a topologically trivial insulator. If the system is in a quantum spin Hall topological insulator, assuming  $\vec{d}$  is ordered along  $\hat{z}$  direction, the quantized flux of  $U(1)$  gauge field  $a_\mu \Gamma_{45}$  would carry spin  $S^z$ , which is a conserved quantity. Usually the instanton of a 2+1d compact  $U(1)$  gauge field will gap out the photon excitation [29]. However, in this case since the quantized gauge flux carries conserved spin, it will suppress the instanton of the compact  $U(1)$  gauge field  $a_\mu$ , thus  $a_\mu$  is in its photon phase (a similar physics was discussed in Ref. [31]). Since the 2+1d photon phase of the  $U(1)$  gauge field is the condensate of gauge flux, the  $U(1)$  spin rotation around  $\hat{z}$  axis is spontaneously broken in the photon phase, thus eventually the spin  $SU(2)$  symmetry is broken down to a discrete subgroup *i.e.* there are in total three Goldstone modes instead of two. If the spinon band insulator has trivial topology, then the system is in an ordinary ferro-quadrupolar phase as discussed in Ref. [9, 10].

Weak spatial nematic order does not open a gap but only splits the quadratic band touching into Dirac fermions at two different momenta (Fig. 1); the original Dirac fermions  $\psi$  also shift. When the nematic order magnitude is very strong, above some critical value, all the Dirac fermions meet and annihilate in pairs, and the spinons become fully gapped.

Spin chirality order, which breaks time-reversal and reflection symmetries, gaps out both the quadratic band touching and the Dirac points. Depending on the sign of  $a_4$  and  $b_4$ , a nonzero spin chirality order can drive the spinons into a topological Chern insulator, or a topologically trivial band insulator with the same symmetry. In the former case, one obtains a chiral spin liquid, in which the  $U(1)$  gauge field  $a_\mu \Gamma_{45}$  acquires a Chern-Simons term after integrating out the fermions. In the topologically trivial band insulator, the  $U(1)$  gauge field will become confined by instanton proliferation.

*Other phases:* For  $S = 1$  spins, we may also consider another state with  $\Delta_1^{(m)}$  and  $\Delta_2^{(m)}$  both nonzero, and  $|\Delta_1^{(m)}| \neq |\Delta_2^{(m)}|$ . In this case, the spinons have two different bands both with quadratic band touching at  $\vec{k} = 0$ , but they have different band curvature:

$$H \sim \eta^\dagger \{ (\partial_x^2 - \partial_y^2) (A\Gamma_{13} + B\Gamma_{25}) + 2\partial_x \partial_y (A\Gamma_{23} - B\Gamma_{15}) \} \eta. \quad (17)$$

$A$  and  $B$  are two linear combinations of pairing amplitudes on nearest and 2nd neighbor links. In this state, the gauge symmetry is broken down to  $Z_4$ :

$$\eta_i \rightarrow Q_i \eta_i, \quad Q_i \in \{\pm 1, \pm \Gamma_4\}. \quad (18)$$

The  $Z_4$  gauge field has a deconfined phase in 2+1 dimension, and this state is thus clearly locally stable. It also exhibits non-zero finite spin susceptibility and  $\gamma = C_v/T$  at zero temperature.

It was shown in material  $\text{Ba}_3\text{NiSb}_2\text{O}_9$  that under magnetic field  $\gamma$  is still a constant at low temperature [14]. An external magnetic field will induce a small Fermi surface for both the  $U(1)\times Z_2$  state and the  $Z_4$  state. With a Fermi surface, the  $U(1)$  gauge field will acquire a standard  $|\omega|/q$  term in its action, which leads to a  $z = 3$  gapless dispersion, while the  $Z_4$  gauge field is still gapped. Thus the current experimental observations are more consistent with the  $Z_4$  state.

A similar  $d + id$  state with quadratic band touching can also be considered for spin-1/2 systems on the triangular lattice. The same mean field Hamiltonian as Eq. 9 applied, but without an orbital index. This state remains time-reversal and reflection invariant, and has  $Z_2$  gauge structure. One might consider this as a candidate state for the spin liquids observed in the compounds  $\kappa\text{-(ET)}_2\text{Cu}_2(\text{CN})_3$ ,  $\text{EtMe}_3\text{Sb}[\text{Pd}(\text{dmit})_2]_2$ , and  $\text{Ba}_3\text{CuSb}_2\text{O}_9$  [2–6, 32].

*Summary and future work:* In this work we developed a new theory of spin liquid for spin-1 quantum magnet, using this theory we proposed two candidate states for the recently discovered material  $\text{Ba}_3\text{NiSb}_2\text{O}_9$ . In Ref. [22, 33], a variational Monte Carlo computation based on the Gutzwiller projected wavefunction for  $S = 1/2$  was used to compare the energy of various mean field spin liquid states. Extension of this method to our generalized  $S = 1$  projected wavefunction, Eq. 7, is a non-trivial and interesting problem for the future.

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