Residual Symmetries for Neutrino Mixing with a Large θ_{13} and Nearly Maximal δ_{D}

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Residual Symmetries for Neutrino Mixing with a Large θ_{13} and Nearly Maximal δ_D

Shao-Feng Ge[1,*], Duane A. Dicus[2,†], and Wayne W. Repko[3,‡]

[1] *Institute of Modern Physics and Center for High Energy Physics, Tsinghua University, Beijing 100084, China*

[2] *Physics Department, University of Texas, Austin, TX 78712*

[3] *Department of Physics and Astronomy, Michigan State University, East Lansing MI 48824*

The residual $\mathbb{Z}_2^s(k)$ and $\overline{\mathbb{Z}}_2^s(k)$ symmetries induce a direct and unique phenomenological relation with $\theta_x(\equiv\theta_{13})$ expressed in terms of the other two mixing angles, $\theta_s(\equiv\theta_{12})$ and $\theta_a(\equiv\theta_{23})$, and the Dirac CP phase δ_D. $\mathbb{Z}_2^s(k)$ predicts a θ_x probability distribution centered around $3^\circ \sim 6^\circ$ with an uncertainty of 2° to 4° while those from $\overline{\mathbb{Z}}_2^s(k)$ are approximately a factor of two larger. Either result fits the T2K, MINOS and Double Chooz measurements. Alternately a prediction for the Dirac CP phase δ_D results in a peak at $\pm 74^\circ$ ($\pm 106^\circ$) for $\mathbb{Z}_2^s(k)$ or $\pm 123^\circ$ ($\pm 57^\circ$) for $\overline{\mathbb{Z}}_2^s(k)$ which is consistent with the latest global fit. We also give a distribution for the leptonic Jarslkog invariant J_ν which can provide further tests from measurements at T2K and NOνA.

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Introduction – The T2K [1] and MINOS [2] experiments indicate a relatively large reactor angle θ_x for neutrino mixing. At the 90% C.L., T2K gives $0.03\,(0.04) < \sin^2 2\theta_x < 0.28\,(0.34)$, with zero Dirac CP phase, δ_D, for normal (inverted) hierarchy while MINOS gives $0.01\,(0.026) < \sin^2 2\theta_x < 0.088\,(0.150)$ and Double Chooz [3] with $\sin^2 2\theta_{13} = 0.085 \pm 0.051$ at 68% C.L..

Many varied theoretical efforts have been made to understand this large θ_x. Discrete groups such as S_3 [4], A_4 [5, 6], S_4 [6–8], and the binary tetrahedral group T' [9] have been quite popular while new possibilities are explored in [10]. Other efforts concentrate on perturbations from some featured zeroth-order mixing such as democratic [11, 12], bimaximal [8, 12, 13], tribimaximal [6, 13, 14], and tetra-maximal [15] patterns. More discussions can be found in [16].

In these papers symmetries or other model assignments are employed. We will show that phenomenological consequences of residual symmetries $\mathbb{Z}_2^s(k)$ and $\overline{\mathbb{Z}}_2^s(k)$ can be readily established predicting not only θ_x to be large, fitting the T2K, MINOS and Double Chooz data, but also δ_D nearly maximal in good agreement with the latest global fits. This provides the first strong and direct evidence for residual symmetries.

Residual Symmetries – The symmetry that directly determines the lepton mixing pattern need not be the same as the full symmetry of the fundamental lagrangian. As the left-handed charged lepton and neutrino reside in a same $SU(2)_L$ doublet, they are governed by a common symmetry which must be broken. Otherwise they would share a same diagonalization matrix [17, 18], leading to trivial leptonic mixing. It is the residual symmetry that determines the mixing matrices, if indeed the mixing is believed to be determined by symmetry.

*Electronic address: gesf02@gmail.com

†Electronic address: dicus@physics.utexas.edu

‡Electronic address: repko@pa.msu.edu

It is convenient to work in the diagonal basis of charged leptons [19]. To completely determine the mixing matrix, a product of two $\mathbb{Z}_2$ symmetries is enough [18, 20]. One is the well-known μ–τ symmetry [21] and the other is $\mathbb{Z}_2^s$ [18] which can be extended to accommodate a general solar angle [22] generated by,

$$G_1(k) = \frac{1}{2+k^2}\begin{pmatrix} 2-k^2 & 2k & 2k \\ 2k & k^2 & -2 \\ 2k & -2 & k^2 \end{pmatrix}. \tag{1}$$

There is another residual $\overline{\mathbb{Z}}_2^s(k)$ represented by $G_2 \equiv G_1 G_3$, where G_3 is the matrix for μ–τ symmetry [18],

$$G_2(k) = \frac{1}{2+k^2}\begin{pmatrix} 2-k^2 & 2k & 2k \\ 2k & -2 & k^2 \\ 2k & k^2 & -2 \end{pmatrix}. \tag{2}$$

Since μ–τ symmetry is just a first order approximation, indicated by the experimental data [1–3] and other considerations [21], it has to be broken. The remaining symmetry would be $\mathbb{Z}_2^s(k)$ or $\overline{\mathbb{Z}}_2^s(k)$ but not both as they are not independent. However, their phenomenological consequences need not be the same as we show below. Note that, since the diagonal mass matrix of charged leptons is not degenerate, G_1 and G_2 only apply to the neutrino sector after the full symmetry is broken down to residual symmetries.

Correlation Between Mixing Angles – With a single $\mathbb{Z}_2^s(k)$, a correlation between the three mixing angles and the Dirac CP phase can be derived. In particular,

$$\cos\delta_D = \frac{(s_s^2 - c_s^2 s_x^2)(c_a^2 - s_a^2)}{4 c_a s_a c_s s_s s_x}, \tag{3a}$$

$$\cos\delta_D = \frac{(s_s^2 s_x^2 - c_s^2)(c_a^2 - s_a^2)}{4 c_a s_a c_s s_s s_x}, \tag{3b}$$

for $\mathbb{Z}_2^s(k)$ [23] and $\overline{\mathbb{Z}}_2^s(k)$ respectively. Note that only physical quantities are involved which gives the possibility of robust physical predictions. By implementing the measured values of the three mixing angles, a prediction of δ_D can be made. Or, (3) can be solved for θ_x,

$$\sin\theta_x = p\left[\pm\sqrt{c_D^2 + \cot^2 2\theta_a} - c_D\right]\tan 2\theta_a (\tan\theta_s)^p, \quad (4)$$

with $c_D \equiv \cos\delta_D$ while $p = \pm 1$ for $\mathbb{Z}_2^s(k)$ or $\overline{\mathbb{Z}}_2^s(k)$. Solutions for the $\pm$ sign within the parenthesis are equivalent through a redefinition $(\theta_x, \delta_D) \to (-\theta_x, \delta_D + \pi)$ leaving no effect on the measured physical quantity $\sin^2\theta_x$. This is also true for the overall p. The difference comes from the exponent p leading to a $(\tan\theta_s)^2 \approx 1/2$ factor between the $\mathbb{Z}_2^s(k)$ and $\overline{\mathbb{Z}}_2^s(k)$ predictions.

The main feature of (4) can be seen by expanding it to the leading order. As the reactor angle $\theta_x \equiv \delta_x$ is small and the atmospheric angle $\theta_a \equiv 45^\circ + \delta_a$ is nearly maximal, (3) reduces to,

$$\frac{\delta_x}{\delta_a} = -p\frac{(\tan\theta_s)^p}{\cos\delta_D}\,. \quad (5)$$

Eqs.(3)–(5) are general and direct. To demonstrate this, three examples are provided. Eq.(5) was first obtained in a minimal seesaw model [22] with μ–τ and CP softly broken and $\mathbb{Z}_2^s(k)$ retained exactly, befitting the situation discussed here. A special case with $k = 2$, which constrains the mixing matrix to be trimaximal, is studied in [6]. Even an "unphysical" bimaximal solution [24] can be covered as a marginal example. Note that the first two examples are obtained in model-dependent and perturbative ways while the last one comes from a pure symmetry analysis.

The ratio (5) of the deviation of the reactor angle from zero and that of the atmospheric angle from maximal is given by the solar angle and the CP phase. Its absolute value is a minimum when δ_D equals 0 or π, $|\delta_x| \geq (\tan\theta_s)^p|\delta_a|$. Alternately (3) can be solved exactly with $\cos\delta_D = \pm 1$ to give an absolute lower bound $\sin\theta_x \geq (\tan\theta_s)^p|c_a - s_a|/|c_a + s_a|$. An upper bound can also be obtained but since $c_a \approx s_a$ it is larger than 1.

Numerical Predictions – A nonzero θ_x has been consistent with global fits for several years. The first hint appears in [25] at only 0.9σ C.L.. It persists in all subsequent global fits [26–30] and increases steadily to about 3σ [31, 32] as summarized in Table I.

	$\sin^2\theta_s$ (θ_s)	$\sin^2\theta_a$ (θ_a)	$\sin^2\theta_x$ (θ_x)
Best Fit	0.306 (33.6°)	0.42 (40.4°)	0.021 (8.3°)
1σ Range	0.291-0.324 (32.7-34.7°)	0.39-0.50 (38.7-45.0°)	0.013-0.028 (6.6-9.6°)

TABLE I: The global fit [32] for the neutrino mixing angles.

The fits can be classified into two catagories depending on the result for the atmospheric angle θ_a which is persistently maximal in [27, 28, 30, 31] while an apparent deviation from 45° is claimed possible in [26, 29, 32] due to a subleading effect [33].

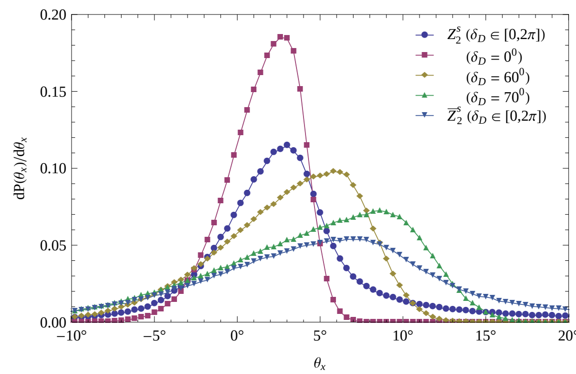


FIG. 1: (Color Online) Predicted distributions of θ_x.

From (4) the distribution of θ_x can be derived by using asymmetric Gaussian distributions $\mathbb{P}$ as,

$$\frac{dP(\theta_x)}{d\theta_x} = \int f_x^p \mathbb{P}(s_a^2)\mathbb{P}(s_s^2) ds_a^2 ds_s^2 \frac{d\delta_D}{2\pi}, \quad (6)$$

where $f_x^p \equiv \frac{1}{2}\delta(\theta_x - \arcsin\bar{s}_x)$ are δ-functions that pick out the predicted value $\bar{s}_x \equiv$ RHS of (4) for θ_x, given a concrete input of p, s_a^2, s_s^2, and δ_D. There is also a mirror contribution for negative θ_x that is not shown – hence the $\frac{1}{2}$ prefactor. We take δ_D to be evenly distributed in $[0, 2\pi)$, as in (6), or replace it with a specific value. The integration (6) can be simulated with scattering points or the delta function can be converted to one for θ_s and the other integrals done numerically. The results are shown in Fig. 1 for both $\mathbb{Z}_2^s(k)$ and $\overline{\mathbb{Z}}_2^s(k)$. We will first discuss the results from $\mathbb{Z}_2^s(k)$.

After averaging over δ_D, the probability distribution peaks around 3° with an asymmetric width from 2° to 4°. This is in sharp contrast with the distribution given by the previous global fit [23], which peaks at 0°. From (5) we can see that δ_x is proportional to δ_a. With θ_a significantly deviating from the maximal value, the predicted δ_x must increase accordingly. In the global fit adopted in [23], the central value of θ_a is about 43° and the maximal value is well within the 1σ range. Hence, there is no apparent nonzero peak in the predicted distribution of θ_x. For the latest global fit [32] of θ_a the central value is about 40.4°, while the maximal value is at the edge of the 1σ region. This significant change in θ_a leads to a clear nonzero prediction of θ_x. As θ_s only contributes as an overall factor in (5), its deviation will not change the conclusion for θ_x. For example, a different treatment of the reactor data leads to 3.7% difference [30] in $\tan\theta_s$. The best fit values of $\sin^2\theta_x(\theta_x)$ vary from 0.02 (8.1°) to 0.04 (11.5°) which are still covered by our predictions. This is also treated in [32] but with a much smaller variation, approximately 20% in $\sin^2\theta_x$.

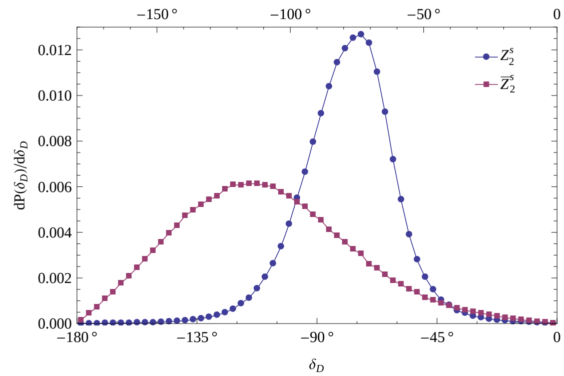
The measured θ_x [1–3] is not independent of the Dirac CP phase δ_D. But this does not affect the matching between the experimental result and the theoretical predictions. Figure 1 shows that averaging over possible δ_D values gives a best fit value and deviation which resemble those with vanishing δ_D. As the experimental fit of θ_x depends only slightly on δ_D while the theoretical prediction is sensitive to it, as shown in (5), varying δ_D can effectively improve the matching. For example, using $\delta_D = 60^\circ$ moves the peak to the MINOS and Double Chooz central values while $\delta_D = 70^\circ$ for that of T2K.

Since $|\delta_x| \geq |\delta_a| \tan\theta_s$, the prediction for θ_x with vanishing δ_D is the most conservative in the sense that it gives the smallest prediction for θ_x. The prediction with δ_D uniformly distributed also peaks at 3° but has an extended tail to higher θ_x. This is because, while δ_D is uniformly distributed, $\cos\delta_D$ is not. Its distribution varies as $(\sin\delta_D)^{-1}$ which is relatively suppressed for small $\cos\delta_D$. Thus, the most conservative region is the most probable one. For example, the probability for $|\cos\delta_D| \leq 0.1\,(0.2, 0.3, 0.4)$ is just 6% (13%, 19%, 26%) corresponding to $\delta_D = 84^\circ\,(78^\circ, 73^\circ, 66^\circ)$ respectively. Most of the significant region lies between $\delta_D = 0^\circ$ and approximately $\delta_D = 60^\circ$. Within this region, the θ_x peak varies from approximately 3° to around 6° and the width changes from roughly $2^\circ \sim 4^\circ$ to almost $4^\circ \sim 8^\circ$. This is the region covered by MINOS result $2.9^\circ(4.6^\circ) < \theta_x < 8.6^\circ(11.4^\circ)$ and $5.3^\circ < \theta_x < 10.8^\circ$ for Double Chooz at 1σ level while T2K has $5.0^\circ(5.8^\circ) < \theta_x < 16.0^\circ(17.8^\circ)$ at 90% C.L..

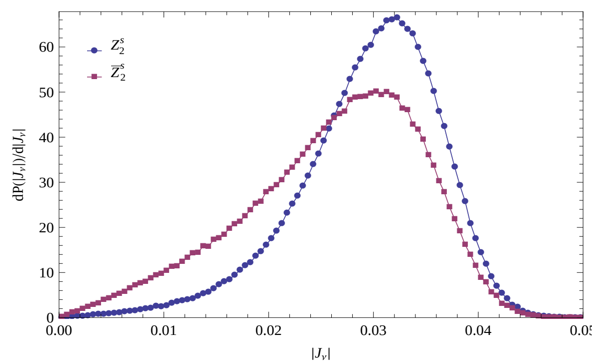
The above discussion also applies to the case of $\overline{\mathbb{Z}}_2^s(k)$. The only difference is the factor of about 2 coming from the exponent p in (4). As $\theta_x < 10^\circ$ can be treated as small perturbation, this will induce approximate factors of 2 in the peak location and 1/2 in its height relative to the predictions from $\mathbb{Z}_2^s(k)$. The result is still in good agreement with the data and the global fits.

This consistency between the data and our prediction of a large θ_x provides the first nontrivial indication of the viability of residual symmetries $\mathbb{Z}_2^s(k)$ or $\overline{\mathbb{Z}}_2^s(k)$. The correlation between the mixing angles (3) is independent of the group parameter k, and is obtained in a direct way, making the result quite robust.

The change in the global fit also alters our prediction of the Dirac CP phase δ_D [23]. As shown in Fig. 2(a) the most probable value of δ_D is no longer maximal. This is also caused by the shifted central value of θ_a. As δ_a deviates further from $\frac{\pi}{4}$, maximal δ_D becomes less probable as indicated by (3). Instead it peaks around $\pm 74^\circ$ for $\mathbb{Z}_2^s$. Notice that a mirror solution in (4) can be obtained through $(\theta_x, \delta_D) \to (-\theta_x, \delta_D + \pi)$ generating another peak around $\pm 106^\circ$. These are in perfect consistency with the indication of $-74^\circ(-110^\circ)$ for inverted (normal) hierarchy [31]. Although no concrete number is provided, a nonzero CP phase also appears in [32]. For $\overline{\mathbb{Z}}_2^s(k)$, the predicted $\cos\delta_D$ (3b) is larger than (3a) by a factor of 2. Consequently, the peak moves to around $\pm 123^\circ$ ($\pm 57^\circ$).



(a) Dirac CP Phase δ_D



(b) Jarlskog Invariant J_ν

FIG. 2: (Color Online) Predicted distributions of (a) the Dirac CP phase δ_D and (b) Jarlskog Invariant J_ν.

The distribution of the leptonic Jarlskog invariant J_ν is shown in Fig. 2(b). These predictions can be tested at T2K [34] and at NOνA [35].

Conclusions – Phenomenological consequences of the residual $\mathbb{Z}_2^s(k)$ and $\overline{\mathbb{Z}}_2^s(k)$ symmetries are compared with data and global fits. Although not independent, their predictions are different. A large reactor angle θ_x peaking around 3° or 6° which is consistent with T2K, MINOS and Double Chooz can be obtained and the Dirac CP phase δ_D has peaks at $\pm 74^\circ$ ($\pm 106^\circ$) or $\pm 123^\circ$ ($\pm 57^\circ$) in excellent agreement with the latest global fits. This provides the first strong and direct support for $\mathbb{Z}_2^s(k)$ and $\overline{\mathbb{Z}}_2^s(k)$ as residual symmetries of neutrino mixing. Further confirmation may come from the measurement of the leptonic Jarlskog invariant J_ν at T2K or NOνA.

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[1] T2K, K. Abe *et al.* [arXiv:1106.2822].
[2] MINOS, P. Adamson et al. [arXiv:1108.0015].
[3] Talk given by H. De Kerrect at LowNu2011 workshop, http://workshop.kias.re.kr/lownu11/
[4] S. Zhou, [arXiv:1106.4808].
[5] E. Ma and D. Wegman, [arXiv:1106.4269]; A. Adulpravitchai and R. Takahashi, [arXiv:1107.3829].
[6] S. F. King and C. Luhn, [arXiv:1107.5332].
[7] H. Ishimori and T. Kobayashi, [arXiv:1106.3604]; Y. Daikoku, H. Okada and T. Toma, [arXiv:1106.4717]. P. S. B. Dev, R. N. Mohapatra and M. Severson, [arXiv:1107.2378].
[8] D. Meloni, [arXiv:1107.0221].
[9] D. A. Eby, P. H. Frampton and S. Matsuzaki, Phys. Lett. B **671**, 386 (2009) [arXiv:0810.4899]; A. Aranda, Phys. Rev. D **76**, 111301 (2007) [arXiv:0707.3661]; A. Aranda, C. Bonilla, R. Ramos, A. D. Rojas, [arXiv:1011.6470]; M. C. Chen and K. T. Mahanthappa, [arXiv:1107.3856].
[10] A. Aranda, C. Bonilla, R. Ramos and A. D. Rojas, Phys. Rev. D **84**, 016009 (2011) [arXiv:1105.6373]; R. d. A. Toorop, F. Feruglio and C. Hagedorn, [arXiv:1107.3486]; Q. H. Cao, S. Khalil, E. Ma and H. Okada, [arXiv:1108.0570].
[11] Z. Z. Xing, [arXiv:1106.3244].
[12] W. Chao and Y. J. Zheng, [arXiv:1107.0738].
[13] D. Marzocca, S. T. Petcov, A. Romanino and M. Spinrath, [arXiv:1108.0614].
[14] X. G. He and A. Zee, [arXiv:1106.4359]; T. Araki, [arXiv:1106.5211]; S. Morisi, K. M. Patel and E. Peinado, [arXiv:1107.0696]; S. Dev, S. Gupta and R. R. Gautam, [arXiv:1107.1125]; Y. H. Ahn, H. Y. Cheng and S. Oh, [arXiv:1107.4549].
[15] H. Zhang and S. Zhou, [arXiv:1107.1097].
[16] H. J. He and F. R. Yin, [arXiv:1104.2654]; J. M. Chen, B. Wang and X. Q. Li, [arXiv:1106.3133]; N. Qin and B. Q. Ma, [arXiv:1106.3284]; J. E. Kim and M. S. Seo, [arXiv:1106.6117]; Y. j. Zheng and B. Q. Ma, [arXiv:1106.4040]; S. N. Gninenko, [arXiv:1107.0279]; N. Haba and R. Takahashi, [arXiv:1106.5926]; S. F. King, [arXiv:1106.4239]; A. B. Balantekin, [arXiv:1106.5021]; X. Chu, M. Dhen and T. Hambye, [arXiv:1107.1589]; G. Altarelli, [arXiv:1107.1980]; S. Antusch and V. Maurer, [arXiv:1107.3728]; W. Rodejohann, H. Zhang and S. Zhou, [arXiv:1107.3970]; T. W. Kephart, P. Leser and H. Pas, [arXiv:1106.6201].
[17] C. S. Lam, Phys. Rev. D **71**, 093001 (2005) [arXiv:hep-ph/0503159].
[18] C. S. Lam, Phys. Rev. Lett. **101**, 121602 (2008) [arXiv:0804.2622]; C. S. Lam, Phys. Rev. D **78**, 073015 (2008) [arXiv:0809.1185].
[19] D. A. Dicus, S. F. Ge and W. W. Repko, Phys. Rev. D **82**, 033005 (2010) [arXiv:1004.3266].
[20] W. Grimus, L. Lavoura and P. O. Ludl, J. Phys. G **36**, 115007 (2009) [arXiv:0906.2689 [hep-ph]].
[21] R. N. Mohapatra and S. Nussinov, Phys. Rev. D **60**, 013002 (1999) [hep-ph/9809415]; C. S. Lam, Phys. Lett. B **507**, 214 (2001) [hep-ph/0104116].
[22] S. F. Ge, H. J. He and F. R. Yin, JCAP **1005**, 017 (2010) [arXiv:1001.0940].
[23] S. F. Ge, D. A. Dicus and W. W. Repko, Phys. Lett. B **702**, 220 (2011) [arXiv:1104.0602].
[24] C. S. Lam, [arXiv:1105.5166].
[25] A. B. Balantekin and D. Yilmaz, J. Phys. G **35**, 075007 (2008) [arXiv:0804.3345].
[26] G. L. Fogli, E. Lisi, A. Marrone, A. Palazzo and A. M. Rotunno, Phys. Rev. Lett. **101**, 141801 (2008) [arXiv:0806.2649].
[27] T. Schwetz, M. A. Tortola and J. W. F. Valle, New J. Phys. **10**, 113011 (2008) [arXiv:0808.2016].
[28] M. Maltoni and T. Schwetz, PoS **IDM2008**, 072 (2008) [arXiv:0812.3161].
[29] M. C. Gonzalez-Garcia, M. Maltoni and J. Salvado, JHEP **1004**, 056 (2010) [arXiv:1001.4524].
[30] T. Schwetz, M. Tortola and J. W. F. Valle, New J. Phys. **13**, 063004 (2011) [arXiv:1103.0734];
[31] T. Schwetz, M. Tortola and J. W. F. Valle, [arXiv:1108.1376].
[32] G. L. Fogli, E. Lisi, A. Marrone, A. Palazzo and A. M. Rotunno, [arXiv:1106.6028].
[33] G. L. Fogli, E. Lisi, A. Marrone and A. Palazzo, Prog. Part. Nucl. Phys. **57**, 742 (2006) [hep-ph/0506083].
[34] T2K, Y. Itow *et al.* [hep-ex/0106019].
[35] NOνA, D. S. Ayres *et al.* [hep-ex/0503053]; O. Mena, S. Palomares-Ruiz and S. Pascoli, Phys. Rev. D **73**, 073007 (2006) [hep-ph/0510182]; D. S. Ayres *et al.*, "*The NOνA Technical Design Report*."