Shastry Replies:

B. Sriram Shastry

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Reply to Comment on Extremely Correlated Fermi Liquids

B. Sriram Shastry
Physics Department, University of California, Santa Cruz, CA 95064, USA

The Letter[1] (L) presents a framework for calculating the properties of extremely correlated Fermi liquids (ECFL) with eliminated double occupancy, where the standard Feynman diagram formalism fails. By adapting Schwinger's method of source fields to generate exact functional differential equations, it overcomes the vexing absence of Wick's theorem for non canonical electrons. Here the electronic Greens function $\mathcal{G}$ is found as the convolution of a canonical Greens function g involving a self energy Φ, and a dynamical spectral weight μ, given in terms of a second "self energy" Ψ. An expansion of the exact equations is given in a parameter λ argued to be equivalent to the particle density n. Equations for Ψ and Φ to first order in λ are written down (Eqs. L(18-20)), giving the first ω dependent corrections to the Fermi gas. These require detailed computations that are underway. Meanwhile for illustrating of the formalism, a further analytical simplification to the above first order equations is made. This "simplified ECFL theory" thereby yields the explicit analytical approximation Eq. L(26) for the one electron spectral function $\rho_{\mathcal{G}}$ of the t-J model in 2-dimension. In [2] this expression is successfully confronted with data on cuprate superconductors.

The Comment (C) questions the positivity of the spectral function of the simplified ECFL theory. As already recognized, and explicitly addressed in the Letter, maintaining positivity of the spectral function within various approximations of this framework can be challenging in general. This aspect of the formalism differs from the standard theory, and one must be prepared to make suitable physical approximations, e.g. by invoking hard or soft cutoffs. As written Eq (26) is non negative, thanks to the explicit cut off in the numerator. On the other hand, the cut off plays almost no role in the most interesting regime of this approximation relevant to ARPES, since *for most occupied states the spectral functions is already positive* (also see L. Ref[13]).

The Comment addresses the $\vec{k}$ dependence of the simplified ECFL theory[3], and compares with a dynamical mean field theory calculation (C. Ref [3]). This remark mistakenly supposes that the simplified ECFL theory is a proposed solution of the t-J model, in the limit of high dimensions. In clarification: *the Letter does not claim to solve the t-J model in high dimensions* by this approximation. The context of the simplified ECFL theory is discussed near Eq. L(24). In brief this approximation is "high dimensional" only to the extent that the two self energies *within the first order in λ equations* Eq. L(18-20) are proportional to each other, if their momentum dependence is neglected. Concerning higher order terms in λ, which must be addressed for a complete solution, we make no such claim. The term "high dimensional approximation" *thus refers only to the relationship between the two self energies in the first order in λ theory Eq. L(18-20).* Renaming the approximation as the "simplified ECFL theory" from our initial nomenclature makes its nature more clear. The ECFL solution of the t-J model for $d \to \infty$ is undoubtedly of interest. If found, it would clarify the role of the crucial variable $\frac{U}{d}$. In ECFL one assumes $\frac{U}{d} \to \infty$, while apparently $\frac{U}{d} \to 0$ in (C. Ref [3]).

The Comment remarks on the nature of the ECFL state and its purported inaccessibility from the Hubbard model. A physically meaningful and rigorous characterization of a quantum fluid is in terms of the symmetry of the ground state, and in the nature of its lowest excitations for small $(\vec{k} - \vec{k}_F, \omega)$ as reflected in $\mathcal{G}$, and as addressed by scaling theories[4]. The technique by which $\mathcal{G}$ is calculated seems peripheral to this issue. The Hubbard model has a Fermi liquid phase at suitable parameters with standard signatures[4]. The ECFL state also represents a Fermi liquid (FL) in the above sense, provided the energies $|\xi_k|, |\omega|$ are smaller than the energy scale $\Delta(\vec{k}_F, 0)$ (Eq. L(22)). Generically $\Delta(\vec{k}_F, 0)$ is non vanishing in ECFL, as in the simplified ECFL theory. Thus we asymptotically again arrive at a FL state, without a fundamental obstruction to adiabatic continuation in U between the two FL states. The theory interestingly also allows for a vanishing $\Delta(\vec{k}_F, 0)$ at special values of parameters, thereby locating possible quantum critical points. It also yields several useful results away from the asymptotic region[2]. This work was supported by DOE under Grant No. FG02-06ER46319.

[1] B. S. Shastry, Phys. Rev. Letts. **107**, 056403 (2011).
[2] G. H. Gweon, B. S. Shastry and G. D. Gu, Phys. Rev. Letts. **107**, 056404 (2011).
[3] The ECFL theory may equally well be represented in terms of a Dyson self energy Σ, as shown in B. S. Shastry, Phys. Rev. B 84, 165112 (2011). For the simplified ECFL theory $\Im m\ \Sigma(\vec{k}, \omega)$ is displayed in Figs. (8,9) of this work. It is a function of $\vec{k}$ only through the single particle energy $\varepsilon_{\vec{k}}$. This seems to be the minimal $\vec{k}$ dependence required to reconcile data with theory[2].
[4] R. Shankar, Rev. Mod. Phys. **66**, 129 (1994), D. Zanchi and H. J. Schulz, Phys. Rev. **B 54**, 9509 (1996).