# Mean Velocity Profile in a Sheared and Thermally Stratified Atmospheric Boundary Layer

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# The mean velocity profile in a sheared and thermally stratified atmospheric boundary layer

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A stability correction function $\phi_m(\zeta)$ that accounts for distortions to the logarithmic mean velocity profile (MVP) in the lower atmosphere caused by thermal stratification was proposed by Monin and Obukhov in the 1950s using dimensional analysis. Its universal character was established from many field experiments. However, theories that describe the canonical shape of $\phi_m(\zeta)$ are still lacking. A previous link between the spectrum of turbulence and the MVP is expanded here to include the effects of thermal stratification on the turbulent kinetic energy dissipation rate and eddy-size anisotropy. The resulting theory provides a novel explanation for the power-law exponents and coefficients already reported for $\phi_m(\zeta)$ from numerous field experiments.

Most human activity and biological processes occur within the lower atmosphere, a thermally stratified region characterized by shear and buoyancy-driven turbulence. Thermal stratification arises because of diurnal heating and cooling resulting in finite sensible heat flux ($H_s$) at the earth's surface, while turbulence is mechanically produced due to the reduced mean velocity near the ground. The co-existence of shear- and buoyancy- generated turbulence leads to many difficulties in describing the flow properties in the lower atmosphere. Even for a stationary, horizontally homogeneous, high Reynolds number flow above an infinite flat and heated (or cooled) surface, the description of elementary flow statistics such as the mean velocity profile (MVP) has resisted complete theoretical treatment. There are inklings of a possible universal behavior in the MVP across a wide range of thermal stratification conditions as demonstrated by the collapse of data from multiple field experiments using dimensional analysis, known as Monin-Obukhov Similarity Theory [1, 2]. Indeed, this dimensional analysis proved so successful that it led some [3] to state that "*with proper non-dimensionalization, all flow statistics in the surface layer can be reduced to a set of universal curves*".

Monin-Obukhov Similarity Theory [1, 2] argues that a non-dimensional MVP is given as

$$\frac{d\overline{u}}{dz}\frac{\kappa_\nu z}{u_*} = \phi_m(\zeta), \tag{1}$$

where $\overline{u}$ is the horizontal mean velocity, over-bar is Reynolds averaging, $u_* = \sqrt{\tau_o/\rho}$ is the friction velocity, $\tau_o$ is the ground shear stress, $\rho$ is the mean air density, $\kappa_\nu$ is the von Karman constant, $z$ is the height from the ground surface, $\zeta = z/L$ is the atmospheric stability parameter, and $L$ is the Obukhov length given as [4] $L = -u_*^3/\kappa_\nu/\left(\frac{g}{T_v}\frac{H_s}{\rho C_p}\right)$ , where $g$ is the gravitational acceleration, $T$ is the mean virtual potential temperature, and $C_p$ is the specific heat capacity of dry air at constant pressure. The $\phi_m(\zeta)$ is a dimensionless stability correction function that cannot generally be inferred from dimensional considerations alone and must be determined from empirical data.

The so-called Businger-Dyer (BD) stability correction function has proved successful in fitting numerous field experiments reporting $\phi_m(\zeta)$, including the results of the classic Kansas experiments [5] shown in Figure 1. These $\phi_m(\zeta)$ functions are used in virtually all climate, atmospheric, hydrologic, and ecological applications or models of land-surface processes when land surface fluxes are to be coupled to the state of the atmosphere [6, 7]. The BD $\phi_m(\zeta)$ is expressed as [8] $\phi_m(\zeta) = (1 - 16\zeta)^{-1/4}$ when $\zeta < 0$ (hereafter referred to as unstable conditions characterized by $H_s > 0$ during daytime) and $\phi_m(\zeta) = (1+4.7\zeta)$ when $\zeta > 0$ (hereafter referred to as stable conditions characterized by $H_s < 0$ as may occur during night-time). For non-thermally stratified yet sheared ($u_* > 0$) flows (e.g., neutral conditions, $H_s = 0$, $L \to \infty$, and therefore $\zeta = 0$),$\phi_m(0) = 1$, thereby recovering the well-known 'log-law of the wall' [2] $d\overline{u}/dz = u_*/(\kappa_\nu z)$. A major critique of the BD $\phi_m(\zeta)$ is their failure to recover the well-known free convection limit [9] given as $\phi_m(\zeta) \sim \zeta^{-1/3}$ so as to cancel out any $u_*$ dependence in the MVP when $-\zeta$ is very large [10]. This critique was partially addressed by the so-called KEYPS or the O'KEYPS equation (after Obukhov, Kazansky, Ellison, Yamamoto, Panofsky, and Sellers) given by [11–13]

$$[\phi_m(\zeta)]^4 - \gamma\zeta\,[\phi_m(\zeta)]^3 = 1, \tag{2}$$

whose analytical solution for $\phi_m(\zeta) > 0$ is given in the supplementary material. The $\gamma$ is an empirical constant that must be inferred from data. First proposed by Obukhov [4, 14], the O'KEYPS equation was intended to be an interpolation function that recovers the BD scaling for mildly unstable conditions (for small $-\zeta$) while ensuring that $\phi_m(\zeta) \sim (-\zeta)^{-1/3}$ for very large $-\zeta$. Theoretical justification for the O'KEYPS equation remained heuristic and assumed a constant heat to momentum eddy-diffusivity (see supplementary material), an assumption negated by the Kansas experiment thereby preventing wide-spread acceptance of the O'KEYPS equation. The aim here is to provide a theoretical framework to predict

FIG. 1. The determination of $\phi_m(\zeta)$ from the Kansas experiment. Note the quasi-linear increase for stable atmospheric conditions ($\zeta > 0$) and the -1/4 power-law dependence for unstable conditions ($\zeta < 0$). The so-called Businger-Dyer (BD) stability correction functions are shown as lines. The $\phi_m(\zeta)$ is expected to scale as $(-\zeta)^{-1/3}$ as the free convection limit is approached, often coinciding with $-\zeta > 5$ (not shown here) as discussed elsewhere [10].

FIG. 2. Derivation of the turbulent shear stress for an isotropic eddy of size $2s$ as in [18]. The characteristic eddy here transfers momentum down at a rate $\rho u(z+s)\upsilon(s)$ and up at a rate $\rho u(z-s)\upsilon(s)$, where variations in $\rho(z)$ were neglected relative to variations in $u(z)$ in the momentum transfer. The most efficient eddy size that transports momentum to the ground is an eddy of size $2s = z$. A departure from the [18] approach is the addition of the ground heating or cooling ($H_s$).

the shape of $\phi_m(\zeta)$, which has thus far remained elusive, lagging behind experiments [3, 10, 15, 16] and numerical simulations [17].

For a stationary, planar homogeneous, high Reynolds flow with negligible subsidence and mean horizontal pressure gradient, the mean longitudinal momentum balance reduces to $\partial\tau_t(z)/\partial z = 0$, where $\tau_t$ is the turbulent stress at height $z$ assumed to represent the total stress at the ground, $\tau_o$, resulting in $\tau_t = \tau_o = \rho u_*^2$. In the absence of any thermal stratification ($|L| \to \infty$), a theoretical linkage between the MVP and the spectrum of turbulence has recently been proposed [18]. Such a framework serves as a starting point in the analysis of heated or cooled boundaries. The momentum flux exchanged by the most effective momentum transporting eddy at level $z$ is given by [18] (Figure 2)

$$u_*^2 = \kappa_\tau \upsilon(s)\left[\overline{u}(s+z) - \overline{u}(s-z)\right] \approx \kappa_\tau \upsilon(s)\left[\frac{d\overline{u}(z)}{dz}2s\right], \tag{3}$$

where $\overline{u}(s+z) - \overline{u}(s-z)$ is the net momentum per unit mass exchanged at height $z$ due to eddies of size $2s$, $\upsilon(s) = |w(x+2s) - w(x)|$ is the turnover velocity characterized by the magnitude of the velocity difference assuming the eddy in Figure 2 is isotropic, and $\kappa_\tau$ is a proportionality constant. The eddy size that contributes most efficiently to $\tau_o$ is an eddy 'touching' the ground surface resulting in $s = z$ and simplifying equation 3 to

$$2\frac{\kappa_\tau}{\kappa_\nu}\frac{\upsilon(z)}{u_*}\left[\frac{du(z)}{dz}\frac{\kappa_\nu z}{u_*}\right] = 2\frac{\kappa_\tau}{\kappa_\nu}\frac{\upsilon(z)}{u_*}\left[\phi_m\right] = 1. \tag{4}$$

An estimate of $\upsilon(z)$, the turnover velocity, is necessary to describe $\phi_m$. This estimate may be provided from Kolmogorov's 4/5 law for the third-order velocity structure function [19, 20], according to which $\upsilon(z) = \left[\kappa_\epsilon \epsilon z\right]^{1/3}$, where $\kappa_\epsilon = 4/5$, and $\epsilon$ is the mean turbulent kinetic energy (TKE) dissipation rate. This estimate of $\upsilon(z)$ is exact for locally homogeneous and isotropic turbulence [20] and can be used in equation (4) provided $\epsilon$ is known. To determine $\epsilon$, the TKE budget subject to the same idealized flow conditions as the mean longitudinal momentum balance reduces to [21]

$$\epsilon = u_*^2\frac{\partial\overline{u}}{\partial z} + \frac{g}{T_v}\frac{H_s}{\rho C_p} + \left(-\frac{1}{2}\frac{\partial\overline{w'e'^2}}{\partial z} - \frac{1}{\rho}\frac{\partial\overline{w'p'}}{\partial z}\right), \tag{5}$$

where $e$ is the TKE, $p'$ is the turbulent pressure, and $w'$ is the turbulent vertical velocity. The first and second terms on the right-hand side of equation (5) are the mechanical production and the buoyant production or dissipation depending on the sign of $H_s$. The $\overline{w'e^2}$ and $\overline{w'p'}$ are the turbulent transport and pressure redistribution of TKE, and these terms do not 'globally' contribute to any generation or destruction of TKE within the entire atmospheric boundary layer because $\int_0^h\left(-\frac{1}{2}\frac{\partial\overline{w'e^2}}{\partial z} - \frac{1}{\rho}\frac{\partial\overline{w'p'}}{\partial z}\right)dz = 0$ , where $h$ is the atmospheric boundary layer height. Upon neglecting any contributions arising from the turbulent TKE transport and pressure redistribution terms *locally* at height $z$ , equation (4) becomes

$$\frac{2\kappa_\tau\kappa_\epsilon^{1/3}}{\kappa_\nu^{4/3}}\left[\phi_m\right]\left[\frac{\kappa_\nu z}{u_*^3}\left(u_*^2\frac{\partial\overline{u}}{\partial z} + \frac{g}{T_v}\frac{H_s}{\rho C_p}\right)\right]^{1/3} = 1. \tag{6}$$

Using the definition of $L$, equation (6) can be expressed as

$$\beta_1\left[\phi_m(\zeta)\right]^4\left[1 - \frac{\zeta}{\phi_m(\zeta)}\right] = 1, \beta_1 = \frac{2^3\kappa_\tau^3\kappa_\epsilon}{\kappa_\nu^4}. \tag{7}$$

Imposing $\phi_m(0) = 1$ results in $\beta_1 = 1$ and $\kappa_\tau = (1/2)(\kappa_\nu^4/\kappa_\epsilon)^{1/3} \approx 0.21$. This linkage between the constant $\kappa_\tau$, the Von Karman constant $\kappa_\nu$, and the Kolmogorov's 4/5 law (via $\kappa_\epsilon$) is an outcome of attached eddies of size $z$ being within the inertial subrange, a range bounded above by the integral length scale of the flow ($I_L$) and below by the Kolmogorov viscous dissipation length scale ($\eta = \nu^{3/4}\epsilon^{-1/4}$, where $\nu$ is the kinematic viscosity). When $H_s > 0$, equation (7) recovers all the scaling exponents observed in $\phi_m(\zeta)$ for small and large $\zeta$. For small values of $-\zeta$, the -1/4 power-law (shown in Figure 1 for unstable conditions) is recovered by noting that the term $[\zeta/\phi_m(\zeta)] \approx \zeta$ so that

$$\left[\phi_m(\eta)\right]^4\left[1 - \zeta\right] \approx 1; \Rightarrow \phi_m(\zeta) \approx (1-\zeta)^{-1/4}. \tag{8}$$

For large $-\zeta$, the -1/3 power-law for convective scaling [9, 16] (not shown in Figure 1) is recovered by noting that the term $[-\zeta/\phi_m(\zeta)] \gg 1$ so that

$$\left[\phi_m(\zeta)\right]^4\left[-\zeta/\phi_m(\zeta)\right] \approx 1; \Rightarrow \phi_m(\zeta) \approx (-\zeta)^{-1/3}. \tag{9}$$

For stable atmospheric conditions and large $\zeta$, $\zeta/\phi_m < 1$ to maintain $\phi_m > 0$, and hence $\phi_m$ must increase proportionately with increasing $\zeta$.

Equation (7) becomes identical to the O'KEYPS equation when $\gamma = 1$. However, the derivation leading to equation (7) makes no assumptions about similarity between heat and momentum transfer and does not employ heuristic gradient-diffusion closure arguments regarding the vertical velocity variance, unlike previous derivations of the O'KEYPS equation [13, 22, 23]. Our analysis leading to equation (7) demonstrates that the canonical form of the O'KEYPS equation (to within a constant $\gamma$) emerges naturally within the atmospheric surface layer when eddies of size $z$ are most efficiently exchanging momentum with the ground and are 'embedded' within the inertial subrange (i.e. $\eta \ll z \ll I_l$).

When fitted to a number of data sets, the parameter $\gamma$ in the O'KEYPS equation is larger than unity. Reported values range from 5 to 18, with $\gamma = 9$ proposed by a number of authors [12, 13]. The derivation here identifies two possible reasons why $\gamma$ is larger than unity and may not be entirely universal: (1) contributions from the turbulent flux transport and pressure re-distribution terms in the TKE budget, and (2) anisotropy of the attached eddy to the ground surface along with concomitant departure from the Kolmogorov scaling, $\upsilon(z) = [\kappa_\epsilon \epsilon z]^{1/3}$ . In the first case, it is known that the sum of the flux-transport and pressure redistribution terms both increase in magnitude (but oppositely in sign) with increasing $H_s$ for unstable conditions. Equation (7) may be expanded to account for this increase via a coefficient $\beta_2$ so that

$$\epsilon = u_*^2 \frac{\partial u}{\partial z} + \frac{g}{T_v}\frac{H_s}{\rho C_p}(1+\beta_2); \beta_2 = \frac{\left(-\frac{1}{2}\frac{\partial \overline{w'e^2}}{\partial z} - \frac{1}{\rho}\partial \overline{w'p'}\partial z\right)}{\frac{g}{T_v}\frac{H_s}{\rho C_p}}. \tag{10}$$

For the simplest case of a constant $\beta_2$, it can be shown that the O'KEYPS equation is recovered if $\gamma = 1 + \beta_2$. However, inclusion of the transport and redistribution terms to the TKE budget in isolation is not sufficient to provide values as large as $\gamma = 9$ (or the factor 16 in BD). In fact, despite large uncertainties in the magnitude of the pressure redistribution term, the values of $\beta_2$ from the Kansas experiments are never much larger than unity [24].

For the non-isotropic eddy scenario, the vertical dimension of the eddy that contributes most efficiently to momentum exchange remains of size $s = z$. However, the longitudinal dimension of the eddy is no longer $z$. To account for such large-scale anisotropy in the calculation of $\upsilon(s')$, it may be assumed that $s' = z f(\zeta)$ , where $f(\zeta)$ is a dimensionless anisotropy function such that $f(0) = 1$ to recover the log-law. The derivation proceeds as before, with $\upsilon(s') = [\kappa_\epsilon \epsilon z f(\zeta)]^{1/3}$ and equation (3) becoming

$$\frac{2\kappa_\tau \kappa_\epsilon^{1/3}}{\kappa_\nu^{4/3}} [\phi_m] \left[\frac{\kappa_\nu z f(\zeta)}{u_*^3}\left(u_*^2 \frac{\partial u}{\partial z} + \frac{g}{T_v}\frac{H_s}{\rho C_p}(1+\beta_2)\right)\right]^{1/3} = 1. \tag{11}$$

FIG. 3. *Left:* Estimation of the anisotropy function $f(\zeta)$ from the measured wavelength corresponding to the vertical velocity spectral peaks in the Kansas experiment. The dashed vertical line indicates the value of $\zeta$ where the $\lambda_w(\zeta)/\lambda_w(0)$ becomes quasi-constant according to the Kansas data. The dotted vertical lines indicate the near-neutral regime ($|-z/L| < 0.1$) *Right:* Comparison between measured (circles) and modeled (solid line) $\phi_m(\zeta)$ with $\beta_e = 1$ and the $f(\zeta)$ shown in the left panel. The classical Businger-Dyer $\phi_m(\zeta)$(dotted line,BD) and predictions from the O'KEYPS equation with $\gamma = 9$ (dashed line) are also shown for reference.

Hence, the revised equation (6) is now given by

$$[\phi_m]^4 \left[\left(1 - (1+\beta_2)\frac{\zeta}{\phi_m}\right)\right] = \frac{1}{f(\zeta)}. \tag{12}$$

An estimate of $f(\zeta)$ can be determined from the ratio of wavelength ($\lambda_w(\zeta)$) corresponding to the vertical velocity spectral peaks relative to their near-neutral ($\lambda_w(0)$) counterpart [15]. From the Kansas experiments, $f(\zeta) = \lambda_w(\zeta)/\lambda_w(0)$ resulting in

$$f(\zeta) = \frac{z(0.55 - 0.38|\zeta|)^{-1}}{z(0.55-0)^{-1}} = \left(1 - \frac{0.38}{0.55}|\zeta|\right)^{-1}, -\zeta < 1; \tag{13}$$

$$f(\zeta) = 3.23, 1 < -\zeta < 0.1h/L. \tag{14}$$

For moderately stable conditions, it is difficult to determine $f(\zeta)$ given the dependence of this wavelength on absolute $L$ and possible independence from z. However, for $\zeta > 1.5$, $f(\zeta)$ can be smaller than 0.1. It is instructive to explore how well $f(\zeta) \neq 1$ connected only to anisotropy in eddy sizes recovers the data in Figure 1. An interpolation formula to equation (14), given as $f(\zeta) = \left(1 - \frac{0.38}{0.55}[1 - \exp(15\zeta)]\right)^{-1}$ was used for $zeta < 0$. This formula guarantees that $f(0) = 1$, recovers a near-constant $f(\zeta)$ for $-\zeta < 1$ , and ensures that this constant limit is approached smoothly at $-\zeta = 1$ (Fig. 3). For stable conditions ($\zeta > 0$), it is assumed that $f(\zeta) = \left(1 + \frac{1}{0.55}\zeta\right)^a$, where the bracketed quantity is inferred from vertical velocity spectral peaks for very mildly stable conditions, and $a \approx -6$ is needed to ensure an approximately smooth $df(\zeta)/d\zeta$ for small $\zeta$ as the flow transitions from stable to unstable conditions. Using these anisotropy functions and assuming a $\beta_2 = 1$, predictions based on equation (12) are compared against the data from the Kansas experiment (Fig. 1) as well as the O'KEYPS equation with $\gamma = 9$, as shown in Fig. 3. It appears that this plausible combination of $\beta_2$ and $f(\zeta)$ does offer a novel explanation for the variations in measured $\phi_m(\zeta)$ with $\zeta$.

The long-surmised link between the stability correction functions for momentum, the Kolmogorov spectrum, and the turbulent kinetic energy budget was established across a wide range of atmospheric stability regimes. The canonical form of the O'KEYPS equation arises from this link when the momentum transporting eddies attached to the ground remain embedded within the inertial subrange. The novelty of the derivation is that no assumptions are made about equality between heat and momentum transfer or about heuristic gradient-diffusion modeling of the vertical velocity variance. Moreover, the proposed derivation suggests that the empirical parameter $\gamma$ of the O'KEYPS equation (or the factor of 16 in the BD equations) is partly linked to contributions arising from the turbulent transport and pressure redistribution terms in the turbulent kinetic energy budget (via $\beta_2$). The $\gamma$ primarily encodes all the information about changes in the spectrum of turbulence in general and the anisotropy in the momentum transporting eddies in particular as atmospheric stability regimes are altered. The anisotropy argument presented only modifies the longitudinal geometry shown in Figure 2 with progressive changes in $H_s$. This argument can be completed by noting that not only the most energetic length, but the entire spectrum of the vertical velocity scale is modified by $\zeta$. More formally, this modification leads to a revised estimate of $\upsilon(s)^2 = \int_{1/s}^{\infty} E_w(\kappa) d\kappa$, where $E_w(\kappa)$ is the energy spectrum of the vertical velocity at wavenumber $\kappa$ [18]. With such a representation, $\upsilon(s) = (\kappa_\epsilon \epsilon s)^{1/3} \sqrt{I_w(\zeta)}$, where $I_w(\zeta)$ is a correction to the Kolmogorov scaling primarily due to the fact that the low-frequency component of $E_w(\kappa)$ is modified by $\zeta$. This correction may be small for unstable conditions given the extensive spread of the inertial subrange in the velocity spectra [15]. For very stable flows, $f(\zeta)$ goes to zero very fast (as shown in Figure 3), and therefore the effect of the anisotropy in equation (12) becomes very large. Accounting for the effect of the eddy-size anisotropy on the full spectrum of the vertical velocity may reduce this rate of growth, as changes in the dissipative regime of the spectrum act as a low-pass filter. Note that in field experiments, $I_w(\zeta)$ may be 'contaminated' by unsteadiness originating from the outer layer due to meso-scale motion (i.e. a violation of the assumption of stationary flow) and may lead to non-universal $\phi_m(\zeta)$.

There are two 'end-member' limits not considered here: The pure convection and the very stable atmospheric limits. As to the former, the framework adopted in Figure 2 no longer applies as finite shear is necessary for such analysis ($\tau_o > 0$). It was shown in some field experiments [10] that as $u_* \to 0$ (no shearing) and when $H_s$ remains high, $\phi_m(\zeta)$ seems to exhibit some increase with increasing $-\zeta/5$ that is not reproduced here. With regards to the very stable stratification, the flow may not be entirely turbulent, and unsteadiness can emerge due to multiple exogenous phenomena [25] not considered here such as passage of clouds. An appreciation as to why $\phi_m(\zeta)$ for such a stably stratified flow condition remains so difficult to predict theoretically is discussed next. We have shown that for $-5 < \zeta < 2$, a close relationship between $\phi_m(\zeta)$ and $\gamma$ characterizing the MVP and the properties of the turbulent energy spectrum (especially the Kolmogorov inertial subrange scaling) was established. This indicates that atmospheric surface layer flows resemble continuous phase transitions (or transitions of the 2nd order) in which the global state (i.e. MVP) is impacted by the statistics of the fluctuations (i.e the power-law spectrum of the turbulent velocity fluctuations). The collapse of the data and the near-universal character of $\phi_m(\zeta)$ across a wide range of $u_*$ and $H_s$ boundary conditions (analogous to external fields) suggests an analogy to the so-called 'Widom scaling' near critical points reviewed by Goldenfeld [26] via the reduced variable $\zeta$. This type of transition differs from the classical (or first-order) transition to turbulence in which a laminar flow is subjected to an instability whose amplitude scales with the Reynolds number. However, for stable atmospheric flows in which thermal stratification entirely dampens the mechanical production of turbulence [25] in equation (10), both types of critical transitions co-exist within a typical Reynolds averaging period. If so, this co-existence may explain why a complete theory for moderately to very stable atmospheric flows is currently beyond reach [27].

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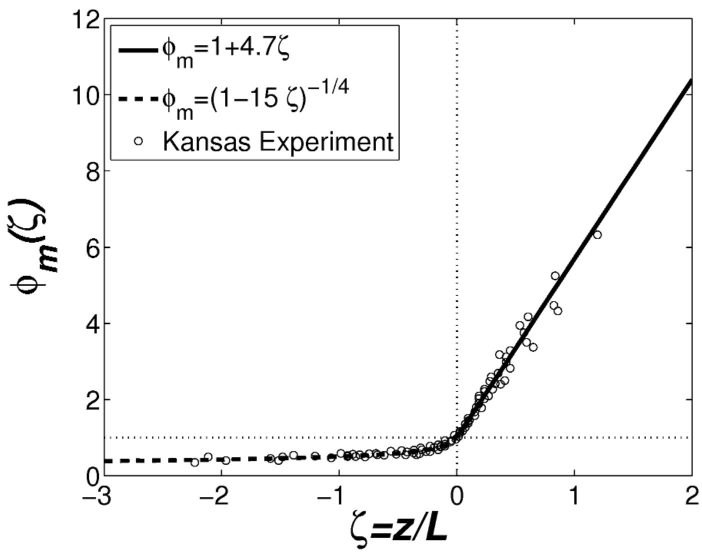


Figure 1 LH12912 05Oct2011

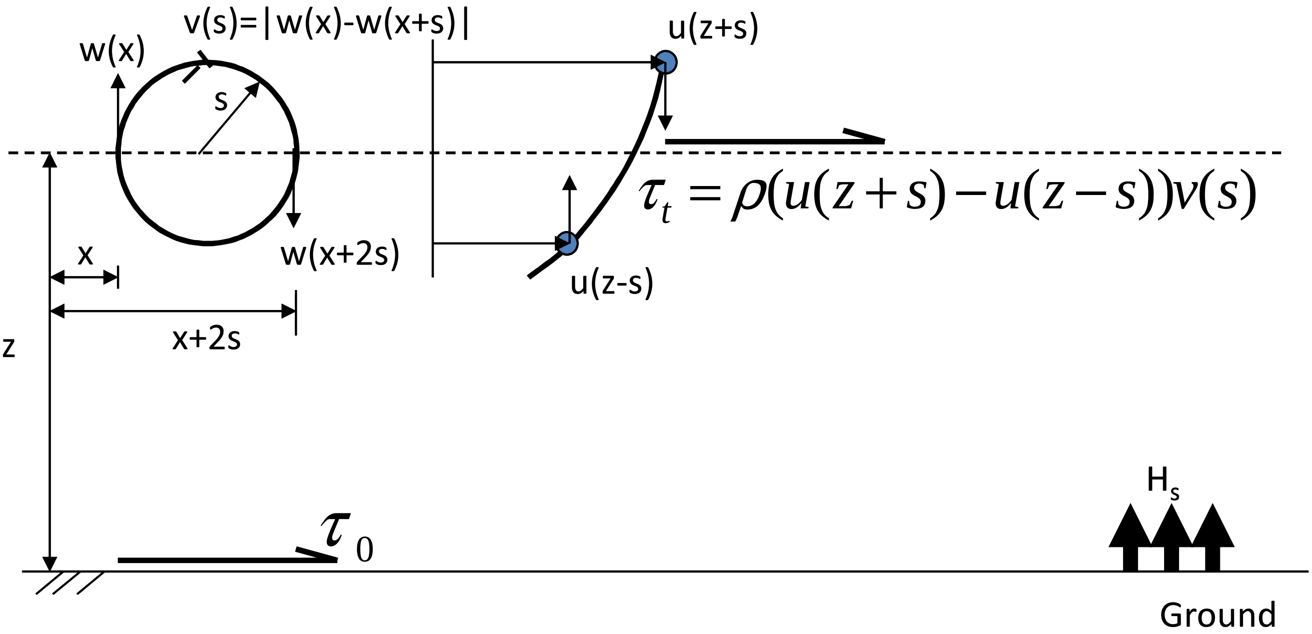


Figure 2 LH12912 05Oct2011

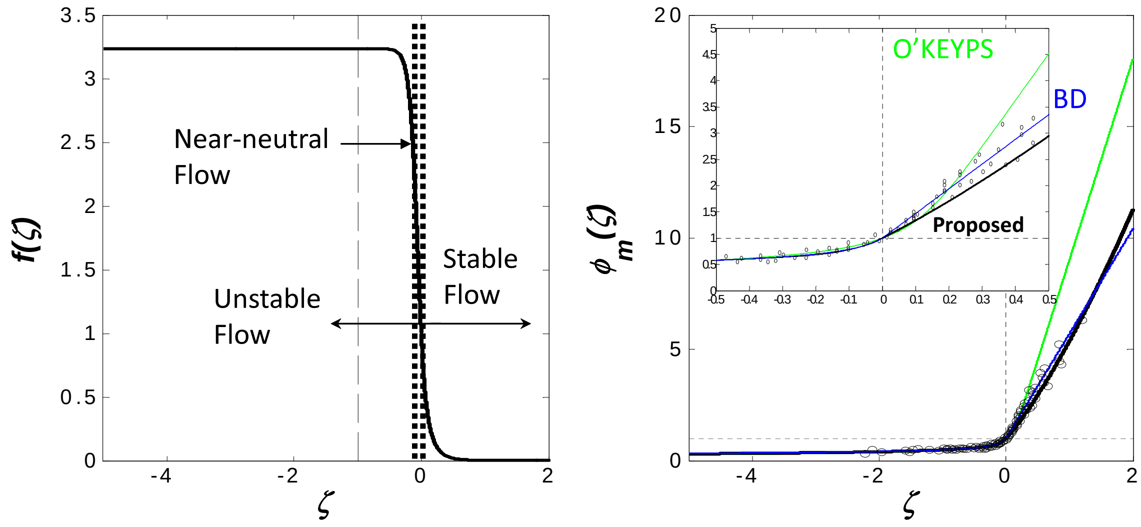


Figure 3 LH12912 05Oct2011