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Rahul Sinha, N. G. Deshpande, Sandip Pakvasa, and Chandradew Sharma

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Determination of Weak Amplitudes using Bose Symmetry and Dalitz Plots

Rahul Sinha,[1] N. G. Deshpande,[2] Sandip Pakvasa,[3] and Chandradew Sharma[4]

[1] *The Institute of Mathematical Sciences, Taramani, Chennai 600113, India*
[2] *Institute of Theoretical Science, University of Oregon, Eugene, OR 94703, USA*
[3] *Department of Physics and Astronomy, University of Hawaii, Honolulu, HI 96822, USA*
[4] *Dept. of Physics,BITS, PILANI-K.K. Birla Goa Campus, Zuarinagar, Goa 403726, India*

We present a new method using Dalitz plot and Bose symmetry of pions that allows the complete determination of the magnitudes and phases of weak decay amplitudes. We apply the method to process like $B \to K^*\pi$, with the subsequent decay of $K^* \to K\pi$. Our approach enables the additional measurement of an isospin amplitude without any theoretical assumption. This advance will help in measuring weak phase and probing for new physics beyond standard model with fewer assumptions.

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Hadronic weak decays are important in extracting CP violating phases and probing the effects of physics beyond the standard model. Our inability to calculate all hadronic effects accurately has made these tasks challenging. While significant progress has been made in estimating hadronic effects, one still needs to use symmetry arguments, such as SU(3), to reduce the number of hadronic parameters to be calculated. An alternative helpful approach is to look for innovative methods that enable obtaining more observables, thereby reducing the dependence on theoretical assumptions. In this letter, we present a new method based on Dalitz plot, isospin, and Bose symmetry that enables the measurement of extra observables and allows for a complete determination of all the weak decay amplitudes and phases. We present the method through the concrete example of $B \to K^*\pi$ where the K^* [15] decays into $K\pi$. We show how the consequences of Bose symmetry between the two final state pions enables one additional measurement–the direct measurement of the the $A_{3/2}$ amplitude, which is crucial in determining the CP violating phase and looking for new physics; our method is the only one not needing any extra assumptions.

The large number of D and B mesons produced in heavy flavor facilities has prompted a revival of Dalitz plot analysis approach to study their decays [2–10]. It is well known that identical bosons obey Bose symmetry in the Dalitz plot distribution, and amplitudes must be written in terms of isospin and spatial parts in such a way that overall symmetry under permutation of identical particles is obeyed. Historically, this fact has been noted in the Dalitz plot study involving three pion decay of mesons [11].

The four decay modes $B^0 \to K^{*+}\pi^-$, $B^+ \to K^{*0}\pi^+$, $B^0 \to K^{*0}\pi^0$ and $B^+ \to K^{*+}\pi^0$ can be described using isospin in a fashion analogous to the decays $B \to K\pi$. The isospin $I = \frac{1}{2}$ initial state decays to a final state that can be decomposed into either $I = \frac{1}{2}$ or $I = \frac{3}{2}$ via a Hamiltonian that allows $\Delta I = 0$ or $\Delta I = 1$ transitions. The transition $\Delta I = 0$ results only in a single amplitude with final state $I = \frac{1}{2}$ labeled as $B_{1/2}$, whereas the transition with $\Delta I = 1$ can results in two amplitudes with $I = \frac{1}{2}$ or $I = \frac{3}{2}$ represented as $A_{1/2}$ and $A_{3/2}$ respectively. The isospin amplitudes $A_{1/2}$, $A_{3/2}$ and $B_{1/2}$ are themselves defined [1] in terms of the Hamiltonian to be:

$$
\begin{aligned}
A_{1/2} &= \pm\sqrt{\tfrac{2}{3}}\langle\frac{1}{2},\pm\frac{1}{2}|\mathscr{H}_{\Delta I=1}|\frac{1}{2},\pm\frac{1}{2}\rangle\ , \\
A_{3/2} &= \sqrt{\tfrac{1}{3}}\langle\frac{3}{2},\pm\frac{1}{2}|\mathscr{H}_{\Delta I=1}|\frac{1}{2},\pm\frac{1}{2}\rangle\ , \\
B_{1/2} &= \sqrt{\tfrac{2}{3}}\langle\frac{1}{2},\pm\frac{1}{2}|\mathscr{H}_{\Delta I=0}|\frac{1}{2},\pm\frac{1}{2}\rangle\ .
\end{aligned}
\tag{1}
$$

The four decays are described in terms of three isospin amplitudes $A_{1/2}$, $A_{3/2}$ and $B_{1/2}$ as follows:

$$
\begin{aligned}
\mathcal{A}^{-+} &= \mathcal{A}(B^0 \to K^{*+}\pi^-) = A_{3/2} + A_{1/2} - B_{1/2}, \\
\mathcal{A}^{+0} &= \mathcal{A}(B^+ \to K^{*0}\pi^+) = A_{3/2} + A_{1/2} + B_{1/2}, \\
\mathcal{A}^{00} &= \sqrt{2}\,\mathcal{A}(B^0 \to K^{*0}\pi^0) = 2A_{3/2} - A_{1/2} + B_{1/2}, \\
\mathcal{A}^{0+} &= \sqrt{2}\,\mathcal{A}(B^+ \to K^{*+}\pi^0) = 2A_{3/2} - A_{1/2} - B_{1/2}.
\end{aligned}
\tag{2}
$$

These amplitudes satisfy the identity $\mathcal{A}^{00} + \mathcal{A}^{-+} = \mathcal{A}^{+0} + \mathcal{A}^{0+}$ and may be represented by a quadrilateral in the complex plane shown in Fig. 1. Unfortunately, much like the description of $B \to K\pi$ decays, the four branching ratios are not enough to fix the three complex isospin amplitudes and assumptions like SU(3) have been used to understand these decays. We will show how Bose symmetry can be used to obtain the isospin amplitudes directly.

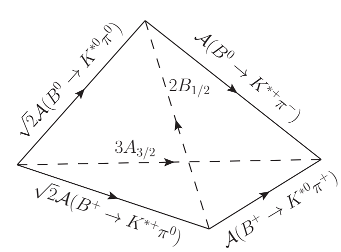

FIG. 1: The four $B \to K^*\pi$ amplitudes related by isospin (see Eq. (2)) . The quadrangle is fixed once $A_{3/2}$ is known.

The $K^*\pi$ final state has the distinct advantage over $K\pi$, as it allows for a model independent measurement of all the isospin amplitudes. To understand better how we achieve this consider for example the decay $B^+ \to K^{*0}\pi^+$, with K^{*0} decaying to $K^0\pi^0$. We hence have the final state $K^0\pi^0\pi^+$. The two pions, $\pi^+\pi^0$ can either have a net isospin $I_{\pi\pi} = 1$ or $I_{\pi\pi} = 2$. The key point we rely on is that Bose symmetry demands that the isospin state $I_{\pi\pi} = 2$ requires that the two pions are in spatially even state. The state $K^0\pi^0\pi^+$ with the pions being in the even state cannot arise from final isospin 1/2 state, and can only arise from final isospin 3/2 state. Thus isolating the state with two pions in the spatially even state is equivalent to isolating the $I_{\pi\pi} = 2$ component, which would provide a measurement of the isospin 3/2 amplitude of $K^0\pi^0\pi^+$ and hence the $I = 3/2$ component of $K^*\pi$, since the strong decay $K^* \to K\pi$ conserves isospin. This provides an *additional observable $A_{3/2}$, thereby enabling the quadrangle depicted in Figure 1 to be completely fixed.* The decays $B \to K\pi\pi$ have been studied earlier [3, 6, 8, 10], but the methods proposed there do not permit determination of all the amplitudes without additional assumptions and (or) modes being considered.

Once $A_{3/2}$ is measured the quadrangle is completely fixed. One has a total of 'eleven' observables: the four decay rates for each B and $\bar{B}$, $A_{3/2}$ and its conjugate mode equivalent $\bar{A}_{3/2}$ and the time dependent asymmetry relating the angle between $A(B \to K^{*0}\pi^0)$ and $A(\bar{B} \to \bar{K}^*\pi^0)$. This provides just enough observables to solve all the 'six' topological amplitudes [12] T, C, P, P_{EW}, P_{EW}^C and A and their 'five' relative phases purely in terms of observables and the weak phase $\gamma(\phi_3)$ which can be measured elsewhere. This would provide valuable information on hadronic parameters and enable clean test of physics beyond the Standard Model. Alternatively, one can measure the weak phase $\gamma(\phi_3)$ [13] with fewer assumptions about hadronic matrix elements, since we have obtained two extra observables.

We now consider the decay $B(P) \to K(p_1)\pi(p_2)\pi(p_3)$ in the Gottfried-Jackson frame with B moving in the $\hat{z}$ axis such that the two pions go back to back with $\pi(p_2)$ at an angle θ with $K(p_1)$. In this frame $\vec{p_2} + \vec{p_3} = \vec{0}$. We define $s \equiv (p_2 + p_3)^2 = (P - p_1)^2$, $t \equiv (p_3 + p_1)^2 = (P - p_2)^2$ and $u \equiv (p_1 + p_2)^2 = (P - p_3)^2$. t and u can be written as:

$$t \equiv a + b\,\cos\theta\ , \qquad (3)$$

$$u \equiv a - b\,\cos\theta\ , \qquad (4)$$

where,

$$a = \frac{M^2 + m_K^2 + 2m_\pi^2 - s}{2} \qquad (5)$$

$$b = \frac{\sqrt{s - 4m_\pi^2}}{2\sqrt{s}}\lambda^{1/2}(M^2, m_K^2, s) \qquad (6)$$

and $\lambda(M^2, m_K^2, s) = (M^4 + m_K^4 + s^2 - 2M^2m_K^2 - 2M^2s - 2m_K^2s)$.

Let us now consider the three body final state $K\pi\pi$. Since the final state carries two pions which respect Bose symmetry, the final state should have an overall symmetry under isospin and space for the two pions, i.e. isospin odd states must be odd under exchange of the two pions and the isospin even states must be even under the exchange of the two pions. We now construct the isospin states of $|K\pi\pi\rangle_I$. Note, we have placed a subscript 'I' to indicate this is just the isospin part of the state and that the state $|K\pi\pi\rangle$ will include the spatial dependence. The isospin states are obtained as follows:

$$|K^0\pi^0\pi^+\rangle_I = \frac{1}{\sqrt{5}}|\frac{5}{2},\frac{1}{2}\rangle_{\rm e} + \sqrt{\frac{3}{10}}|\frac{3}{2},\frac{1}{2}\rangle_{\rm e} - \frac{1}{\sqrt{6}}|\frac{3}{2},\frac{1}{2}\rangle_{\rm o} - \frac{1}{\sqrt{3}}|\frac{1}{2},\frac{1}{2}\rangle_{\rm o}, \qquad (7)$$

where the subscript 'e' and 'o' mean that the two pions in the state are in an 'even' and 'odd' state respectively. We note that these subscripts are introduced only to take note of the isospin symmetry of the two pions.

The complete state $|K^0\pi^0\pi^+\rangle$ resulting from B decay is then easily written as:

$$|K^0\pi^0\pi^+\rangle = \Big(\frac{1}{\sqrt{5}}|\frac{5}{2},\frac{1}{2}\rangle_{\rm e} + \sqrt{\frac{3}{10}}|\frac{3}{2},\frac{1}{2}\rangle_{\rm e}\Big)X - \Big(\frac{1}{\sqrt{6}}|\frac{3}{2},\frac{1}{2}\rangle_{\rm o} + \frac{1}{\sqrt{3}}|\frac{1}{2},\frac{1}{2}\rangle_{\rm o}\Big)Y\cos\theta, \qquad (8)$$

where X and $Y\cos\theta$ are introduced to take care of the spatial and kinematic contributions as is seen from the discussion above (see Eqns. (3) and (4)). In general, X and Y can be arbitrary even functions of $\cos\theta$. We retain the subscripts e and o merely to track the even or odd isospin state of the two pion in the three-body final state.

We define $C_{1/2}$, $C_{3/2}$ and $D_{1/2}$ as the three-body isospin amplitudes analogously to the two-body amplitudes $A_{1/2}$, $A_{3/2}$ and $B_{1/2}$ defined in Eq. (1). We further add a superscript 'e' or 'o' for amplitudes arising from even or odd isospin states respectively. The amplitudes for the decays $B^+ \to K^0\pi^0\pi^+$ is:

$$A(B^+ \to K^0\pi^0\pi^+) = \frac{3}{\sqrt{10}}C_{3/2}^{\rm e}\,X - \Big[\frac{1}{\sqrt{2}}C_{3/2}^{\rm o} + \frac{1}{\sqrt{2}}(C_{1/2}^{\rm o} + D_{1/2}^{\rm o})\Big]\,Y\cos\theta\ . \qquad (9)$$

The other charged B decay amplitudes are:

$$A(B^+ \to K^+\pi^-\pi^+) = \Big[-\frac{1}{\sqrt{5}}C_{3/2}^{\rm e} + \frac{1}{\sqrt{2}}(C_{1/2}^{\rm e} + D_{1/2}^{\rm e})\Big] X + \Big[\frac{1}{2}(C_{1/2}^{\rm o} + D_{1/2}^{\rm o}) - C_{3/2}^{\rm o}\Big]Y\cos\theta, \qquad (10)$$

$$A(B^+ \to K^+\pi^0\pi^0) = -\Big[\frac{2}{\sqrt{5}}C_{3/2}^{\rm e} + \frac{1}{\sqrt{2}}(C_{1/2}^{\rm e} + D_{1/2}^{\rm e})\Big]X \qquad (11)$$

$$A(B^+ \to K^0\pi^+\pi^0) = \frac{3}{\sqrt{10}} C^{\rm e}_{3/2}\, X + \Big[\frac{1}{\sqrt{2}} C^{\rm o}_{3/2} + \frac{1}{\sqrt{2}}(C^{\rm o}_{1/2} + D^{\rm o}_{1/2})\Big]\, Y \cos\theta \ . \quad (12)$$

The amplitudes for the neutral B decay modes can analogously be written. We emphasize again that the amplitudes expressed in Eq. (9)–(12) are explicitly Bose symmetric. One may also note that while we have considered B^+ decays explicitly, the same analysis could equally well have been done with B^0 decays.

We now discuss an alternate approach, where we consider the decay as a two step process. The decays $B \to K^*\pi$ are described by the amplitudes given in Eq. (2). The K^{*0} resonance decays by strong interaction into two modes: $K^0\pi^0$ and $K^+\pi^-$. Using isospin the states $|K^{*0}\pi^+\rangle$ and $|K^{*+}\pi^0\rangle$ may hence be expressed in terms of the three body finals states as,

$$|K^{*0}\pi^+\rangle = \sqrt{\frac{1}{3}}\,|[K^0\pi^0]\pi^+\rangle - \sqrt{\frac{2}{3}}\,|[K^+\pi^-]\pi^+\rangle, \quad (13)$$

$$|K^{*+}\pi^0\rangle = -\sqrt{\frac{1}{3}}\,|[K^+\pi^0]\pi^0\rangle + \sqrt{\frac{2}{3}}\,|[K^0\pi^+]\pi^0\rangle \ . \quad (14)$$

Here the square bracket denotes that the particles arise from a K^* resonance. The matrix element for the two step decay $B^+ \to [K^0\pi^0]\pi^+$ is then given by:

$$\mathcal{M}(B^+ \to [K^0\pi^0]\pi^+) = \frac{g_{K^*K\pi}}{\sqrt{3}}\,(A_{3/2} + A_{1/2} + B_{1/2}) \times (P+p_3)^\mu (p_1-p_2)^\nu \frac{\Big(-g_{\mu\nu} + \frac{(p_1+p_2)_\mu (p_1+p_2)_\nu}{m^2_{K^*}}\Big)}{u - m^2_{K^*} + i m_{K^*}\Gamma_{K^*}}$$

where $g_{K^*K\pi}$ takes care of the couplings and other proportionality terms in the expression for the amplitude. The term $(P+p_3).(p_1-p_2)$ is easily to be $t-s$. Hence,

$$\mathcal{M}(B^+ \to [K^0\pi^0]\pi^+) = \frac{g_{K^*K\pi}}{\sqrt{3}}(A_{3/2} + A_{1/2} + B_{1/2}) \times \left(\frac{s-t+c}{u - m^2_{K^*} + i\, m_{K^*}\Gamma_{K^*}}\right). \quad (15)$$

The amplitude corresponds to a K^{*0} resonance at $u = m_{K^*}$ on the Dalitz plot. Note that the amplitude can be separated into two parts – the isospin amplitude and the spatial part of the amplitude given by the large round bracket. Finally, the constant c is given by

$$c = \frac{(M^2 - m_\pi^2)(m_K^2 - m_\pi^2)}{m^2_{K^*}} \ . \quad (16)$$

Similarly with an intermediate K^{*+} resonance one obtains,

$$\mathcal{M}(B^+ \to [K^0\pi^+]\pi^0) = \frac{g_{K^*K\pi}}{\sqrt{3}}(2A_{3/2} - A_{1/2} - B_{1/2}) \times \left(\frac{s-u+c}{t - m^2_{K^*} + i\, m_{K^*}\Gamma_{K^*}}\right) \ , \quad (17)$$

as the amplitude corresponding to the resonance K^{*+} resonance at $t = m_{K^*}$.

Clearly Eqns. (15) and (17) taken separately are not of the form depicted in Eqns. (9) and (12) and do not respect overall Bose symmetry as is required. The two body even and odd isospin amplitudes for the mode $B^+ \to K^{*0(+)}\pi^{+(0)}$ are given by the sum and difference of the amplitudes for $B^+ \to K^{*0}\pi^+$ and $B^+ \to K^{*+}\pi^0$, and are defined to be A_e and A_o respectively [16]. We then have,

$$A_e = \frac{g_{K^*K\pi}}{\sqrt{3}}\frac{3}{2}\,A_{3/2}, \quad (18)$$

$$A_o = \frac{g_{K^*K\pi}}{\sqrt{3}}\frac{1}{2}(-A_{3/2} + 2\,A_{1/2} + 2\,B_{1/2}) \quad (19)$$

The sum of the matrix element of the two contributing modes is Bose symmetric and may be written in an explicitly symmetric form as:

$$\mathcal{M}(B^+ \to [K\pi]\pi) = \Bigg[A_e\Big(\frac{s-t+c}{u - m^2_{K^*} + i\, m_{K^*}\Gamma_{K^*}} + \frac{s-u+c}{t - m^2_{K^*} + i\, m_{K^*}\Gamma_{K^*}}\Big) + A_o\Big(\frac{s-t+c}{u - m^2_{K^*} + i\, m_{K^*}\Gamma_{K^*}} - \frac{s-u+c}{t - m^2_{K^*} + i\, m_{K^*}\Gamma_{K^*}}\Big)\Bigg]. \quad (20)$$

We note that using Eqns. (18) and (19) we recover the sum of Eqns. (15) and (17). It now follows that:

$$|\mathcal{M}(B^+ \to [K\pi]\pi)|^2 = \frac{f_1|A_e|^2 + f_2{\rm Re}(A_e A_o^*) + f_3{\rm Im}(A_e A_o^*) + f_4|A_o|^2}{\Big((m^2_{K^*} - t)^2 + m^2_{K^*}\Gamma^2_{K^*}\Big)\Big((m^2_{K^*} - u)^2 + m^2_{K^*}\Gamma^2_{K^*}\Big)} \quad (21)$$

where the denominator can be expanded as

$$\big((m_{K^*}^2-t)^2+m_{K^*}^2\Gamma_{K^*}^2\big)\big((m_{K^*}^2-u)^2+m_{K^*}^2\Gamma_{K^*}^2\big) = \big((a-m_{K^*}^2)^2+m_{K^*}^2\Gamma_{K^*}^2\big)^2-2b^2\cos^2\theta((a-m_{K^*}^2)^2-m_{K^*}^2\Gamma_{K^*}^2\big)+b^4\cos^4\theta,$$

and after some simplification we find,

$$f_1 = 4\,(-3a+c+Q)^2\big((a-m_{K^*}^2)^2+m_{K^*}^2\Gamma_{K^*}^2\big)+8b^2(a-m_{K^*}^2)(3a-c-Q)\cos^2\theta+4b^4\cos^4\theta \quad (22)$$

$$f_2 = 8b\big((3a-c-Q)\big(m_{K^*}^2\Gamma_{K^*}^2+(m_{K^*}^2-a)(-4a+c+m_{K^*}^2+Q)\big)-b^2(-4a+c+m_{K^*}^2+Q)\,\cos^2\theta\big)\,\cos\theta \quad (23)$$

$$f_3 = 8\,b\,m_{K^*}\Gamma_{K^*}\big(-(3a-c-Q)^2+b^2\cos^2\theta\big)\,\cos\theta \quad (24)$$

$$f_4 = b^2\cos^2\theta\big((-4a+c+m_{K^*}^2+Q)^2+m_{K^*}^2\Gamma_{K^*}^2\big), \quad (25)$$

where $Q = M^2+m_K^2+2m_\pi^2$. It is obvious that the even part of $|\mathcal{M}(B^+\to[K\pi]\pi)|^2$ can be obtained by adding to itself the same term with t and u interchanged. This can be carried out in the Dalitz plot by reflecting the data around $t=u$ line and adding it. By fitting this to a polynomial in $\cos^2\theta$, it is straightforward to extract $|A_e|$ and thus $|A_{3/2}|$ using Eq. (18).

We have been able to extract the even and odd parts by symmetrization, achieved by adding the amplitudes of the contribution from two resonances on the Dalitz plot that are related by the exchange of two pions. The reader may wonder how the even and odd parts for the mode $B^+\to[K^+\pi^-]\pi^+$ could be separated, since there exists no resonance if π^+ and π^- are exchanged. Note that on the K^* resonance the two-body amplitudes $A_{1/2}$, $A_{3/2}$ and $B_{1/2}$ are related to the three-body amplitudes $C^{\rm e,o}_{1/2}$, $C^{\rm e,o}_{3/2}$ and $D^{\rm e,o}_{1/2}$. Comparing Eqns. (18) and (19) with Eqns. (9) or (12) and since $B\to[K^+\pi^0]\pi^0$ is purely even we derive:

$$C^{\rm e}_{3/2}=\sqrt{\frac{5}{6}}A_{3/2},\; C^{\rm e}_{1/2}=-\frac{A_{1/2}}{\sqrt{3}},\; D^{\rm e}_{1/2}=-\frac{B_{1/2}}{\sqrt{3}},$$
$$C^{\rm o}_{3/2}=\frac{A_{3/2}}{\sqrt{6}},\; C^{\rm o}_{1/2}=-\sqrt{\frac{2}{3}}A_{1/2},\; D^{\rm o}_{3/2}=-\sqrt{\frac{2}{3}}B_{1/2}.$$

Using these relations in Eq. (10) which is already symmetric we find that the even and odd parts of the isospin contribution to $B^+\to[K^+\pi^-]\pi^+$ are equal, and thus even the one surviving pole satisfies Bose symmetry.

To summarize, we have shown how Dalitz plot, isospin, and Bose symmetry can be used to obtain extra observables without any theoretical assumptions. We demonstrate the usefulness of this observation by developing the method to determine completely all the weak decay amplitudes for $B\to K^*\pi$. Our new approach would provide valuable information on hadronic parameters, enable clean test of physics beyond the Standard Model and also help in measuring the weak phase $\gamma(\phi_3)$ all with fewer assumptions about hadronic matrix elements. The method has several further applications in three body decays of D and B mesons [14].

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[1] H. J. Lipkin, Y. Nir, H. R. Quinn and A. Snyder, Phys. Rev. D **44**, 1454 (1991).

[2] E. Barberio *et al.* [Heavy Flavor Averaging Group], arXiv:0808.1297 [hep-ex], and online update at `http://www.slac.stanford.edu/xorg/hfag`

[3] N. G. Deshpande, N. Sinha and R. Sinha, Phys. Rev. Lett. **90**, 061802 (2003) [arXiv:hep-ph/0207257].

[4] M. Gronau and J. L. Rosner, Phys. Lett. B **564**, 90 (2003) [arXiv:hep-ph/0304178], M. Gronau and J. L. Rosner, Phys. Rev. D **72**, 094031 (2005) [arXiv:hep-ph/0509155].

[5] H. Y. Cheng, C. K. Chua and A. Soni, Phys. Rev. D **72**, 094003 (2005) [arXiv:hep-ph/0506268].

[6] M. Ciuchini, M. Pierini and L. Silvestrini, Phys. Rev. D **74**, 051301 (2006) [arXiv:hep-ph/0601233].

[7] M. Gaspero, B. Meadows, K. Mishra, A. Soffer, Phys. Rev. **D78**, 014015 (2008). [arXiv:0805.4050 [hep-ph]].

[8] M. Gronau, D. Pirjol and J. L. Rosner, Phys. Rev. D **81**, 094026 (2010) [arXiv:1003.5090 [hep-ph]].

[9] B. Bhattacharya and J. L. Rosner, Phys. Rev. D **82**, 114032 (2010) [arXiv:1010.1770 [hep-ph]].

[10] N. L. Lorier, M. Imbeault and D. London, arXiv:1011.4972 [hep-ph], M. Imbeault, N. L. Lorier and D. London, arXiv:1011.4973 [hep-ph].

[11] C. Zemach, Phys. Rev. **133**, B1201 (1964); C. Kacser, S. P. Rosen, Phys. Rev. **D18**, 3427 (1978); C. Kacser, S. P. Rosen, Phys. Rev. **D18**, 3434 (1978).

[12] M. Gronau, O. F. Hernandez, D. London and J. L. Rosner, Phys. Rev. D **50**, 4529 (1994).

[13] M. Neubert, J. L. Rosner, Phys. Lett. **B441**, 403-409 (1998); M. Gronau, Phys. Rev. **D62**, 014031 (2000).

[14] Rahul Sinha, Tom Browder, N. G. Deshpande, Alexei Garmash and Sandip Pakvasa, under preparation.

[15] By K^* we mean $K^*(892)$ or $K^*(1430)$ and similar resonances. The detailed analysis presented here assumes $J^P=1^-$ K^* resonance.

[16] We can include $K^0\rho^+$ contribution, but since we are only interested in A_e, and ρ^+ only gives pions in the odd state, we do not include this contribution here. Our extraction of A_e is still unaffected. A_e does, however, receive contributions from finalstates such as $K^+f_0(980)$, at fixed val-

ues of s. The contributions arising from such resonances in the $\pi\pi$ channel populate different parts of Dalitz plot with different θ distribution compared with K^* and can be easily excluded by conventional techniques. Studying the distribution in terms of θ does not increase the uncertainty in the extraction of the amplitudes.