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Test of Sum Rules in Nucleon Transfer Reactions

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The quantitative consistency of nucleon transfer reactions as a probe of the occupancy of valence orbits in nuclei is tested. Neutron-adding, neutron-removal and proton-adding transfer reactions were measured on the four stable, even Ni isotopes, with particular attention to the cross section determinations. The data were analyzed consistently in terms of the distorted wave Born approximation to yield spectroscopic factors. Valence-orbit occupancies were extracted, utilizing the Macfarlane-French sum rules. The deduced occupancies are consistent with the changing number of valence neutrons, as are the vacancies for protons, both at the level of <5%. While there has been some debate regarding the true 'observability' of spectroscopic factors, the present results indicate that empirically they yield self-consistent results.

The understanding of nuclear structure in terms of the shell model involves a number of approximations, but the model has been remarkably successful in describing many of the observed features of nuclei. Nucleon transfer and knockout reactions have been essential in relating these models to experimentally measurable quantities, and specifically single-particle overlaps. The energies of single-particle states and their occupancies have been mapped out by measurements of nucleon-adding and nucleon-removing transfer reactions, assuming the validity of the Macfarlane and French [1] sum rules. These sum rules express how the single-particle overlaps (spectroscopic factors, that are essentially reduced cross sections) are related to the number of vacancies or particles in an orbit with angular-momentum j^{π}. Absolute spectroscopic factors, particularly in $(e, e'p)$ reactions, have been shown to be lower than naive expectations [2], which is understood in terms of short-range correlations between nucleons—in other words the limitations of the shell model. This quenching appears to be a uniform property and does not vary appreciably from orbit to orbit, nor between neighboring nuclei.

A test of the sum rules can be made if both the nucleon-adding and nucleon-removing reactions are measured on the same target nucleus. For a given set of quantum numbers, the vacancies (derived from the summed spectroscopic factors for adding a nucleon) added to the occupancies (from the sum of spectroscopic factors for removal) should be independent of the target, and proportional to the degeneracy of the orbit in question. Although a great deal of work has been done on transfer reactions using the sum rules, their validity has not been tested very quantitatively. A more quantitative and systematic study of the internal consistency of such measurements is timely, since recently there have been several papers questioning whether spectroscopic factors are 'observables' in the formal sense, and whether the occupation numbers derived from them are meaningful [3]. The present work reports on a set of measurements on the four stable even Ni isotopes aimed at such a test.

The doubly-magic nucleus ^{56}Ni may be well described as the closure of the $0f_{7/2}$ shell with both 28 protons and neutrons. In the four stable even isotopes of Ni the neutron orbits $1p_{3/2}$, $0f_{5/2}$, and $1p_{1/2}$ are all at low excitation energy and as the number of neutrons is increased from 30 to 36 these orbits are expected to fill more or less in parallel, rather than sequentially. The subshell of 40 nucleons is not very strongly defined, and the $0g_{9/2}$ state, at slightly higher energy, may or may not participate appreciably at the Fermi surface. Earlier experiments [4] have established the rate of filling approximately, often with isolated measurements, using a variety of different instrumentation and varying assumptions in the analysis.

Experimental procedure. Precision accelerators with the requisite energies and suitable magnetic spectrographs for this type of measurement are on the verge of extinction. The present experiment was done at the Yale tandem accelerator with the Enge split-pole spectrograph and gas-filled focal plane detector. Isotopically enriched self-supporting Ni targets with nominal thickness of 200 μg/cm^2 were used. The techniques follow closely those adopted in previous work (for example, Ref. [5]). To obtain absolute cross sections, the product of target thickness and spectrograph aperture was calibrated using sub-Coulomb α-particle scattering at an incident energy of 9 MeV. Measurements were made under identical experimental conditions using the same spectrograph aperture, target, beam collimation, and beam integrator settings to minimize systematic errors.

Neutron-transfer (d, p) and (p, d) measurements were carried out with 10-MeV deuterons and 28-MeV protons, sufficiently above the Coulomb barriers to give the distinctive angular distributions and ensure a direct reaction mechanism, yet low enough to give reasonable energy resolution in the spectrograph. Helium-induced neutron transfer reactions with better momentum matching for high angular-momentum transfer were also measured, to ensure reliable information for the $\ell = 4$ transitions. The beam energies for the $(\alpha, ^3\text{He})$ and $(^3\text{He}, \alpha)$ reactions were 38 and 25 MeV, respectively.

The Ni isotopes are well studied, and the states that are significantly populated in these reactions are known with most of their spins determined [4]. The purpose of the present measurement therefore is to get a complete set of accurate cross sections that may be analyzed in a consistent manner. The quantities of interest, spectroscopic factors, are best determined from the cross sections at the maxima of the angular distributions, where the approximations in the reaction theory, the distorted wave Born approximation (DWBA), are best satisfied.

The measurements were carried out at the laboratory angles where these maxima are known to occur; for $\ell = 1$ transitions, relevant to the $p_{3/2}$ and $p_{1/2}$ orbits, 15° and 10° were used in the (d, p) and (p, d) reactions respectively, and the $\ell = 3$ measurements, relevant to the $f_{5/2}$ orbit, were taken at 35° and 25°. For the helium-induced reactions, for ℓ=3 and 4 relevant to the $f_{5/2}$ and $g_{9/2}$ orbits, the ideal angle is 0°; the actual measurements were rate limited by scattered beam and therefore carried out as far forward as possible, at 7° and 5° for $(\alpha, ^3\text{He})$ and $(^3\text{He}, \alpha)$ respectively. There is some variation in the peak cross sections due to bombarding energies and Q values; the angles chosen for the (d, p) and (p, d) reactions were different for this reason. Any additional variations were assumed to be correctly accounted for in the DWBA calculations. For the helium-induced reactions, DWBA calculations indicate that the cross section at the angles used differs from that at the peak by only a few percent.

A similar set of data was obtained for the proton-adding $(^3\text{He}, d)$ reaction at 10° and 25° for $\ell = 1$ and 3, respectively, at a beam energy of 18 MeV, as well as the (α, t) reaction at 38 MeV measured at 5°. Proton removal was not measured, partly for technical reasons and prior work indicates that $Z = 28$ is a rather good closed shell.

Normalization procedure. To obtain an accurate measure of the neutron occupancies, the spectroscopic factors need to be normalized empirically to allow for the effect of correlations [2]. We achieve this by summing the spectroscopic factors for both nucleon adding and removing reactions and choosing a normalization that requires the result to add up to the degeneracy of each orbit $(2j+1)$. With data for four Ni isotopes, four independent values of the normalization are obtained and provide some measure of consistency. The DWBA cross sections were calculated using the finite-range code PTOLEMY [6] using the Reid form for the deuteron wave function. The combination of the global potentials of Ref. [7] for protons and Ref. [8] for deuterons was used, though other potentials were also explored. The α, ^{3}He and triton potentials were those of Ref. [9].

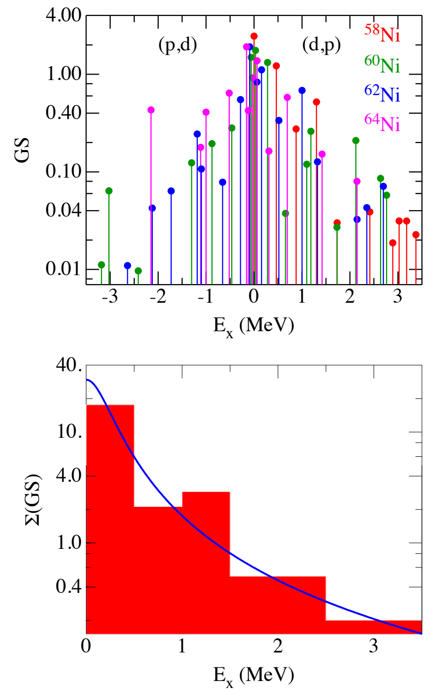

FIG. 1. (Color online) The upper figure shows the distribution of spectroscopic strengths for $\ell = 1$ for the four targets. The hole states are shown at negative energies. The lower figure shows the same data binned in absolute excitation energy. The line represents a Lorentzian shape that provides an estimate of missed strength, as discussed in the text.

According to the Macfarlane and French sum rules,

$$H = \Sigma G_+ S_{adding} \quad \text{and} \quad P = \Sigma G_- S_{removing} \qquad (1)$$

where H and P are the numbers of holes or particles, $G_+ = (2j+1)C^2$ and $G_- = C^2$, the sum is over all transitions with a given total angular momentum transfer j (for a spin 0 target $j = J$), and C^2 is the isospin-coupling Clebsch-Gordan coefficient.

To estimate possible missed strength, in Fig. 1 we plot the distribution of spectroscopic strengths for $\ell = 1$ transitions for all four isotopes, including data for both $1/2^-$ and $3/2^-$ states. The hole strengths are plotted at negative excitation energy. The shift of the single-particle centroids in excitation energy as well as the spin-orbit splitting are ignored, both these would have the effect of slightly broadening the plotted distribution. In the lower part of the figure the strengths are binned, taking both particle and hole strengths as positive excitations, and fit to a Lorentzian shape. The area under this curve above 3.5-MeV excitation is 2.8% of the total, providing an estimate of the strength that would have been missed in the present measurement.

For neutrons, we define the normalization $N_j \equiv S'/S$,

TABLE I. Normalization factors for neutron transfer.

Nucleus	$N_{\ell=1}$	$N_{\ell=3}$	$N_{\ell=3,\alpha}$
^{58}Ni	0.527	0.528	0.518
^{60}Ni	0.548	0.503	0.464
^{62}Ni	0.558	0.554	0.471
^{64}Ni	0.566	0.480	0.433
Mean	0.550±0.015	0.517±0.028	0.471±0.030

where $S' \equiv \sigma_{exp.}/\sigma_{DWBA}$.

$$N_j \equiv (\Sigma G_+ S'_{adding} + \Sigma G_- S'_{removing})/(2j+1) \qquad (2)$$

The values of the normalization are listed in the second column of Table I for the combined $\ell = 1$ strengths, since some of the spin assignments are ambiguous. They are consistent to a few percent, even though the occupancies are changing for these orbits. Since the ratio of $\ell = 1$ and $\ell = 3$ cross sections depends slightly on the choice of radii for the bound state, the normalizations for the two values were considered separately. The sensitivity to changes in the parameters specifying the bound state, including the spin-orbit term, was explored, particularly that of the ratios of DWBA cross sections for different j values. These ratios vary slightly within the range of radii used to specify the bound-state potential; for reasonable parameter choices the variation is less than about 10%. The column labeled $N_{\ell=3,\alpha}$ refers to the normalization factor for the α and ^{3}He-induced reactions for $5/2^-$ states.

There is some sensitivity with different distorting potentials. For $\ell = 1$, the normalization required to satisfy the sum rule with the first choice parameters for the distorting optical potentials [7, 8] is 0.550(15), while it is 0.641(45) for the Perey global potentials [10] or 0.567(36) for the combination of the proton parameters from Ref. [7] and deuterons from Ref. [11]. However, the relative spectroscopic factors derived from each of these different sets of optical potentials are consistent within a few percent. The various normalizations themselves are all around 0.5 - 0.6, which is gratifyingly close to the quenching deduced from $(e, e'p)$ measurements [2].

Neutron Occupancies. Using the above procedure, the mean normalizations listed in Table I were used to obtain the occupancies and vacancies from the neutron-removing and neutron-adding reactions and the results are shown in Fig. 2 for the $\ell = 1$ and 3 transitions. The filling of the orbits is evident, while the sums of these two separate measurements remain constant across the isotopes.

The $0g_{9/2}$ orbit is somewhat problematic because these states appear around 3-MeV excitation energy, where the level density is relatively high and the admixture of unobserved fragments into more complicated states is likely. No clear $\ell = 4$ transitions are observed in neutron removal from ^{58}Ni or ^{60}Ni, while the summed strength

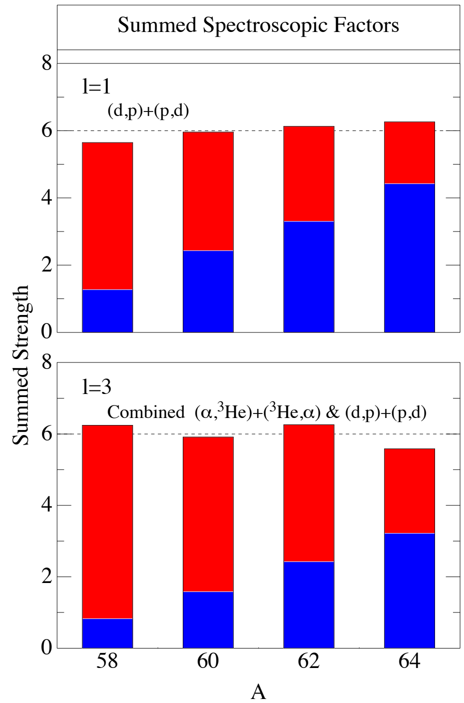

FIG. 2. (Color online) The strengths for the total $\ell = 1$ ($j = 1/2^-$ and $3/2^-$) and 3 ($j = 5/2^-$) spectroscopic factors for neutron adding and removing reactions summed according to Equation 1. Spectroscopic factors from neutron transfer are shown in the upper and lower boxes respectively. The partition between the occupancy (blue) from neutron removal, and vacancy (red) from neutron-adding is shown.

TABLE II. Neutron Occupancies

Nucleus	$1p_{3/2}$	$0f_{5/2}$	$1p_{1/2}$	$0g_{9/2}$	Total
^{58}Ni	0.96	0.67	0.40	0	2.03
^{60}Ni	1.74	1.61	0.71	0	4.06
^{62}Ni	2.31	2.31	0.93	0.34	5.89
^{64}Ni	3.17	3.41	1.07	0.66	8.31

for adding a $g_{9/2}$ neutron changes from 6.0 in ^{58}Ni to 9.5 in ^{64}Ni, suggesting that a substantial fraction of the strength is missing, at least in the lighter Ni isotopes.

Occupancies can be extracted in two ways, either from the neutron-removing reactions directly, or from an independent set of measurements, from the vacancies obtained using the neutron-removing reactions and subtracting these from the $2j+1$ degeneracy of the orbit. We took the average of the two, which amounts to taking the *difference*. The occupancies derived from our data are summarized in Fig. 3 and Table II. The $g_{9/2}$ occupancies given are derived only from the removal reaction, using the normalization obtained for the $5/2^-$ states from the α and ^{3}He induced reactions.

Proton Vacancies. The measurements of the proton-

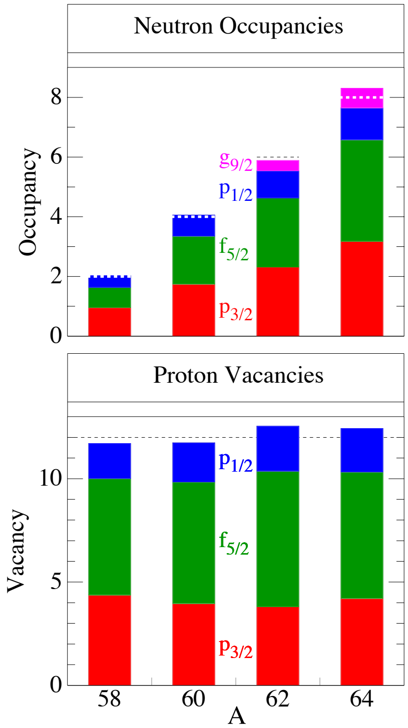

FIG. 3. (Color online) The neutron occupancies and the proton vacancies of the four Ni isotopes as derived from the summed (normalized) spectroscopic strengths. The dashed lines indicate the expected values.

adding (^{3}He,d) and (α, t) reactions were carried out in the same experimental run as the neutron-transfer reactions with the same targets and apertures. The analysis was completed with a similar consistent normalization procedure for the four target nuclei. The normalizations obtained, taking $Z = 28$ to be a closed shell so that the valence orbits are effectively vacant, were 0.63(4) and 0.51(7) for the $\ell = 1$ and 3 (^{3}He,d) transitions, respectively, and 0.90(6) for the $\ell = 3$ transitions in (α, t). The upper isospin component in the sums [12] was not measured directly, but was deduced from the neutron-adding measurements discussed above. The summed vacancies in the four isotopes are very nearly constant at 12.0(3), and the ratio between the different j values is very close to expectations as is shown in Fig. 3. The $g_{9/2}$ strength is again at higher excitation energy and apparently not fully covered in these measurements.

Discussion. Uncertainties in the occupancies and vacancies are difficult to estimate; the statistical uncertainties are small compared to systematic effects, such as possible missed states or the effect of multi-step mechanisms contributing to the reactions. As was pointed out by Ref. [3], the model-dependencies imply that the spectroscopic factors are perhaps not rigorous observables.

Empirically, however, the nucleon occupancies extracted from the measured spectroscopic factors do behave as expected. The summed neutron occupancies of 2.0, 4.1, 5.9, and 8.3 are consistent with the expected 2, 4, 6, and 8 across the Ni isotopes. Similarly the proton vacancies should remain equal to 12 and the measured values of 11.7, 11.7, 12.5, and 12.4 are consistent with this. The rms deviations with a fixed normalization procedure are a few percent. For the neutron normalization, we have relied only on the summed addition plus removal strengths. All the fp neutron orbits seem to be filling more or less in parallel but the $g_{9/2}$ is lagging behind, and becomes apparent only starting with ^{62}Ni.

The data indicate that even though spectroscopic factors may not strictly be true 'observables', this treatment of reaction cross sections does seem to provide a self-consistent description of occupancies, as two independent checks indicate:

1. The sum rules are satisfied in a consistent way over a series of isotopes where the neutron occupancies change. They are also consistent for protons where the occupancy remains the same.

2. The difference between neutron holes and particles changes in a way consistent with the expected populations.

The method of extracting overlaps with single-particle states using an internally consistent normalization procedure seems to work satisfactorily. Apparently spectroscopic factors *do* provide valuable and consistent information on the structure of nuclei. A better understanding of why this empirical treatment works rather well needs to be clarified in terms of the approximations that are made in the reaction theory. A more complete publication of these data is in preparation.

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