Linearly Polarized Gluons and the Higgs Transverse Momentum Distribution
Daniël Boer, Wilco J. den Dunnen, Cristian Pisano, Marc Schlegel, and Werner Vogelsang
DOI: 10.1103/PhysRevLett.108.032002
Linearly Polarized Gluons and the Higgs Transverse Momentum Distribution

Daniel Boer,1 Wilco J. den Dunnen,2 Cristian Pisano,3 Marc Schlegel,4 and Werner Vogelsang4

1Theory Group, KVI, University of Groningen, Zernikelaan 25, NL-9747 AA Groningen, The Netherlands
2Department of Physics and Astronomy, Vrije Universiteit Amsterdam, NL-1081 HV Amsterdam, The Netherlands
3Dipartimento di Fisica, Università di Cagliari, and INFN, Sezione di Cagliari, I-09042 Monserrato (CA), Italy
4Institute for Theoretical Physics, Universität Tübingen, Auf der Morgenstelle 14, D-72076 Tübingen, Germany

We study how gluons carrying linear polarization inside an unpolarized hadron contribute to the transverse momentum distribution of Higgs bosons produced in hadronic collisions. They modify the distribution produced by unpolarized gluons in a characteristic way that could be used to determine whether the Higgs boson is a scalar or a pseudoscalar particle.

PACS numbers: 12.38.-t; 13.85.Ni; 13.88.+e

It is sometimes said that the LHC is a ‘gluon collider’, because at high energies the gluon density inside a proton becomes dominant over the quark densities. Higgs production, in particular, predominantly arises from gluon–gluon ‘fusion’ $gg \to H$ through a triangular top quark loop. QCD corrections to this process have been calculated with increasing precision [1–7], making it well understood. It is not commonly known however that the LHC is actually also to some extent a polarized gluon collider, since gluons can in principle be linearly polarized inside an unpolarized proton. Their corresponding distribution, here denoted by $h^\pm_g$ and first defined in Ref. [8], requires the gluons to have a nonzero transverse momentum with respect to the parent hadron. It corresponds to an interference between $+1$ and $-1$ helicity gluon states that would be suppressed without transverse momentum.

So far the function $h^\pm_g$ has not been studied experimentally, and consequently nothing is known about its magnitude. Only a theoretical upper bound has been given [8, 9]. Recently, several ways of probing $h^\pm_g$ have been put forward, namely in heavy quark pair or dijet production [9], or in photon pair production [10], where in all cases the transverse momentum of the pair is measured. One way in which linearly polarized gluons can manifest themselves in these processes is through azimuthal asymmetries. However, it was found that they can also generate a term in the cross section that is independent of azimuthal angle. This happens when two linearly polarized gluons, one from each hadron, participate in the scattering. In this way they can also contribute to production of a scalar particle, such as a scalar or pseudoscalar Higgs boson, when its transverse momentum $q_T$ is measured. It has in fact been shown [11, 12] that such a contribution is generated perturbatively. In other words, if at tree level gluons are taken to be unpolarized, at order $\alpha_s$ they will become to some extent linearly polarized.

In light of this, we will investigate in this letter how the distribution of linearly polarized gluons may affect the transverse momentum distribution of Higgs bosons for $q_T \ll m_H$, where $m_H$ is the Higgs mass. We shall observe that linearly polarized gluons may in fact provide a tool to uncover whether the Higgs boson is a scalar or a pseudoscalar particle. Thus far, relatively few suggestions to this end have been put forward for the LHC, typically using azimuthal distributions, for example in Higgs + jet pair production [13] or in $\tau$ pair decays [14]. The suggestion we put forward here does not involve measurements of any angular distributions. Instead, we will show that linear polarization of gluons simply leads to a modulation of the Higgs transverse momentum distribution that depends on the nature of the Higgs particle.

Transverse momentum dependent distribution functions (TMDs) of gluons in an unpolarized hadron are defined through a matrix element of a correlator of the gluon field strengths $F^{\mu\nu}(0)$ and $F^{\mu\nu}(\xi)$, evaluated at fixed light-front (LF) time $\xi^+ = \xi^-n = 0$, where $n$ is a lightlike vector conjugate to the parent hadron’s four-
momentum $P$. Decomposing the gluon momentum as $p = x P + p_T + p_T^-$, the correlator is given by [8]
\[
\Phi^\mu\nu_g(x, p_T) = \frac{n_\rho n_\sigma}{(p^\mu n^\nu - p^\nu n^\mu)} \frac{1}{2\pi^3} \int d(\xi P) d^2\xi T e^{ip\xi}
\times \langle P| \mathcal{T} F^{\rho\sigma}(0) F^{\mu\nu}(\xi) |P\rangle_{\text{LF}}
= -\frac{1}{2\pi} \left\{ g_\mu^\nu f_1^g - \left( \frac{p_T^2}{M^2} + g_\mu^\nu \frac{p_T^2}{2M^2} \right) h_1^g \right\}.
\]
with $p_T^2 = -p_T^2$, $g_\mu^\nu = g^{\mu\nu} - P^\mu n^\nu / P \cdot n - n^\mu P^\nu / P \cdot n$, and $M$ the proton mass. $f_1^g(x, p_T^2)$ represents the unpolarized gluon distribution and $h_1^g(x, p_T^2)$ the distribution of linearly polarized gluons. In (1) we have omitted a Wilson line that renders the correlator gauge invariant. As any TMD, $h_1^g$ will receive contributions from initial and/or final state interactions, which make the gauge link process-dependent. Therefore, despite the fact that it is $T$-even, $h_1^g$ can receive non-universal contributions, and its extraction can be hampered for processes where factorization does not hold, such as dijet production in hadron-hadron collisions [15–17]. Higgs production, on the other hand, is expected to allow for TMD factorization, just like the Drell-Yan process. A more detailed study of this remains to be carried out.

The calculation of the Higgs production cross section in the TMD framework closely follows Refs. [15, 18]. The generic contribution by $gg \to H$ reads
\[
\frac{\mathcal{E}_H}{d^3q} \bigg|_{q_T < m_H} = \frac{\pi x_a x_b}{16m_H^2S} \times
\int d^2p_{aT} \int d^2p_{bT} \mathcal{F}^\mu^\nu (x_a, p_{aT})
\times \Phi^\mu\nu_g (x_b, p_{bT}) \left( \hat{\mathcal{M}}^\mu\nu (\mathcal{M}^\mu\nu)^* | p_{aT} \rightarrow p_{bT} \right) \mathcal{O} \left( \frac{q_T}{m_H} \right).
\]
For now, we assume on-shell production of the Higgs particle, with $\bar{q}$ and $E_H$ its momentum and energy. $P_a$ and $P_b$ are the momenta of the colliding protons, $S = (P_a + P_b)^2$, and $x_a (b) = q^2 / (2P_a (b) \cdot q)$. To lowest order, the hard partonic amplitude $\mathcal{M}$ is given by the well-known formula [4] for the $gg \to H$ triangle diagram:
\[
\frac{\mathcal{E}_H}{d^3q} \bigg|_{q_T < m_H} = \frac{\pi x_a x_b}{16m_H^2S} \times
\int d^2p_{aT} \int d^2p_{bT} \mathcal{F}^\mu^\nu (x_a, p_{aT})
\times \Phi^\mu\nu_g (x_b, p_{bT}) \left( \hat{\mathcal{M}}^\mu\nu (\mathcal{M}^\mu\nu)^* | p_{aT} \rightarrow p_{bT} \right) \mathcal{O} \left( \frac{q_T}{m_H} \right).
\]
For a scalar Standard Model (SM) Higgs boson $H^0$, where we only consider top quarks in the triangle, and where $G_F$ is the Fermi constant, $\alpha_s$ the strong coupling constant, $\tau = m_H^2 / (4m_T^2)$ with the top mass $m_t$, and $A_H(\tau) = 2(\tau + (\tau - 1)J(\tau)) / \tau^2$ with
\[
J(\tau) = \left\{ -\frac{1}{4} \left( \ln \frac{1 + \sqrt{1-\tau}}{1 - \sqrt{1-\tau}} - i\pi \right)^2, \quad \tau > 1 \right\}
\]
valid for all $x$ and $p_T$. The maximally possible effect will be generated when this bound is saturated. Models may also shed light on the size of $h_1^{\perp g}$. In the simple perturbative quark target model of gluon TMDs of Ref. [20] the function $h_1^{\perp g}$ is found to possess the same characteristic $1 / x$ increase as the distribution of unpolarized gluons $f_1^g$, which suggests that linearly polarized gluons may be as relevant at small $x$ as unpolarized ones. Another recent model calculation [21] shows saturation of the positivity bound for $h_1^{\perp g}$ in heavy nuclei in certain transverse momentum regions (Weisz"acker-Williams model) or even over the full momentum range (dipole model). This suggests that saturation of the positivity bound at least locally in $x$ or $p_T$ might not be an unrealistic assumption.
We follow a standard approach for TMDs in the literature (see [22]) and assume a simple Gaussian dependence of the gluon TMDs on transverse momentum:

$$f_{1}^{g}(x, p_{T}^{2}) = \frac{G(x)}{\pi p_{T}^{2}} \exp \left(-\frac{p_{T}^{2}}{2p_{0}^{2}}\right),$$

where $G(x)$ is the collinear gluon distribution and the width $(p_{0}^{2})$ is assumed to be independent of $x$. The bound (9) is directly satisfied by the form

$$h_{1}^{\perp g}(x, p_{T}^{2}) = \frac{M^{2}G(x)}{2\pi (p_{T}^{2})^{2}} \frac{2e(1-r)}{r} \exp \left(-\frac{p_{T}^{2}}{2r(p_{0}^{2})}\right).$$

We choose $r = 2/3$. The left panel of Fig. 2 shows the $p_{T}$-dependence of $f_{1}^{g}$ and $h_{1}^{\perp g}$ for two values of the Gaussian width: $(p_{T}^{2}) = 1$ GeV$^{2}$ and $(p_{T}^{2}) = 7$ GeV$^{2}$. The latter value may be more appropriate at $Q = m_{H}$, cf. the Gaussian fit to $f_{1}^{g}(x, p_{T}^{2})$ evolved to $Q = M_{Z}$ of Ref. [23].

![Figure 2](image-url)

**FIG. 2:** Left: Gaussian distributions for $f_{1}^{g}$ and $h_{1}^{\perp g}$ (divided by $G(x)$) as functions of $p_{T}$ for two different values of $(p_{T}^{2})$. Right: Resulting ratio $R = C[w_{H} h_{1}^{\perp g} h_{1}^{\perp g}] / C[f_{1}^{g} f_{1}^{g}]$.

It is straightforward to compute the convolution integrals appearing in Eq. (6) analytically:

$$C[f_{1}^{g} f_{1}^{g}] = \frac{G(x_{a})G(x_{b})}{2\pi (p_{T}^{2})^{2}} \exp \left(-\frac{q_{T}^{2}}{2(p_{0}^{2})}\right),$$

$$C[w_{H} h_{1}^{\perp g} h_{1}^{\perp g}] = \frac{G(x_{a})G(x_{b})}{36\pi (p_{T}^{2})^{2}} \times \left[\frac{2}{3} - \frac{q_{T}^{2}}{p_{T}^{2}} + \frac{3q_{T}^{2}}{4p_{T}^{2}}\right] \exp \left(-\frac{3q_{T}^{2}}{4p_{0}^{2}}\right).$$

Their ratio $R$ is shown in the right panel of Fig. 2. It is a measure of the relative size of the contribution by linearly polarized gluons and shows the anticipated double node of $C[w_{H} h_{1}^{\perp g} h_{1}^{\perp g}]$. It is evident that at least within our simple model, linearly polarized gluons have a sizable effect on the Higgs $q_{T}$ distribution. We stress again that the effect enters scalar and pseudoscalar Higgs production with opposite sign. If the effect is at or near the level shown by Fig. 2, it should easily allow to determine the parity of the Higgs boson, provided a sufficiently fine scan in $q_{T}$ is possible in experiment, to resolve both nodes.

So far we have considered only the production of an on-shell Higgs boson. In reality the Higgs will decay into some observed final state, and there will be background reactions contributing to this final state that are not related to the Higgs. These backgrounds may themselves be sensitive to linearly polarized gluons. We will now briefly consider one example of this, the Higgs decay into a photon pair. We reserve a more detailed study of final states such as $\gamma Z$, $ZZ$ or $WW$ for a future publication.

After production in $gg \to H$, the two-photon decay of a SM Higgs occurs through a top quark or $W$-boson triangular loop. The decay of a pseudoscalar Higgs is instead described by physics beyond the SM and hence is model-dependent. There are often no tree-level couplings to $W$-bosons in this case [24], so here we consider only the top quark coupling. For both a scalar or pseudoscalar Higgs, the lowest-order amplitude can be written as [25]

$$\hat{M}_{\gamma\gamma} = \sqrt{2}G_{F}\alpha_{s}\alpha s^{2} A_{H(A)}(\tau) F_{H(A)\to \gamma\gamma}(s),$$

where $\alpha$ is the electromagnetic coupling and $\Gamma_{H}$ the Higgs decay width. For a scalar Higgs, $\hat{M}_{\mu\nu} = g_{\mu\nu}^{\tau} \delta_{\lambda_{\alpha}\lambda_{\beta}}$, whereas $r^{\mu\nu} = \lambda_{\alpha}^{\mu} b_{T}^{\lambda_{\beta}} \delta_{\lambda_{\alpha}\lambda_{\beta}}$ in the pseudoscalar case, with $\lambda_{\alpha}, \lambda_{\beta}$ the photon helicities. $A_{H}(\hat{\tau})$ and $A_{A}(\hat{\tau})$ are given in Eqs. (3) and (5) with $\hat{\tau} = s/(4m^{2}_{H})$, where $s = (p_{a} + p_{b})^{2} \approx x_{a}x_{b}S$ for gluon momenta $p_{a}, p_{b}$. Finally,

$$F_{H\to \gamma\gamma}(s) = \mathcal{W}(\tau_{W}) - \frac{4}{9} N_{A_{H}(\hat{\tau})},$$

with $\tau_{W} = s/(4m^{2}_{W})$ and $\mathcal{W}(\tau) = -(2\tau^{2} + 3\tau + 3(2\tau - 1)J_{\tau})/\tau^{2}$ describes the contribution by the $W$ triangular loop. We assume $F_{A\to \gamma\gamma}(s) = \frac{4}{3} N_{A_{A}(\hat{\tau})}$. In the following we consider a relatively light Higgs mass $m_{H} = 120$ GeV with a small total width $\Gamma_{H} \approx 5 \times 10^{-3}$ GeV [26].

As is well known, an important QCD background to photon pair production at high energies is generated by $gg \to \gamma\gamma$ via a quark box [27]. This subprocess was studied recently in the context of TMD factorization in Ref. [10]. Using its results, we add the two lowest-order amplitudes describing the box diagram and the Higgs resonance and extract the azimuthal-angle independent cross section:

$$\int d\phi \frac{d\sigma_{gg}}{dq^{2} d\Omega} = F_{1} C[f_{1}^{g} f_{1}^{g}] + F_{2} C[w_{H} h_{1}^{\perp g} h_{1}^{\perp g}].$$

Here $q = q_{a} + q_{b}$ is the momentum of the photon pair. $d\Omega = d\phi d\cos \theta$ denotes the solid angle element for each photon, with the angles $\phi, \theta$ defined in the Collins-Soper frame [10]. $F_{1}$ and $F_{2}$ are calculated functions of $\theta$ and the pair mass $Q = \sqrt{s}$ that we will not give here. They depend on the box and Higgs amplitudes.

We find that the box contribution dominates the process except when the photon pair mass is close to the Higgs mass. Figure 3 shows the effect of the box-Higgs interference on the ratio $F_{2}/F_{1}$ as a function of $Q$ around
$m_H = 120$ GeV for a scalar or pseudoscalar Higgs. Away from $Q = m_H$ (by a few hundred MeV) we find $F_1 \gg F_2$, such that the additional term from linearly polarized gluons contributes at most 10% to the cross section, but on average around 1% or less. However, near $Q = m_H$ where the Higgs contribution dominates, we find $F_1 \approx \pm F_2$. The ratio of the second to first term in Eq. (16) then becomes approximately the ratio $\pm R$ of Fig. 2. Figure 3 suggests that a distinction between a scalar and pseudoscalar Higgs is possible, if the experimental resolution of the photon pair mass $Q$ is sufficiently good. Higgs bosons in extensions of the SM which typically have larger widths, would required less fine Q-binning. Also, for heavier Higgs bosons other final states such as $W W$- or $Z Z$-production may allow for a better $Q$-resolution. In any case the $Q$-bin size around the Higgs mass is to be chosen as small as possible to maximize the effects caused by linearly polarized gluons.

We conclude that the effect of linearly polarized gluons on the Higgs transverse momentum distribution can in principle be used to determine the parity of the Higgs boson, provided $h_\perp^+ g$ is of sufficient size. Of course, it could turn out that $h_\perp^+ g$ is in reality smaller than in our model or that it exhibits nodes in $x$ or $p_T$, complicating the analysis. Our results thus provide additional motivation for experimental studies of $h_\perp^+ g$ using different probes, such as dijet and heavy quark or photon pair production. We stress that perturbative gluon-radiation effects will alter the $q_T$ distributions expected on the basis of our simple Gaussian model. Their inclusion will require merging our model with the soft-gluon resummation techniques described in [11, 12, 23, 28, 29]. This will also affect the eventual size of the contribution by linearly polarized gluons. A full study of this is needed.

We thank John Collins and Feng Yuan for useful discussions. C.P. is supported by Regione Autonoma della Sardegna under grant PO Sardegna FSE 2007-2013, L.R. 7/2007. This research is part of the FP7 EU-programme Hadron Physics (No. 227431) and part of the research program of the “Stichting voor Fundamenteel Onderzoek der Materie (FOM)” which is financially supported by the “Nederlandse Organisatie voor Wetenschappelijk Onderzoek (NWO)”. W.V.’s work is supported by the U.S. Department of Energy (contract DE-AC02-98CH10886).

FIG. 3: Ratio $F_2/F_1$ as a function of pair mass squared in a region around $m_H = 120$ GeV for various angles $\theta$.

- $\theta = \pi/2$
- $\theta = 3\pi/8$
- $\theta = \pi/4$
- $\theta = \pi/8$

$m_H = 119.7$ GeV for a scalar, and $120.3$ GeV for a pseudoscalar.