Avoidance of Majorana Resonances in Periodic Topological Superconductor-Nanowire Structures

Jay D. Sau, Chien Hung Lin, Hoi-Yin Hui, and S. Das Sarma

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Majorana resonances and how to avoid them in periodic topological superconductor-nanowire structures

Jay D. Sau[1],* Chien Hung Lin[1], Hoi-Yin Hui[1], and S. Das Sarma[1]
[1] *Condensed Matter Theory Center and Joint Quantum Institute, Department of Physics, University of Maryland, College Park, Maryland 20742-4111, USA*

Semiconducting nanowires in proximity to superconductors are promising experimental systems for Majorana fermions which may ultimately be used as building blocks for topological quantum computers. A serious challenge in the experimental realization of the Majorana fermion in these semiconductor-superconductor nanowire structures is tuning the semiconductor chemical potential in close proximity to the metallic superconductor. We show that, presently realizable structures in experiments with tunable chemical potential lead to *Majorana resonances*, which are interesting in their own right, but do not manifest non-Abelian statistics. This poses a central challenge to the field. We show how to overcome this challenge, thus resolving a crucial barrier to the solid state realization of a topological system containing the Majorana fermion. We propose a new topological superconducting array structure where introducing the superconducting proximity effect from adjacent nanowires generates Majorana fermions with non-Abelian statistics.

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Introduction: Majorana fermions (MF) have been the subject of intense recent study, both due to their fundamental interest as a new type of particle with non-Abelian statistics and their potential application in topological quantum computation (TQC)[1–4]. Topological superconductors are promising candidates for the practical solid state realization of MFs [5–11]. A simple topological superconducting (TS) systems supporting MFs, which has attracted serious experimental attention [3], consists of a semiconductor nanowire in a magnetic field placed on an ordinary superconductor [9–11]. The superconducting pair-potential is induced in the nanowire by proximity-effect from the s-wave superconductor (SC) in contact with the nanowire. It has been shown that such a nanowire can be driven into a TS phase with Majorana end modes [7, 10, 11]. The s-wave proximity effect on a InAs quantum wire, which also has a sizable SO coupling, may have already been realized in experiments[12]. Therefore, it seems that a MF-carrying TS state in a semiconductor quantum wire may be within experimental reach. This has created a great deal of interest in the physics communtiy [3, 4].

In this Letter, we introduce the new theoretical concept of a Majorana resonance, in contrast to the well-studied Majorana bound state. Such Majorana resonances, which is an excitation that is closely related to MFs and is likely occur in experimental set-ups designed to detect such MFs, do not manifest non-Abelian statistics. Here we consider directly the SC-semiconductor nanowire structures being currently explored in experiments [13], which have the schematic form shown in Fig. 1(a) and are similar to the experiment that demonstrated the gate-tunable proximity-effect in InAs nanowires [12]. We show that such structures [12, 13], would produce Majorana resonances rather than the non-Abelian MFs. We then introduce a periodic structure which would lead to true MFs as required for TQC.

Nanowire Hamiltonian: The Majorana end modes together with other quasiparticle excitations of the superconducting semiconductor nanowire system (Fig. 1)[9–11] are described by a Bogoliubov-de Gennes(BdG) Hamiltonian which can be written as

$$H_{BdG} = (-\frac{\hbar^2}{2m^*}\partial_y^2 - \mu(y))\tau_z + V_z\boldsymbol{\sigma}\cdot\hat{\boldsymbol{B}} + \imath\alpha\partial_y\hat{\boldsymbol{\rho}}\cdot\boldsymbol{\sigma}\tau_z + \Delta(y)\tau_x. \quad (1)$$

Here the unit vector $\hat{\boldsymbol{B}}$ gives the direction of the effective Zeeman field (V_Z), the unit vector $\hat{\boldsymbol{\rho}}$ characterizes the spin-orbit coupling (α) and m^* and $\Delta(y)$ are, respectively, the effective mass and proximity-induced pairing potential. The 2×2 Pauli matrices $\sigma_{x,y,z}$ and $\tau_{x,y,z}$ represent the spin and particle-hole degrees of freedom of the Bogoliubov quasiparticles. Such a semiconducting nanowire can be tuned across a phase transition separating a TS and non-topological superconducting (NTS) phase (i.e. an SC not containing any Majorana) simply by tuning either the chemical potentual μ via a gate voltage or the Zeeman splitting V_Z via an in-plane magnetic field. In the TS phase, which is reached by tuning μ and V_Z to satisfy $|\mu| < \mu_c = \sqrt{V_Z^2 - \Delta^2}$ [7, 9–11], such a nanowire supports a pair of Majorana zero energy modes (i.e. the non-Abelian MFs) at each end. The MFs, by virtue of their non-Abelian statistics, can be used for TQC [1, 18–22]. In fact, MFs have been shown to exist in nanowires which are not strictly one-dimensional and several bands are occupied [23, 24].

While the simplicity of the recent proposals for realizing MFs has attracted a significant experimental and theoretical effort, several experimental hurdles towards realization of the MFs remain. One of the key challenges is the requirement of control of the chemical potential μ

which has been realized in free-standing wires by the application of a gate voltage. Nanowires which form ohmic contacts with metals, instead of forming Schottky barriers, have a large density of electrons when in contact with the metal. Gating of such metallic nanowire segments that are directly in physical contact with an SC is ineffective because of strong electrostatic screening both from electrons in the underlying SC and the nanowire. Therefore, experimental attempts [12, 13] for the solid state realization of TS systems induce superconductivity in gated segments of nanowire by the proximity effect from adjacent superconducting nanowire segments (see Fig. 1(a)) that are directly above the SC. In this paper, that proposed signatures of MFs are qualitatively weakened by such experimentally reasonable adaptations of the original proposals allowing suitable gating of the nanowire. In fact, the current experimental structures lead to Majorana resonance states as the end modes in the wires. These resonances are propagating states in the semiconductor sides of the nanowires, and as such, are not bound states with non-Abelian properties! We propose a new periodic structure which should be ideal for the experimental MF realization.

The recently proposed topological insulator (TI) nanowires, [14] if realized in the absence of bulk dopants, can exist in a TS state over a large range of chemical potential μ and might avoid the generic complications discussed in this paper. However, it is likely that the discussion in this letter is relevant to the TI systems as well. The presence of bulk carries in the present experimental realization of TI nanowires, can be expected to lead to Majorana resonances as well. Moreover, the chemical potential μ needs to be reduced substantially for TI nanowires contacted to an SC so that the bulk states are not populated and more stringently, the minigap states are separated from the MFs by a substantial gap of Δ^2/μ [6, 25].

Zero-bias anomaly: The MFs that are predicted to occur in the TS phase of the semiconducting nanowire described by Eq. 1 are associated with zero-energy end modes. These zero-energy modes can in principle be detected as a zero-bias conductance peak in the tunneling current at the end of the wire[9, 15]. This proposal constitutes one of the simplest signatures to test the existence of MFs in SCs. This tunneling experiment may be realized without gating the nanowire directly above the SC by using the set-up shown in Fig. 1(a) where superconductivity is induced on a gated nanowire by the 1D proximity-effect from the SC on the left. A zero energy Majorana mode is expected to occur at the interface of the SC and gated region, if the right half of the nanowire is gated to be effectively in the TS phase (i.e. $\Delta^2 + \mu^2 < V_Z^2$) while the left half of the nanowire above the SC is in the NTS phase. Since, strictly speaking, the pairing potential Δ vanishes in the gated segment of the wire, we provide an alternative explanation for the

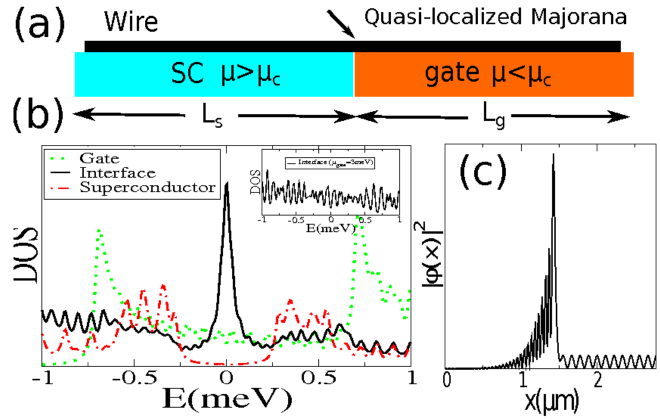

FIG. 1: (a) Geometry to observe zero-bias tunneling signature associated with MFs in gated semiconductor nanowire (thick black line) in the topological regime ($\mu < \mu_c$). Superconductivity is induced from the side of the gated nanowire region by the superconducting(blue box) layer on the left. (b) Tunneling density of states (DOS) of nanowire system shows broadened zero-bias tunneling peak at the interface for large Zeeman potential ($V_Z = 0.75$ meV $> \Delta_0 = 0.5$ meV), gate chemical potential $\mu_g = 0$ meV and spin-orbit energy $E_{SO} = \frac{m^* \alpha^2}{2} = 100\,\mu$eV. Inset shows that the near-zero-energy peak in the interface tunneling DOS goes away for a gate voltage in the non-topological regime $\mu_g = 1.0$ meV. The chemical potential μ_s directly next to the SC is taken to be large ($\mu_s = 5$ meV $\gg \mu_c \approx 0.5$ meV). (c) Zero-bias tunneling density of states shows quasilocalized states at interface that decays in SC and propagates in the gated region.

Majorana resonances below.

Majorana resonances: To quantitatively verify whether such a zero-bias peak exists in the set-up in Fig. 1(a) we calculate the tunneling density of states for the BdG Hamiltonian of a nanowire (Eq. 1) with periodic boundary conditions and length ($L = L_s + L_g$), such that the interval $0 < x < L_s$ is in contact with a SC (length L_s) and $L_s < x < L_s + L_g$ is gated (length L_g). The proximity-induced superconducting order parameter $\Delta(x)$ is taken to be $\Delta(x) = \Delta_0$ for $0 < x < L_s$ which is in direct contact with the SC [9, 25] and taken to be $\Delta(x) = 0$ otherwise. The chemical potential in the gated region can be controlled such that $\mu(x) = \mu_g < \mu_c = V_Z$. The chemical potential elsewhere is taken to be $\mu(x) = \mu_s \gg \mu_c$ since the electron density in the immediate vicinity of the SC could be large. The junction of the superconducting and gated regions at $x = L_s$ is expected to be analogous to the interface of nanowires in the TS phase (since $\mu_g < \mu_c$) and an NTS phase (since $\mu_s > \mu_c$) and thus is expected to support an MF [10, 11].

The tunneling conductance at a given position on the nanowire, x, and at a given bias voltage V relative to the SC, is proportional to the local density of states ($\rho(x; V) = \sum_{\sigma, E_n \approx V} |u_{n,\sigma}(x)|^2$) in the weak tunneling regime[9]. Here E_n and $(u_{n,\sigma}(x), v_{n,\sigma}(x))$ are the BdG eigenvalues and eigenstates respectively of the continuum BdG Hamiltonian in Eq. (1) which is calculated using a lattice approximation. The results for the tunneling calculations in the gated region ($L_s < x < L_s + L_g$), the interface ($x = L_s$) and the superconducting region ($0 < x < L_s$) are shown in Fig. 1(b). The tunneling density of states at the junction is peaked near zero-bias as predicted [9] for MFs in topological wires while the density of states in the superconducting region shows a characteristic superconducting gap and that in the gated region shows a uniform density of states characteristic of a metal. The inset in Fig. 1(b) shows that the Majorana resonance disappears for larger values of μ_g.

The density of states in Fig. 1(b) can be understood from the multi-channel picture of MFs in quasi-one dimensional wires [16, 23, 24]. For large magnetic fields, $V_Z > \Delta$, the wire above the superconductor in Fig. 1(a) behaves like a multi-channel p-wave superconductor. Some of these channels get terminated at the interface by the gate voltage. The termination of an odd number of channels leads to a Majorana bound state [16].

The peak associated with the MF is broadened in this set-up compared to previous predictions (where thermal broadening , neglected in our work, is the dominant contribution). This is a consequence of hybridization of the MFs with the gapless excitations in the gated nanowire which is not directly in contact with the SC. This is clear from Fig. 1(c), where the wave-function of the state is found to be propagating (i.e. not decaying) in the gapless gated segment of the nanowire. Therefore the broadening of the MF is entirely analogous to the broadened Fano resonance that occurs when a localized impurity is in contact with a bulk metal. Thus the MF here is a resonance, not a bound state - this is indeed a Majorana resonance!

Fractional Josephson effect: A definitive signature of MFs in TS nanowires is the fractional Josephson effect [16, 17]. The fractional Josephson effect not only probes the zero-energy character of the MFs but is also a signature of its non-Abelian statistics [16]. It has been shown that a semiconducting nanowire in the TS phase ($\mu < \mu_c$), placed on a superconducting ring in the geometry shown in Fig. 2(a), would show a current versus phase relation which is $2\Phi_0 = hc/e$ periodic in flux, Φ, instead of the conventional Φ_0 flux periodicity [10, 11]. The Andreev bound state (ABS) spectrum (E vs. Φ) in the junction (gap in Fig.2(a)), shown in the left panel of Fig.2(a), determines the current-phase relation of the junction. The calculated function $E(\Phi)$ for the BdG Hamiltonian in Eq. 1 for $\mu(x) < \mu_c$ and $\Delta(x) = \Delta_0$ shows the $2\Phi_0$ periodic $E(\Phi)$ in agreement with previous calculations [10, 11].

Absence of Josephson Fractionalization: Since it is difficult to apply a gate voltage to the nanowire directly above an SC, we consider a geometry (shown in Fig. 2(b)) so that $\mu(x) = \mu_g$ in the gated region inside the junction ($L_s < x < L_s + L_g$), while $\mu(x) = \mu_s > \mu_c$ in the nanowire segment in contact with the superconductor ($0 < x < L_s$). The Josephson effect geometry involves a small junction in a long superconductor, so we take the superconducting length $L_s \gg \xi$ where $\xi \sim 200$ nm is the coherence length of the superconductor. Therefore the ABS spectrum (shown in Fig. 2(b)) in the Josephson junction is periodic in the flux quantum Φ_0. For the modified geometry in Fig. 2(b), the ABS spectrum does not cross $E = 0$ at $\Phi = \Phi_0/2$. Therefore the corresponding current versus phase relation similarly only shows Φ_0 periodicity. The Josephson effect fractionalization signature for MF is destroyed unless the chemical potential in contact with the superconductor μ_s can be tuned such that $\mu_s < \mu_c$. This seems to be a disaster since the Majorana resonance would not manifest any non-Abelian statistics!

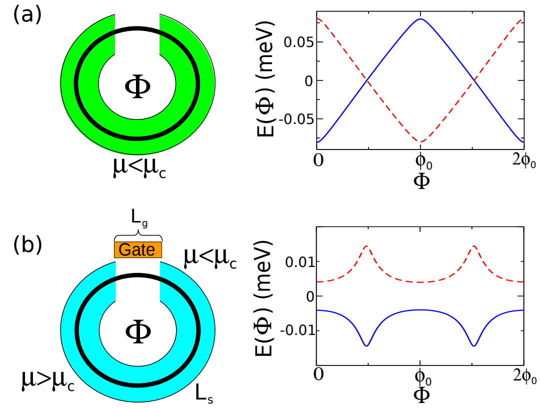

FIG. 2: (a) Junction in a ring topological superconductor structure with chemical potential control over entire structure (such that $\mu \sim 0$ meV $< \mu_c$) shows a fractional Josephson effect. Flux dependence of ABS energies and the corresponding Josephson current in junction show $2\Phi_0$ periodicity. (b) Experimental adaptations modify geometry so that $\mu_s > \mu_c$ in superconductor (length $L_s = 1.5\,\mu$m) with gate-induced chemical potential control only in junction ($\mu_g < \mu_c$) (length $L_g = 600$ nm). ABS spectrum show a conventional Josephson effect in this case despite tunneling signature of MFs in Fig. 1.

It is clear that simple structures where superconductivity is proximity-induced from adjacent nanowires cannot support true Majorana bound-state zero modes and instead support only Majorana resonances which cannot be used for TQC. These structures are thus at best 'half-topological-superconductors' (HTS) which can support Majorana resonance but not MF. In what follows, we propose a new TS structure that alternates between superconductor and gated regions (i.e. a periodic HTS system which is fundamentally inhomogeneous) that can realize true MFs with non-Abelian statistics.

Periodic topological invariants: The identification of spatially inhomogeneous topological structures requires the use of topological invariants that are more complex than the simple criterion ($\mu_g, \mu_s < \mu_c$) [27]. For structures that are periodic in space, the topological character of the superconductor can be identified by calculating a Pfaffian topological invariant [10, 16, 28]

$$Z = Sign(i^n Pf(H(k=0)\Lambda))Sign(i^n Pf(H(k=\pi)\Lambda)) \tag{2}$$

of the Bloch Hamiltonian $H(k)$ of the unit-cell of repetition. Here $\Lambda = \sigma_y \tau_y$ and k is the Bloch wave-vector along the periodic structure. Structures with $Z = -1$ are in a TS phase with MFs. In fact, the fractional Josephson effect also occurs in a ring-geometry (shown in Fig.2) if the corresponding periodic system has $Z = -1$. The periodic structure corresponding to the ring-geometry is obtained by cutting the ring structure at any point, straightening it out and then using this as the unit-cell of the periodic structure as shown in Fig. 3(a) and (c).

Fractional Josephson effect in rings with short superconductors: To realize a periodic structure which is topological it is necessary to find a system in a ring geometry that has a fractional Josephson effect. The fractional Josephson effect is attributed to quasiparticle tunneling around the ring as opposed to Cooper pair tunneling responsible for the conventional Josephson effect[16, 17]. Since Cooper pairs have electric charge $2e$, the energy levels associated with Cooper pairs in a ring are sensitive to flux in the ring with a period of $\frac{hc}{2e} = \Phi_0$. In contrast, the flux periodicity of quasiparticle (with charge e) energy levels is twice as large ($\frac{hc}{e} = 2\Phi_0$) . Therefore the $2\Phi_0$ flux periodicity, which defines the fractional Josephson effect and is characteristic of TS nanowires, is natural for the persistent current in non-superconducting mesoscopic rings [26]. This suggests that the fractional Joesephson effect in Fig. 2(b) can be restored in a ring geometry with a superconducting segment of length, L_s, that is smaller than the coherence length such that tunneling of quasiparticles around the ring is still allowed while maintaining a superconducting gap. This can be seen from the plots of the ABS energies E vs. Φ in Fig. 3(a). A small value of L_s (lower plot) realizes a $2\Phi_0$ flux-periodicity characteristic of the fractional Josephson effect, while consistent with Fig. 2 (b), large L_s yields only a conventional Josephson effect. As seen in Fig. 3(b) the presence of a superconducting segment, ensures that the structure is indeed gapped to quasiparticle excitations and is a *bona fide* TS. The resulting one-dimensional TS can be tuned across a TS-NTS transition using a gate voltage and supports true zero-energy MFs at TS-NTS boundaries. Therefore all the schemes [18–22] proposed for TQC using TS nanowires [10, 11] can be adapted to implement TQC in the proposed structure.

Phase-diagram and uneven structures: The dependence of the TS properties of the periodic HTS structure in Fig. 3(a) on the parameters L_s, μ_g is shown in Fig. 3(e). The color in the phase diagram in Fig. 3(e) only indicates the gap in the TS phase so that the dark

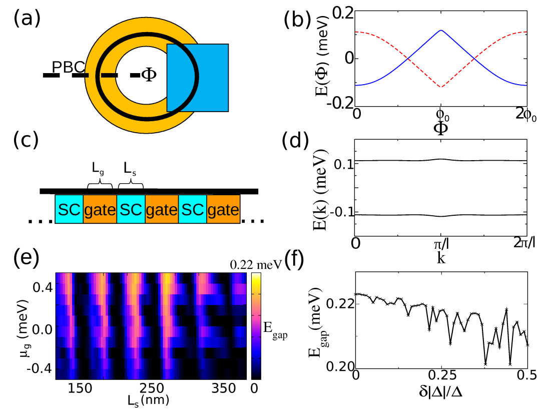

FIG. 3: (a) Short-superconductor geometry ($L_s \lesssim \xi \sim 200$ nm compared to Fig. 2(b)) to obtain fractional Josephson effect. (b) Corresponding ABS spectrum shows $2\Phi_0$ periodicity in current for ($L_s = 100$ nm, $L_g = 350$ nm and $\mu_g = 0$ meV). (c) Periodic HTS structure corresponding to (a) is in TS phase because of fractional Josephson effect. (d) Band gap versus wave-vector in TS phase of periodic HTS structure. (e) Minimum band-gap in TS (with $Z = -1$ shown as bright) phase as a function of L_s and μ_g. The dark regions are either non-topological (i.e. $Z = 1$) or have small band gaps. (f) Minimum band-gap as a function of proximity-induced Δ magnitude fluctuations.

regions are either nearly gapless or non-topological in nature. The phase diagram suggests that to obtain an optimal TS one must engineer structures with the appropriate values of (L_s, μ_g). The oscillatory appearance of the topological phase in Fig. 3(e) is a result of the periodic L_s-dependence of the relative phases between the normal and Andreev transmission across the superconducting nanowire segments. Present experimental techniques allow the fabrication of structures with reasonably accurate feature length control. However the tunneling induced proximity effect, which depends on tunneling strength, can vary. Our calculation of the topological gap for an optimal parameter set(shown in Fig. 3(f)) indicates that the topological gap is robust to such disorder.

Conclusion: Experimental set-ups for realizing MFs and non-Abelian statistics in semiconductor nanowires are likely to be restricted to geometries where the chemical potential μ is only tunable in free-standing segments of the nanowire. We have found that this restriction qualitatively affects two of the simplest proposals for detecting MFs and forbids the realization of true non-Abelian MFs instead producing Majorana resonance modes. We resolve this problem by proposing a spatially inhomogenous periodic structure that can be engineered to support MFs at its ends. Our work thus resolves a key difficulty in the solid state realization of the MFs by providing an experimentally realizable architecture where the conflicting dichotomy of semiconductor gating and superconductor proximity effects could coexist harmoniously leading to non-Abelian particles potentially capable of carrying out

TQC. We have introduced in this work the new concept of a Majorana resonance mode which is generically present in the currently studied superconducting-semiconductor structures [7, 10, 11] and topological insulator systems [6, 14].

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* Present address: Department of Physics, Harvard University, Cambridge, Massachusetts.

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