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Constraint on parity-violating muonic forces

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Using the nonobservance of missing mass events in the leptonic kaon decay $K \to \mu X$, we place a strong constraint on exotic parity-violating gauge interactions of the right-handed muon. By way of illustration, we apply it to an explanation of the proton size anomaly that invokes such a new force; scenarios in which the gauge boson decays invisibly or is long-lived are constrained.

In the standard model (SM), the right-handed charged lepton field ℓ_R is a gauge singlet, and the chiral muon field μ_R is an example of such a field. It is straightforward to add a new $U_{\mu_R}(1)$ gauge interaction without modifying the SM gauge group structure, and simultaneously evade many phenomenological constraints. Recently, this possibility has been entertained [1] to explain a measurement of the proton radius obtained from the Lamb shift of muonic hydrogen [2] , that is 5σ smaller than that determined from ordinary hydrogen or e-p scattering data [3]. While the new interaction alone would be in conflict with measurements of the muon anomalous magnetic dipole moment $g_\mu - 2$ [4], one can arrange a delicate cancellation from another sector of new physics, such as a new scalar boson associated with the Higgs mechanism. Although unnatural, such fine tuning is conceivable.

An explicit example of such a cancellation can be found in the model of Ref. [1] which has a $U_{\mu_R}(1)$ vector gauge boson V and a complex scalar field, both with mass of tens of MeV. The Lamb shift correction in muonic hydrogen is accounted for by a modest gauge coupling $g_R \approx 0.01$ and a small kinetic mixing amplitude $\kappa \sim 0.002$ between V and the photon field. The large V-exchange contribution to $g_\mu - 2$ is cancelled at the 0.1% level by the contribution of the scalar.

In this Letter, we examine an important constraint on the g_R gauge coupling to μ_R in the context of the leptonic kaon decay, $K \to \mu\nu$ [5]. If V is lighter than 100 MeV, it can be radiated from the muon line of the above process. If V is stable, the combined recoiling system forms a missing mass for which there is no experimental evidence. In fact, the size of g_R that accommodates the Lamb shift of muonic hydrogen [1] is not allowed by leptonic kaon decay provided V decays invisibly or does not decay inside the detector.

Note that in the minimal version of the model of Ref. [1], V decays promptly into e^+e^- pairs via kinetic mixing with the photon, and our constraint does not apply.[1] More baroque realizations, in which there are new particles that are charged under $U_{\mu_R}(1)$ and lighter than $m_V/2$, are strongly constrained unless these particles decay to the SM.

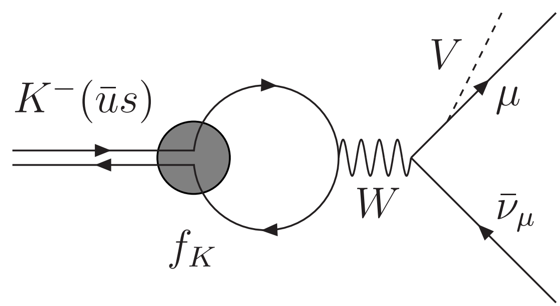

FIG. 1. V bremsstrahlung in $K^- \to \mu^- \bar{\nu}_\mu$ decay.

For the sake of generality, we assume that a light vector particle V and the right-handed muon interact via the Lagrangian term,

$$g_R \bar{\mu}_R \not{V} \mu_R \ . \tag{1}$$

It is possible to produce a V boson by radiation in $K \to \mu\nu$ decay as long as the V boson is lighter than about 100 MeV; see Fig. 1.

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[1] Measurements of $K^+ \to \mu^+ \nu e^+ e^-$ have been made with e^+e^- invariant masses above 145 MeV [6], so that they are relevant only for $m_V > 145$ MeV.

However, a recent search for V in the decay chain $\phi \to \eta V$, $\eta \to \pi^+\pi^-\pi^0$, $V \to e^+e^-$, by the KLOE-2 collaboration [7] excludes the kinetic mixing parameters corresponding to the points with $(m_V, g_R) = (50\ \mathrm{MeV}, 0.05)$ and $(100\ \mathrm{MeV}, 0.07)$ in Ref. [1]. The $(m_V, g_R) = (10\ \mathrm{MeV}, 0.01)$ point of Ref. [1] yields a proton-muon interaction that is incompatible with measurements of the muonic $3D_{5/2} - 2P_{3/2}$ X-ray transition in ^{24}Mg and ^{28}Si [8]. Other points of the minimal scheme that survive these constraints may exist, but this requires a parameter space scan.

In the process $K^- \to \mu^- V \bar{\nu}_\mu$, the relevant hadronic weak-current matrix element is $\langle 0|\bar{u}\gamma^\alpha(1-\gamma_5)s|K^-\rangle = f_K p_K^\alpha$, where p_K^α denotes the momentum of the decaying kaon and $f_K = 156.1$ MeV [9]. The amplitude for the process is then

$$\mathcal{M} = \frac{\sqrt{2} g_R\, G_F\, f_K\, m_\mu\, \sin\theta_C}{(p_\mu + p_V)^2 - m_\mu^2} \left[\bar{u}_\mu \not{\epsilon}_V \not{p}_K \frac{1-\gamma_5}{2} v_\nu \right] , \quad (2)$$

where θ_C is the Cabibbo angle and ϵ_V^μ is the polarization vector of the V boson. The spin-summed squared amplitude is given by

$$\sum |\mathcal{M}|^2 = \frac{4 g_R^2 G_F^2 f_K^2 m_\mu^2 \sin^2\theta_C}{(m_V^2 + 2 p_V\cdot p_\mu)^2} \Big[2 p_K\cdot p_\mu\, p_K \cdot p_\nu - m_K^2 p_\mu \cdot p_\nu + \frac{2 p_V \cdot p_\mu}{m_V^2} (2 p_K \cdot p_V\, p_K\cdot p_\nu - m_K^2 p_V \cdot p_\nu) \Big] . \quad (3)$$

In the rest frame of the kaon, energy conservation in terms of the scaling variables,

$$x_\alpha = 2E_\alpha/m_K = 2 p_K \cdot p_\alpha / m_K^2 , \qquad \alpha = \mu, \nu, V$$

dictates $x_\mu + x_\nu + x_V = 2$. We have for the scalar products,

$$\begin{aligned} p_\mu \cdot p_\nu &= \frac{m_K^2}{2}(1 - x_V + \delta_V - \delta_\mu) , \\ p_\mu \cdot p_V &= \frac{m_K^2}{2}(1 - x_\nu - \delta_V - \delta_\mu) , \\ p_\nu \cdot p_V &= \frac{m_K^2}{2}(1 - x_\mu - \delta_V + \delta_\mu) , \end{aligned} \quad (4)$$

with $\delta_V = m_V^2/m_K^2$ and $\delta_\mu = m_\mu^2/m_K^2$. We thus derive the differential decay rate

$$\frac{d\Gamma(K^- \to \mu^- V \bar{\nu}_\mu)}{dx_\mu dx_\nu} = \frac{m_K}{256\pi^3} \sum |\mathcal{M}|^2 , \quad (5)$$

with $\sum|\mathcal{M}|^2$ in Eq. (3) written in terms of $x_{\mu,\nu,V}$ and $\delta_{\mu,V}$. The range of x_μ is $\left[2\sqrt{\delta_\mu}, 1 + \delta_\mu - \delta_V\right]$. x_ν is bounded by the following upper and lower limits:

$$\frac{1}{2(1 - x_\mu + \delta_\mu)} \Big[(2 - x_\mu)(1 - x_\mu + \delta_\mu + \delta_V) \pm \sqrt{x_\mu^2 - 4\delta_\mu}(1 - x_\mu + \delta_\mu - \delta_V) \Big] . \quad (6)$$

It is useful to normalize our result in Eq. (5) with respect to the standard two-body decay rate,

$$\Gamma(K^- \to \mu^- \bar{\nu}_\mu) = \frac{G_F^2}{8\pi} m_K m_\mu^2 f_K^2 \sin^2\theta_C \left(1 - \frac{m_\mu^2}{m_K^2}\right)^2 \quad (7)$$

to get the dimensionless formula

$$\frac{1}{\Gamma(K^- \to \mu^- \bar{\nu}_\mu)} \frac{d\Gamma(K^- \to \mu^- V \bar{\nu}_\mu)}{dx_\mu dx_V} = \frac{g_R^2/(1-\delta_\mu)^2}{16\pi^2 (1 - \delta_\mu - x_\nu)^2} \Big[x_\mu x_\nu - 1 + x_V - \delta_V + \delta_\mu + \frac{1}{\delta_V}(1 - x_\nu - \delta_V - \delta_\mu)(x_V x_\nu - 1 + x_\mu + \delta_V - \delta_\mu) \Big] . \quad (8)$$

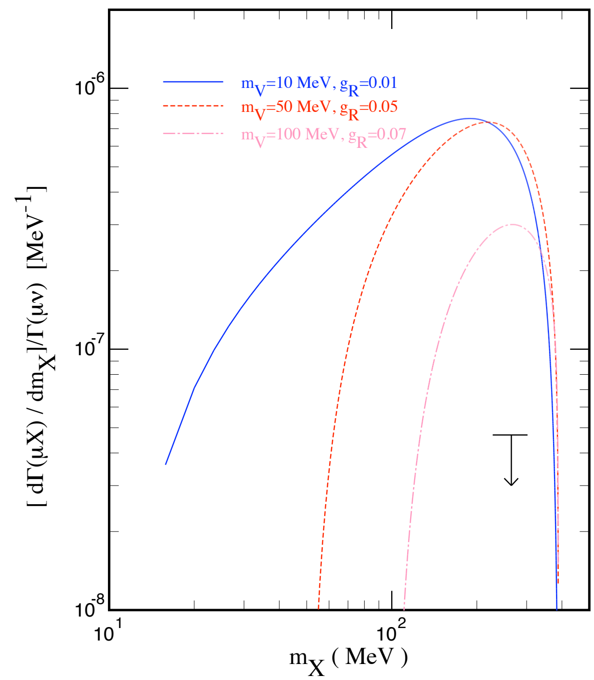

FIG. 2. Differential decay rate of muonic kaon decay with V bremsstrahlung as a function of the missing mass, normalized to the standard two-body muonic kaon decay. The 90% CL upper limit in the mass range $227.6 \le m_X \le 302.2$ MeV is marked by a short horizontal line. The distributions for the three benchmark points shown violate the upper limit. We remind the reader that the bound is evaded by the minimal model of Ref. [1], since V decays promptly to e^+e^-; model extensions in which V decays invisibly or is long-lived are strongly constrained.

After integrating over x_ν, the resulting energy distribution in x_μ can be confronted by the search for a missing recoiling mass in muonic kaon decay. To compare with experiment, we need $\frac{1}{\Gamma(K^-\to\mu^-\bar{\nu}_\mu)} \frac{d\Gamma(K^-\to\mu^- X)}{dm_X}$ versus m_X, with X denoting the missing energy. Since $p_X = p_V + p_\nu$, we get $m_X^2 = m_K^2(1 - x_\mu + \delta_\mu)$, and

$$\frac{d\Gamma}{dm_X} = \frac{2\sqrt{1 - x_\mu + \delta_\mu}}{m_K} \frac{d\Gamma}{dx_\mu} . \quad (9)$$

A null result for missing mass in such decays was obtained with a sensitivity of 10^{-7} MeV^{-1} [5]. The experimental acceptance of the muon kinetic energy is in the range, 60 MeV to 100 MeV, that corresponds to a missing mass m_X of 302.2 MeV to 227.6 MeV, a mass interval of 74.6 MeV. The nonobservation of a signal sets a 90% CL upper limit on the branching fraction of 3.5×10^{-6} in this

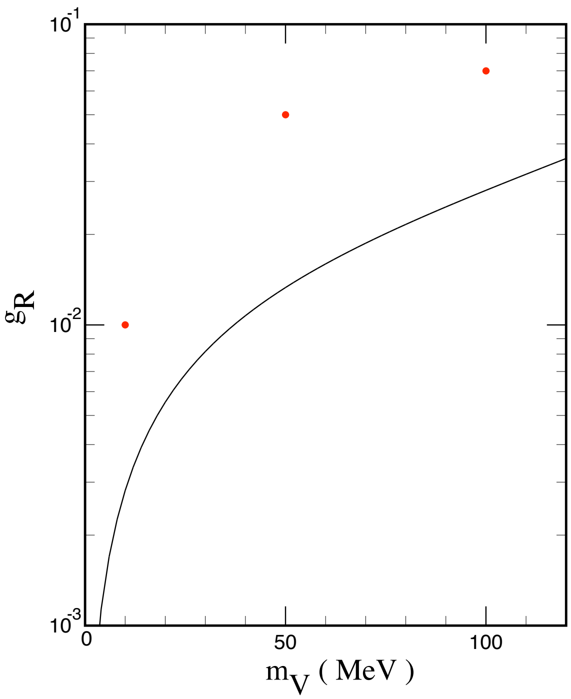

FIG. 3. The (m_V, g_R) parameter space above the solid curve is excluded at the 90% CL. The three red dots are the benchmark points in Fig. 2 and are disallowed if V decays invisibly or is long-lived.

mass interval, corresponding to a normalized differential fraction 4.7×10^{-8} MeV^{-1}. In previous work, this limit has been used to constrain the Majoron model [10].

In Fig. 2, we show the normalized differential decay rate of $K \to \mu V \nu$ as a function of the missing mass. The short horizontal line marks the 90% confidence level (CL) upper limit in that mass range. We also show the differential decay rate curves corresponding to three benchmark choices of (m_V, g_R) for the model of Ref. [1] with the assumption that V has a long enough lifetime that it does not decay inside the detector, or that it decays invisibly. The 90% CL upper limit on g_R is shown in Fig. 3. The three benchmark choices of Fig. 2 indicated by red dots are disallowed.

In conclusion, we pointed out a constraint on a new gauge interaction that couples to the right-handed muon and has a gauge boson mass less than about 100 MeV. This light gauge boson can be copiously produced by bremsstrahlung off the muon line in $K \to \mu\nu$ decays. The lack of experimental evidence for missing mass events constrains the size of the coupling and variants of a model [1] proposed to explain the proton size anomaly.

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