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Amir H. Safavi-Naeini, Jasper Chan, Jeff T. Hill, Thiago P. Mayer Alegre, Alex Krause, and Oskar Painter

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Observation of quantum motion of a nanomechanical resonator

Amir H. Safavi-Naeini,¹ Jasper Chan,¹ Jeff T. Hill,¹ Thiago P. Mayer Alegre,¹ Alex Krause,¹ and Oskar Painter^{1,*}

¹Thomas J. Watson, Sr., Laboratory of Applied Physics, California Institute of Technology, Pasadena, CA 91125

In this work we use resolved sideband laser cooling to cool a mesoscopic mechanical resonator to near its quantum ground state (phonon occupancy 2.6 ± 0.2), and observe the motional sidebands generated on a second probe laser. Asymmetry in the sideband amplitudes provides a direct measure of the displacement noise power associated with quantum zero-point fluctuations of the nanomechanical resonator, and allows for an intrinsic calibration of the phonon occupation number.

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Experiments with trapped ions and neutral atoms [1-3], dating back several decades, utilized techniques such as resolved sideband laser cooling and motional sideband absorption and fluorescence spectroscopy to cool and measure a single trapped particle in its vibrational quantum ground state. These experiments generated significant interest in the coherent control of motion and the quantum optics of trapped atoms and ions[4], and were important stepping stones towards the development of ion-trap based quantum computing [5, 6]. Larger scale mechanical objects, such as fabricated nanomechanical resonators, have only recently been cooled close to their quantum mechanical ground state of motion [7-14]. In a pioneering experiment by O'Connell, et al. [11], a piezoelectric nanomechanical resonator has been cryogenically cooled $(T_b \sim 25 \text{ mK})$ to its vibrational ground state and strongly coupled to a superconducting circuit qubit allowing for quantum state preparation and read-out of the mechanics. An alternate line of research has been pursued in circuit and cavity optomechanics [15], where the position of a mechanical oscillator is coupled to the frequency of a high-Q electromagnetic resonance allowing for back-action cooling [16, 17] and continuous position read-out of the oscillator. Such optomechanical resonators have long been pursued as quantum-limited sensors of weak classical forces [9, 15, 18-20], with more recent studies exploring optomechanical systems as quantum optical memories and amplifiers [21-24], quantum nonlinear dynamical elements [25], and quantum interfaces in hybrid quantum systems [26-29].

Despite the major advances in circuit and cavity optomechanical systems made in the last few years, all experiments to date involving the cooling of mesoscopic mechanical oscillators have relied on careful measurement and calibration of the motionally scattered light to obtain the average phonon occupancy of the oscillator, $\langle n \rangle$. Approach towards the quantum ground state in such experiments is manifest only as a weaker measured signal, with no evident demarcation between the classical and quantum regimes of the oscillator. A crucial aspect of zero-point fluctuations of the quantum ground state is that they cannot supply energy, but can only contribute to processes where energy is absorbed by the mechanics. This is different from classical noise, and techniques that attempt to measure zero-point motion without being sensitive to this



FIG. 1: (a) A scanning electron micrograph of the silicon nanobeam optomechanical cavity. Finite-element-method (FEM) numerical simulations of the electric field amplitude of the (b) first- and (c) second-order optical modes of the cavity which are used for cooling and probing the mechanical motion, respectively. (d) FEM numerical simulation showing the displacement amplitude of the coupled breathing mechanical mode.

aspect (i.e., standard continuous linear position detection) can always be interpreted classically and described by some effective temperature.

A more direct method of thermometry and characterization of the quantized nature of a mechanical oscillator, one particularly suited to small $\langle n \rangle$ and utilized in the above-mentioned trapped atom experiments [1-3], is referred to as motional sideband spectroscopy. This method relies on the fundamental asymmetry in the quantum processes of phonon absorption from (proportional to $\langle n \rangle$) and emission into (proportional to $\langle n \rangle + 1$) the mechanical oscillator. In the case of atomic systems, this asymmetry can be measured in the motionallygenerated Stokes and anti-Stokes sidebands in either the fluorescence or absorption spectrum of the atom. The ratio of the Stokes to anti-Stokes sideband amplitudes $((\langle n \rangle + 1)/\langle n \rangle)$ deviates significantly from unity as the quantum ground state is reached ($\langle n \rangle \rightarrow 0$), and provides a self-calibrated reference for the phonon occupancy. In the present experiment we cool a nanomechanical resonator to near its quantum ground state, and measure the asymmetry in the motional sidebands utiliz-



FIG. 2: Schematic of the experimental set-up. Two narrowband lasers (linewidth \sim 300 kHz) are used to independently cool and read out the motion of the breathing mechanical mode of the OMC cavity. The 1500 nm (read-out) and 1400 nm (cooling) laser beams are passed through variable optical attenuators (VOAs) to set the laser power, and combined at a wavelength multiplexer (λ -MUX) before being sent into the cryostat through an optical fiber. Transmission of the 1500 nm read-out beam through the OMC cavity, collected at the output end of the optical fiber, is filtered from the 1400 nm cooling beam light via a bandpass filter, pre-amplified by an Erbium-doped fiber amplifier (EDFA), and detected on a high-speed photodetector (PD2) connected to a real-time spectrum analyzer (RSA). An optical wavemeter (λ -meter) is used to monitor both the cooling and readout laser frequencies. The optical reflection from the cavity is used to perform EIT-like spectroscopy [22] on both the read-out and cooling cavity modes. Other components are: amplitude-modulation (a-m) and phase-modulation (ϕ -m) electro-optic modulators, fiber polarization controller (FPC), swept frequency radio-frequency signal generator (rf-sg), lock-in amplifier (lock-in), and optical switches (SW).

ing a form of resolved sideband spectroscopy based upon the filtering properties of a high-*Q* optical cavity with linewidth narrower than the mechanical frequency.

The cavity optomechanical system studied in this work consists of a patterned silicon nanobeam which forms an optomechanical crystal (OMC) [30] capable of localizing both optical and acoustic waves (see Fig. 1). The cavity is designed to have two optical resonances, one for cooling and one for read-out of mechanical motion. The cooling mode is chosen as the fundamental mode of the patterned nanobeam cavity, with a frequency $\omega_c/2\pi = 205.3$ THz and a corresponding free-space wavelength of $\lambda_c = 1460$ nm. The read-out mode is the second-order mode of the cavity with $\omega_r/2\pi = 194.1$ THz ($\lambda_r = 1545$ nm). An in-plane mechanical breathing mode at $\omega_m/2\pi = 3.99$ GHz, confined at the center of the nanobeam due to acoustic Bragg reflection, couples via radiation pressure to both optical resonances.

An illustration of the experimental apparatus used to cool and measure the OMC nanomechanical oscillator is shown in Fig. 2. In order to pre-cool the oscillator, the silicon sample is mounted inside a Helium flow cryostat. For a sample mount temperature of 6.3 K, the thermal bath temperature of the mechanical mode is measured to be 18 K (thermal phonon occupation of $n_b = 94$ phonons) through optical measurements described below. At this temperature the breathing mode damping rate to the thermal bath is found to be $\gamma_i/2\pi = 43$ kHz. The optical resonances of the OMC cavity are measured to have total damping rates of $\kappa_c/2\pi = 390$ MHz and $\kappa_r/2\pi = 1.0$ GHz for the cooling and read-out modes, respectively. An optical fiber taper is used to evanescently couple light to and from the OMC cavity. Utilizing piezoelectric stages, the taper is positioned to the side of the nanobeam cavity and placed in contact with the surface of the silicon microchip surrounding the suspended nanobeam. In this scheme, the fiber taper runs approximately parallel to the nanobeam, and can be rigidly mounted at a prescribed nanoscale gap from the nanobeam. For the taperto-nanobeam gap used here (≤ 200 nm), the coupling rate to the fiber taper waveguide is approximately $\kappa_{e,c}/2\pi = 46$ MHz for the cooling mode and $\kappa_{e,r}/2\pi = 300$ MHz for the read-out mode.

A Hamiltonian describing the coupled OMC cavity system is given by $\hat{H} = \hbar (\omega_r + g_r \hat{x}/x_{zpf}) \hat{a}^{\dagger} \hat{a} + \hbar (\omega_c + g_c \hat{x}/x_{zpf}) \hat{c}^{\dagger} \hat{c} + \hbar \omega_m \hat{b}^{\dagger} \hat{b}$, where $\hat{c} (\hat{c}^{\dagger})$ and $\hat{a} (\hat{a}^{\dagger})$ are the annihilation (creation) operators for photons in the cooling and read-out modes, respectively, and $\hat{x} \equiv x_{zpf}(\hat{b}^{\dagger} + \hat{b})$ is the displacement operator of the breathing mode with $\hat{b}^{\dagger} (\hat{b})$ the phonon creation (annihilation) operator. x_{zpf} , the mode's zero-point fluctuation amplitude, is estimated to be 2.7 fm from FEM simulations. The zero-point optomechanical coupling rates are determined from measurements of the optically-induced damping of the mechanical mode [13] to be $g_c/2\pi = 960$ kHz and $g_r/2\pi = 430$ kHz for the cooling and read-out modes, respectively.

As alluded to above, resolved sideband cooling in optomechanical cavities follows physics which is formally similar to the Raman processes used to cool ions to their motional ground state [1]. A cooling laser, with frequency $\omega_l = \omega_c - \omega_m$, is tuned a mechanical frequency red of the cooling cavity resonance of the OMC, giving rise to an intra-cavity photon population n_c at frequency ω_l . Motion of the mechanical oscillator causes scattering of the intra-cavity cooling beam laser light into Stokes and anti-Stokes sidebands at $\omega_c - 2\omega_m$ and ω_c , respectively. Since the anti-Stokes sideband is resonant with the cavity at ω_c , and $\kappa_c < \omega_m$, the anti-Stokes optical up-conversion process is greatly enhanced relative to the Stokes down-conversion process, leading to cooling of the mechanical mode. Assuming a deeply resolved sideband system $(\kappa_c/\omega_m \ll 1)$, the back-action cooled mechanical mode occupancy is approximately given by $\langle n \rangle_c = \gamma_i n_b / (\gamma_i + \gamma_c)$ [16, 17].

Optical scattering of the intra-cavity light field can also be used to read out the motion of the coupled mechanical oscillator. For a quantum harmonic oscillator, the noise power spectral density (PSD) of the oscillator's position is equal to [20],

$$S_{xx}(\omega)/x_{zpf}^{2} = \frac{\gamma \langle n \rangle}{(\omega_{m} + \omega)^{2} + (\gamma/2)^{2}} + \frac{\gamma (\langle n \rangle + 1)}{(\omega_{m} - \omega)^{2} + (\gamma/2)^{2}}, \qquad (1)$$

where γ is the total mechanical damping rate. The asymmetric zero-point motion contribution to $S_{xx}(\omega)$ (illustrated in Fig. 3a) arises from the non-commutivity of position and momentum operators in quantum mechanics. This absorption-



FIG. 3: (a) Displacement noise PSD, S_{xx} , of a quantum simple harmonic oscillator, plotted against $-\omega$ for clarity. (b) Scheme for measurement of the down-converted (Stokes) motional sideband. Here the read-out laser (green vertical arrow; frequency ω_{lr}) is detuned a mechanical frequency above that of the read-out cavity resonance (green solid curve). (c) Corresponding scheme for measurement of the up-converted (anti-Stokes) motional sideband. The linewidth of the readout cavity (κ_r) and the mechanical resonance (γ) are indicated. Insets to (b) and (c) show a zoomed-out spectra indicating the relative frequency of the cooling cavity mode and cooling laser.

emission asymmetry has no classical analogue; of course, at high phonon occupation numbers where $\langle n \rangle \approx \langle n \rangle + 1$, the classically symmetric spectral density is recovered. Since the optical cavity frequency is linearly coupled to the position of the mechanical oscillator, the displacement noise spectrum is imprinted on the photons leaving the cavity and can be measured optically.

Specifically, consider a read-out laser with frequency ω_{lr} and detuning $\Delta \equiv \omega_r - \omega_{lr}$ from the read-out cavity mode. The optical power spectrum of the motional sidebands of the transmitted read-out beam leaving the cavity is given by [16],

$$S(\omega + \omega_{lr}) = \frac{\kappa_{e,r}}{2\pi\kappa_r} \frac{A_-^{(r)}\gamma\langle n \rangle}{(\omega_m - \omega)^2 + (\gamma/2)^2} + \frac{\kappa_{e,r}}{2\pi\kappa_r} \frac{A_+^{(r)}\gamma(\langle n \rangle + 1)}{(\omega_m + \omega)^2 + (\gamma/2)^2}, \quad (2)$$

Here $A_{\pm}^{(r)}$ and $A_{\pm}^{(r)}$ are the detuning-dependent anti-Stokes and Stokes motional scattering rates, respectively, of the read-out laser, given by $A_{\pm}^{(r)} = g_r^2 \kappa_r n_r / ((\Delta \pm \omega_m)^2 + (\kappa_r/2)^2)$.

As illustrated in Fig. 3b and c, the optical read-out cavity can be used to selectively filter the positive or negative frequency components of $S(\omega)$. For a detuning $\Delta = -\omega_m$ for the read-out laser, $A_+^{(r)} \gg A_-^{(r)}$, resulting in a Lorentzian signal with area I_- proportional to $\langle n \rangle + 1$. Conversely, a detuning of $\Delta = \omega_m$ results in $A_-^{(r)} \gg A_+^{(r)}$, producing a signal of area I_+ proportional to $\langle n \rangle$. Comparison of the area under the Lorentzian part of the measured photocurrent PSD of the transmitted read-out laser for detunings $\Delta = \pm \omega_m$, can then be used to infer the mechanical mode occupancy,

$$\eta \equiv I_{-}/I_{+} - 1 = \frac{1}{\langle n \rangle}.$$
(3)

This simple argument neglects the back-action of the readout beam on the mechanical oscillator. In particular, the mechanical damping rate becomes detuning dependent, with $\gamma_{\pm} \equiv$ $(\gamma_i + \gamma_c)(1 \pm C_r)$ for $\Delta = \pm \omega_m$. Here $C_r \equiv |A_+^{(r)} - A_-^{(r)}|/(\gamma_i + \gamma_c)$ is the effective cooperativity of the read-out beam in the presence of the strong cooling beam, and can be found from the measured spectra by the relation, $C_r = (\gamma_+ - \gamma_-)/(\gamma_+ + \gamma_-)$. The back-action of the read-out beam also results in a corresponding change in the phonon occupancy, given by $\langle n \rangle_{\pm} =$ $\langle n \rangle_c / (1 \pm C_r)$ for $\Delta = \pm \omega_m$. Here $\langle n \rangle_c$ is the mechanical mode occupancy due back-action from the cooling beam only. Adding in a correction for the read-out laser back-action, one finds the following relation between the measured motional sidebands and the phonon occupancy of the cooled mechanical oscillator,

$$\eta' \equiv \frac{I_{-}/I_{+}}{1+C_{r}} - \frac{1}{1-C_{r}} = \frac{1}{\langle n \rangle_{c}},$$
(4)

where for $C_r \ll 1$ we recover the standard relation given in Equation (3).

Figure 4 summarizes the measurement results of the calibrated mechanical mode thermometry and motional sideband asymmetry for the silicon OMC cavity. These measurements are performed with the cooling laser locked a mechanical frequency to the red of the fundamental mode of the OMC cavity, and the cooling laser power swept from $n_c \sim 1$ to 330 (maximum input power of 250 μ W). A much weaker read-out laser $(C_r \ll 1)$ is used to both estimate the mechanical mode phonon occupancy and to compare the motional sideband amplitudes. Locking of the cooling and read-out lasers utilizes a high resolution wavemeter (10 MHz resolution) to set the absolute laser frequency and a weak probe beam to determine the laser-cavity detuning. Here the weak probe beam is generated from the cooling or read-out laser via electro-optic modulation, similar to the EIT-like spectroscopy described in Ref. [22]. The laser-cavity lock for both cooling and read-out lasers is performed every few minutes, multiplexed in time between measurements of the phonon occupancy. With the read-out laser set to a detuning $\Delta = \omega_m$ from the read-out cavity, a Lorentzian spectrum with linewidth γ_+ and integrated area I_+ is measured in the read-out laser photocurrent PSD, from which a mode occupancy of $\langle n \rangle_+$ is inferred from a careful calibration of the optomechanical cavity and photodetection system parameters [13]). Similarly, by placing the read-out laser at $\Delta = -\omega_m$ we obtain spectra with linewidth γ_- and integrated



FIG. 4: (a) Plot of the cooperativity of the read-out beam as a function of damped mechanical linewidth. (b) Plot of the measured ratios γ_-/γ_+ (blue \circ) and $\langle n \rangle_+ / \langle n \rangle_-$ (pink \circ). (c) Plot of the mechanical mode phonon occupancy, $\langle n \rangle_c$, as a function of the optically damped mechanical linewidth, $\bar{\gamma}$. The dashed line is the predicted phonon number $\gamma_i n_b / \bar{\gamma}$ from an ideal back-action cooling model. Vertical error bars in (b) and (c) indicate uncertainty in the calibrated phonon occupancy due to uncertainty in the system parameters and a 95% confidence interval on the Lorentzian fits to spetra. (d) Plot of the asymmetry (η') in the measured Stokes and anti-Stokes sidebands of the read-out laser for each calibrated measurement of $\langle n \rangle_c$. The horizontal error bars arise from a 2% uncertainty in the transmitted read-out laser beam power between detunings $\Delta = \pm \omega_m$, and a 95% confidence interval in the Lorentzian fits to the measured spectra. The vertical error bars in $\langle n \rangle_c$ are the same as in (c). The classical (blue curve) and quantum mechanical (pink curve) relations for the sideband asymmetry are also plotted. (e-g) Plot of the measured Stokes (red curve) and anti-Stokes (blue curve) read-out beam spectra for (from top to bottom) $\langle n \rangle_c = 85$, 6.3, and 3.2 phonons. For clarity, we have divided out the read-out back-action from each spectra by multiplying the measured spectra at detunings $\Delta = \pm \omega_m$ by γ_{\pm} . Additionally, we have plotted the horizontal axis in units of γ , and rescaled the vertical axis for different $\langle n \rangle_c$ to keep the areas directly comparable. The difference in the Stokes and anti-Stokes spectra, which arises due to the quantum zero-point fluctuation of the mechanical system, is shown as a shaded region.

area $I_{-} \propto \langle n \rangle_{-} + 1$, from which we estimate $\langle n \rangle_{-}$.

Figure 4a plots the read-out cooperativity C_r , calculated from the measured γ_{\pm} , versus the mechanical damping rate $\bar{\gamma} = (\gamma_+ + \gamma_-)/2$. The ratios $\xi \equiv \gamma_-/\gamma_+$ and $\langle n \rangle_+/\langle n \rangle_-$ are plotted in Fig. 4b. From $\langle n \rangle_{\pm}$, the laser cooled phonon occupation number, $\langle n \rangle_c$, is calculated and plotted in Fig. 4c versus $\bar{\gamma}$. As expected, $\langle n \rangle_c$ drops approximately linearly with $\bar{\gamma}$, reaching a minimum value of approximately 2.6 ± 0.2 phonons. Further cooling below a single phonon has been achieved in similar devices [13], however in this case cooling is limited by the available power of the 1400 nm cooling laser. Also evident in Fig. 4c, at the higher cooling powers, is an increased scatter and deviation of $\langle n \rangle_c$ from the ideal cooling curve (dashed curve). This can be attributed to optical absorption in the silicon nanobeam [13], which in this case produces a powerdependent variation in n_b and γ_i due to both the read-out and cooling laser beams.

In Fig. 4d, the measured values of the expression η' are plotted versus the calibrated value of $\langle n \rangle_c$. Also plotted are the classical and quantum values of this expression, 0 and $1/\langle n \rangle_c$, respectively. A clear divergence from the classical result of $\eta' = 0$ is apparent, agreeing with the deviation due to zeropoint fluctuations of the mechanical oscillator. This devia-

tion is directly apparent in the measured spectra, shown for $\langle n \rangle_c = 85$, 6.3, and 3.2 phonons in Fig. 4e-g, with the shaded region corresponding to the noise power contribution due to zero-point motion. At the largest powers, we measure asymmetry in the motional sideband amplitudes of ~ 40% in agreement with the inferred $\langle n \rangle_c = 2.6$ phonons from calibrated thermometry.

While the quantum nature of a mechanical resonator will come as little surprise to most physicists, its observation through the zero-point motion is a significant step towards observing and controlling the quantum dynamics of mesoscopic mechanical systems. By demonstrating the fundamentally quantum behavior of an engineered mechanical nanostructure, we have shown that realizable optomechanical systems have the sensitivity and environmental isolation required for such quantum mechanical investigations.

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