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Ramsey Fringes and Time-domain Multiple-Slit Interference from Vacuum

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Sequences of alternating-sign time-dependent electric field pulses lead to coherent interference effects in Schwinger vacuum pair production, producing a Ramsey interferometer, an all-optical time-domain realization of the multiple-slit interference effect, directly from the quantum vacuum. The interference, obeying fermionic quantum statistics, is manifest in the momentum dependence of the number of produced electrons and positrons along the linearly polarized electric field. The central value grows like N^2 for N pulses [i.e., N "slits"], and the functional form is well-described by a coherent multiple-slit expression. This behavior is generic for many driven quantum systems.

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Double-slit experiments form a cornerstone of interferometry in optics and in quantum mechanics. The double-slit equivalent in the time domain constitutes Ramsey interferometry [1], and has been widely studied in atomic systems [2–5]. Here we propose a new realization of Ramsey interferometry using the Schwinger effect, namely the non-perturbative production of electron-positron pairs when an external electric field is applied to the quantum electrodynamical (QED) vacuum [6, 7]. The analogy between double-slit interference and the Schwinger effect was suggested in [8], and a spatial realization of an all-optical double-slit experiment using vacuum polarization effects has been proposed [9]. A multiple-slit analogy has also been made for finite plane-wave pulses in stimulated laser pair production [10]. The elusive Schwinger effect has attracted recent renewed interest, prompted by the possibility of experimental realization in ultra-intense laser field systems [11, 12]. It has been realized that the "Schwinger limit" laser intensity of $4\times10^{29} W/\mathrm{cm}^2$ is not necessarily a strict limit, and might be lowered by several orders of magnitude by manipulation of the form of the laser pulses [13–17]. Here we propose a temporal pulse sequence set-up that acts as a Ramsey interferometer and leads, for the number of pairs created, to an N^2 enhancement factor for N pulses, due to coherent quantum interference. Our description relies on a general quantitative method which applies to a broad range of similar interference phenomena for quantum fields of different quantum statistics, driven by time-dependent perturbations. Interference phenomena are familiar from strong-field atomic and molecular physics, in the theory of atomic ionization [18], and form the basis for the interpretation of photoionization spectra as time-domain realizations of the double-slit experiment [3], for pulses having maximal carrier phase offset. Thus, similar ideas apply directly to a wide variety of physical systems involving time-dependent tunneling [19], Landau-Zener effect [20–22], driven atomic systems [23], chemical reactions [24, 25], Hawking radiation [26], cosmological particle production [27], heavy ion collisions [28, 29], and the dynamical Casimir effect [30].

Consider the QED vacuum subject to a linearly polarized time-dependent electric field $\vec{E}=(0,0,E(t))$, with vector potential $\vec{A}=(0,0,A(t))$, and $E(t)=-\dot{A}(t)$. For such an applied field, spatial momentum is a good quantum number, so we decompose the spinor quantum field operators into modes labelled by their spatial momenta. A Bogoliubov transformation from the initial time-independent basis of fermionic particle/antiparticle creation and annihilation operators, $a_{\mathbf{k}}$ and $b^\dagger_{-\mathbf{k}}$, to a time-dependent basis, $\tilde{a}_{\mathbf{k}}(t)$ and $\tilde{b}^\dagger_{-\mathbf{k}}(t)$, is [31]:

$$\begin{pmatrix}\tilde{a}_{\mathbf{k}}(t)\\ \tilde{b}^\dagger_{-\mathbf{k}}(t)\end{pmatrix}=\begin{pmatrix}\alpha_{\mathbf{k}}(t) & -\beta^*_{\mathbf{k}}(t)\\ \beta_{\mathbf{k}}(t) & \alpha^*_{\mathbf{k}}(t)\end{pmatrix}\begin{pmatrix}a_{\mathbf{k}}\\ b^\dagger_{-\mathbf{k}}\end{pmatrix} \qquad (1)$$

The fermionic anti-commutation relations are preserved by the unitarity condition, $|\alpha_{\mathbf{k}}(t)|^2+|\beta_{\mathbf{k}}(t)|^2=1$, and the time evolution is:

$$\begin{aligned}\dot{\alpha}_{\mathbf{k}} &= \Omega_{\mathbf{k}}(t)\, e^{2i\int^t \mathcal{E}_{\mathbf{k}}(t')dt'}\,\beta_{\mathbf{k}}(t)\\ \dot{\beta}_{\mathbf{k}} &= -\Omega_{\mathbf{k}}(t)\, e^{-2i\int^t \mathcal{E}_{\mathbf{k}}(t')dt'}\,\alpha_{\mathbf{k}}(t)\end{aligned} \qquad (2)$$

where

$$\begin{aligned}\mathcal{E}^2_{\mathbf{k}}(t) &= m^2+k_\perp^2+(k-A(t))^2\\ \Omega_{\mathbf{k}}(t) &= E(t)\sqrt{m^2+k_\perp^2}/(2\mathcal{E}^2_{\mathbf{k}}(t))\ .\end{aligned} \qquad (3)$$

with the notation $\mathbf{k}=(\mathbf{k}_\perp,k)$. It is useful to re-express the time evolution (3) as a two-level problem. For each mode $\mathbf{k}$, define a two-level system by $c^0_{\mathbf{k}}=\alpha_{\mathbf{k}}\, e^{-i\int^t \mathcal{E}_{\mathbf{k}}}$, $c^p_{\mathbf{k}}=\beta_{\mathbf{k}}\, e^{i\int^t \mathcal{E}_{\mathbf{k}}}$, with time evolution:

$$i\frac{d}{dt}\begin{pmatrix}c^0_{\mathbf{k}}\\ c^p_{\mathbf{k}}\end{pmatrix}=\begin{pmatrix}\mathcal{E}_{\mathbf{k}}(t) & i\Omega_{\mathbf{k}}(t)\\ -i\Omega_{\mathbf{k}}(t) & -\mathcal{E}_{\mathbf{k}}(t)\end{pmatrix}\begin{pmatrix}c^0_{\mathbf{k}}\\ c^p_{\mathbf{k}}\end{pmatrix} \qquad (4)$$

The off-diagonal matrix elements $\Omega_{\mathbf{k}}(t)$ are proportional to the electric field $E(t)$ and can be interpreted as a Rabi frequency. Note that the energies $\mathcal{E}_{\mathbf{k}}(t)$ depend also on the field and all matrix elements depend parametrically on $\mathbf{k}$.

The physical quantity we wish to evaluate is the expectation value $\mathcal{N}_{\mathbf{k}}$ of the number of pairs produced from vacuum into the momentum mode $\mathbf{k}$. It is given by

$$\mathcal{N}_{\mathbf{k}} = |\beta_{\mathbf{k}}(t=+\infty)|^2 = |c^p_{\mathbf{k}}(t=+\infty)|^2 \tag{5}$$

Time evolution through a single pulse is described by an S-matrix written as a rotation characterized by an angle $\phi_{\mathbf{k}}$, so that $\mathcal{N}_{\mathbf{k}} = \sin^2\phi_{\mathbf{k}}$. For two successive pulses, as shown in Fig.1, there are two amplitudes, $A_1 = e^{i\theta_{\mathbf{k}}}\cos\phi_2\sin\phi_1$ and $A_2 = e^{-i\theta_{\mathbf{k}}}\cos\phi_1\sin\phi_2$, for producing a pair with momentum $\mathbf{k}$, where $2\theta_{\mathbf{k}}$ is a phase accumulated between the two pulses. For two identical pulses of opposite sign we have $\phi_1 = -\phi_2$, and quantum interference leads to:

$$\mathcal{N}^{\text{2-pulse}}_{\mathbf{k}} = |A_1 + A_2|^2 \;\approx\; 4\sin^2\theta_{\mathbf{k}}\,\mathcal{N}^{\text{1-pulse}}_{\mathbf{k}} \tag{6}$$

assuming $\mathcal{N}_{\mathbf{k}} \ll 1$. An explicit expression for the interference angle $\theta_{\mathbf{k}}$ is obtained below (see (12)).

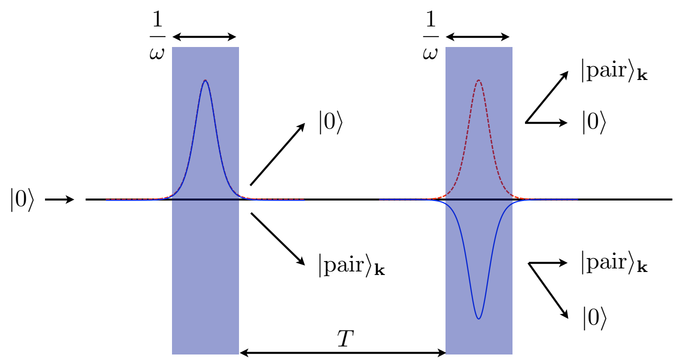

FIG. 1: Two-pulse Ramsey interferometer. A sequence of two identical electric field pulses of width $1/\omega$, separated by time delay T is applied to the vacuum. We consider the symmetric and antisymmetric situations where the pulses have the same or opposite signs.

A qualitative physical understanding of this quantum interference can be given in terms of avoided level-crossings between the instantaneous eigenvalues, $\lambda_\pm = \pm\sqrt{\mathcal{E}^2_{\mathbf{k}} + \Omega^2_{\mathbf{k}}}$, from (4). The maximum pair production occurs for the level-crossings at which $\mathcal{E}_{\mathbf{k}} \approx 0$ and the Rabi frequency $\Omega_{\mathbf{k}}$ is correspondingly large. For example, for a constant electric field, $A(t) = -E\,t$, there is an avoided crossing at $t_0 = -\frac{k}{eE} \pm i\frac{m}{eE}$. The imaginary part leads to the exponential behavior of the pair number, $\mathcal{N}_{\mathbf{k}} \sim \exp\left[-m^2\pi/(eE)\right]$, in analogy with the Landau-Zener argument. The real part, $\mathrm{Re}\,t_0 = -\frac{k}{eE}$, depends on the momentum k. It indicates the existence of an avoided crossing and creation of pairs of momentum k only. For two successive electric field pulses, we must distinguish between the symmetric and antisymmetric configurations of the two pulses as displayed in Fig. 1. As shown in Fig. 2, the antisymmetric configuration allows for two distinct level-crossings for the same momentum k, and therefore we have interference. On the other hand, for the symmetric configuration, we cannot have two different level-crossings for the same momentum k, so there is no interference.

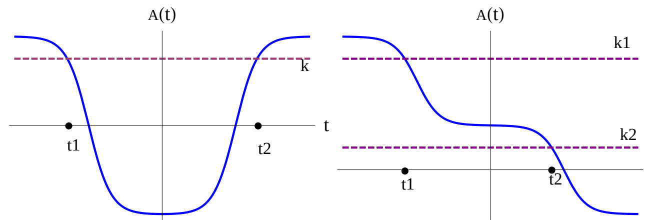

FIG. 2: Plots of the gauge field $A(t)$. The first plot describes the situation of two electric pulses in the antisymmetric configuration. It has two avoided crossings, corresponding to the same k value, and interference occurs. The second plot, with a monotonic $A(t)$ and correspondingly two identical electric pulses (symmetric configuration), has crossings at different k, so no interference takes place.

Building on this qualitative description, we now give a precise quantitative treatment. The time-evolution in (4) can be converted to a Riccati equation for the ratio $R_{\mathbf{k}} = \beta_{\mathbf{k}}/\alpha_{\mathbf{k}} = \left(c^p_{\mathbf{k}}/c^0_{\mathbf{k}}\right) e^{-2i\int^t \mathcal{E}_{\mathbf{k}}(t')dt'}$:

$$\dot{R}_{\mathbf{k}} \;=\; -\Omega_{\mathbf{k}}\left(e^{-2i\int^t \mathcal{E}_{\mathbf{k}}(t')dt'} + R^2_{\mathbf{k}}\, e^{2i\int^t \mathcal{E}_{\mathbf{k}}(t')dt'}\right) \tag{7}$$

The amplitude $R_{\mathbf{k}}$ is a convenient quantity since, according to (5), the corresponding probability taken at $t = +\infty$ is, for $\mathcal{N}_{\mathbf{k}} \ll 1$,

$$|R_{\mathbf{k}}(\infty)|^2 = |\beta_{\mathbf{k}}(\infty)/\alpha_{\mathbf{k}}(\infty)|^2 \approx \mathcal{N}_{\mathbf{k}}\,. \tag{8}$$

This relation allows to describe $\mathcal{N}_{\mathbf{k}}$ as the reflection probability of an associated time-domain scattering problem [31, 32]. While (7) can be solved numerically, deeper physical insight is gained from a semiclassical approximation [33]. A similar-style analysis for Landau-Zener is given in [34]. The turning points (namely the avoided crossings) obtained for $\mathcal{E}_{\mathbf{k}}(t) = 0$, lie in the complex t-plane, and since $A(t)$ is real, they occur in complex conjugate pairs. Then, the semiclassical amplitude $R_{\mathbf{k}}$ is a sum over contributions from different turning points [35]

$$R_{\mathbf{k}}(\infty) \;\approx\; \sum_{t_p} (-1)^p\, e^{i\,\pi/2}\, e^{-2i\int_{-\infty}^{t_p} \mathcal{E}_{\mathbf{k}}(t)\,dt} \tag{9}$$

The exponents in (9) have both real and imaginary parts, so there can be interference effects for $\mathcal{N}_{\mathbf{k}}$, depending on the distribution of turning points. The alternating sign in (9) is from the fermionic statistics.

As a quantitative illustration, consider first a single pulse $E(t) = E\,\mathrm{sech}^2(\omega\,t)$, using $A(t) = -E/\omega\,\tanh(\omega t)$. We set $k_\perp = 0$, as the dominant production is along the field direction. There is an infinite tower of turning points (t_p, t^*_p), given by $\omega t_p = \mathrm{arctanh}(i\,m - k) + ip\pi$, but the dominant contribution comes clearly from the pair closest to the real t axis. There is no interference, and the number of pairs created in momentum k is well-described by the familiar expression

$$\mathcal{N}_{\mathbf{k}}^{\text{1-pulse}} \approx \exp\left[-2K_{\mathbf{k}}\right] \quad , \quad K_{\mathbf{k}} = \left| \int_{t_0}^{t_0^*} \mathcal{E}_{\mathbf{k}}(t) \right| \qquad (10)$$

This expression agrees well with the numerical result, and it is shown in Figs. 3 and 5 as a smooth envelope function.

Now consider two such linearly polarized pulses, of opposite sign (antisymmetric configuration), separated by a time delay T, namely $E(t) = E\,\text{sech}^2(\omega\,(t - T/2)) - E\,\text{sech}^2(\omega\,(t + T/2))$, with $A(t) = E/\omega[1 + \tanh(\omega(t - T/2)) - \tanh(\omega(t + T/2))]$. The turning point structure is now more complicated, but the dominant turning points form two complex conjugate pairs $t_\pm$ and $t_\pm^*$, whose locations are well approximated by

$$t_\pm(k) = \pm T/2 + \frac{1}{2\omega} \ln\left(\frac{E + \omega(k + im)}{E - \omega(k + im)}\right) \qquad (11)$$

These turning points move as functions of longitudinal momentum k, but always form a rectangular array of two complex conjugate pairs with imaginary part of equal magnitude. The integral between the real parts of the different turning points $t_\pm$ yields a quantitative expression for the interference angle $\theta_{\mathbf{k}}$ appearing in (6) [35]:

$$\theta_{\mathbf{k}} \;=\; \int_{Re(t_-)}^{Re(t_+)} \mathcal{E}_{\mathbf{k}}(t)dt \qquad (12)$$

These approximate expressions (6, 12) are shown in Fig. 3, in excellent agreement with the exact numerical result. Notice the characteristic oscillatory form of a double-slit Ramsey interference pattern, underneath an envelope that is 2^2 times the single-pulse result in (10).

Starting from (9), we can also analyze the case of two identical electric pulses in the symmetric configuration. Since the momentum k is fixed, the turning point structure involves now only one pair of (complex conjugate) turning points for the dominant contribution, as for a single pulse, thus leading to no interference. This result is in complete agreement with the more qualitative picture leading to (6) and explained in Fig. 2.

We propose now to generalize the results (6, 12), for the antisymmetric set-up, to build an interferometer by applying to the QED vacuum a sequence of equally-spaced alternating-sign electric field pulses. For a fixed momentum mode k, the number of pairs created depends on the time-delay T between pulses via the standard Fabry-Perot form, as shown in Fig. 4.

For such a field, there are N dominant complex conjugate pairs of turning points, all equally distant from the real axis, given approximately by the expressions (11), displaced by steps of T along the real axis. Thus, when the pulses are well-separated compared to their width, $T \gg 1/\omega$, all the $K_{\mathbf{k}}^{(p)}$-type integrals are approximately

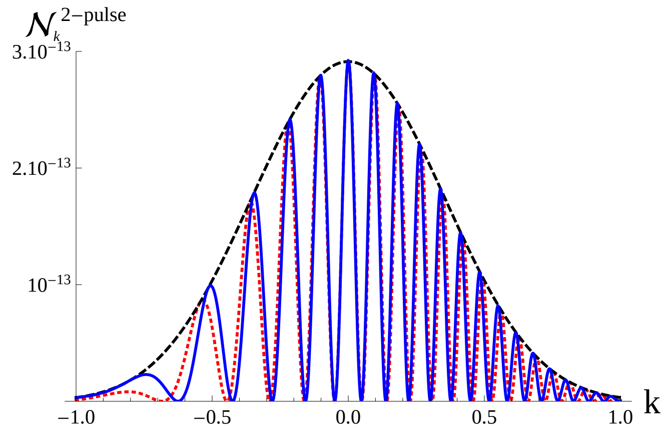

FIG. 3: Number of pairs created $\mathcal{N}_{\mathbf{k}}^{\text{2-pulse}}$, as a function of longitudinal momentum k, for the antisymmetric configuration of the two electric pulses. Here $E = .1$, $\omega = .04$, and $T = 200.2$, all in units where $m = 1$. The solid [blue] curve is the exact result, the dotted [red] curve is the approximate two-slit expression (6), and the dashed [black] envelope curve is 2^2 times the single-slit expression (10).

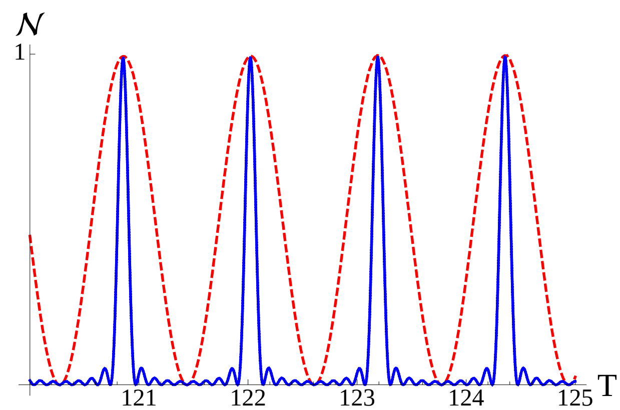

FIG. 4: The number of pairs created at the central peak value of momentum, normalized by N^2 times the single pulse result, as a function of the time delay T. The dashed [red] curve is for $N = 2$, and the solid [blue] curve is for $N = 10$.

equal for each set of turning points, and given by $K_{\mathbf{k}}$ in (10). Moreover, the phase integrals $\theta_{\mathbf{k}}^{(p,p')}$ between the real parts of the different turning points are approximately integer multiples of the phase integral $\theta_{\mathbf{k}}$ for the two-pulse case given in (12). Therefore, the sum over all turning points in (9) is coherent, leading to a simple expression for the number of created pairs,

$$\mathcal{N}_{\mathbf{k}}^{\text{N-pulse}} \approx \begin{cases} \mathcal{N}_{\mathbf{k}}^{\text{1-pulse}} \sin^2\left[N\theta_{\mathbf{k}}\right] / \cos^2\left[\theta_{\mathbf{k}}\right] & , \quad N \text{ even} \\ \mathcal{N}_{\mathbf{k}}^{\text{1-pulse}} \cos^2\left[N\theta_{\mathbf{k}}\right] / \cos^2\left[\theta_{\mathbf{k}}\right] & , \quad N \text{ odd} \end{cases} \qquad (13)$$

This result has the expected form of a Fabry-Perot interference pattern, with the single-pulse number $\mathcal{N}_{\mathbf{k}}^{\text{1-pulse}} = \exp\left[-2K_{\mathbf{k}}\right]$ from (10) playing the role of the single-slit intensity distribution, modulated by the interference term for N equally spaced slits.

Fig. 5 shows this approximate multiple-slit result (13) compared to the numerical result for the $N = 10$ antisymmetric configuration of pulses. The first observation is that the envelope does indeed behave as N^2 times the single-pulse profile, behavior characteristic of multiple-slit interference, resulting in a 100-fold increase of the central peak for the ten-slit configuration. Furthermore, we see clearly the narrowing of the central peaks, another feature of multiple-slit interference. Beyond these qualitative comments, the quantitative agreement between the semiclassical result (13) and numerics is also surprisingly good, especially for the central peaks.

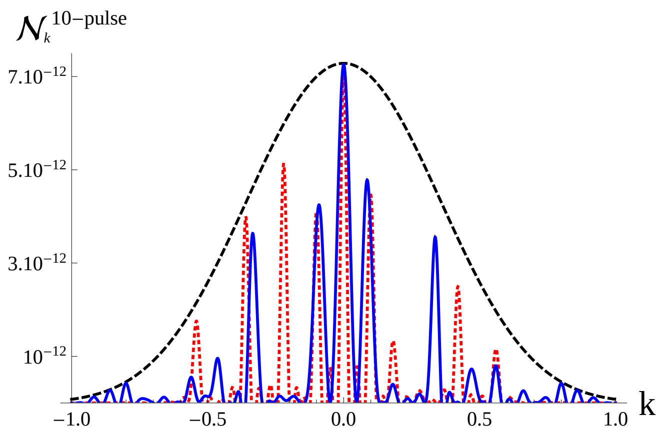

FIG. 5: As in Fig. 3, now for the $N = 10$ antisymmetric configuration of equally spaced pulses. The solid [blue] curve is the exact result, the dotted [red] curve is the approximate $N = 10$ multiple-slit expression (13), and the dashed [black] envelope is 10^2 times the single-slit expression (10).

To conclude, in this paper we have described a Ramsey multiple-time-slit interference effect for pairs created from the QED vacuum. We have shown that interference occurs for a sequence of alternating sign pulses of the electric field and we have proposed a qualitative description based on a study of avoided crossings in a two-level system. We have presented a quantitative semi-classical description which gives approximate results in excellent agreement with the exact numerical solutions. The resulting Ramsey interference leads to a coherent enhancement, which may be viewed as another route towards the Schwinger effect. The physical description in terms of quantum interference and avoided-level-crossings is versatile, and suggests that it would be worthwhile studying other more complex pulse sequences, such as periodic, quasi-periodic or disordered, that might lead to even stronger (exponential) localization of modes [36]. While our QED discussion here was for fermions, both the ideas and analysis generalize straightforwardly to bosons, suggesting potential applications to driven Bose-Einstein condensates or superfluids.

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