Evolution of the 7/2 Fractional Quantum Hall State in Two-Subband Systems

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Evolution of the 7/2 fractional quantum Hall state in two-subband systems

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We report the evolution of the fractional quantum Hall state (FQHS) at total Landau level (LL) filling factor $\nu = 7/2$ in wide GaAs quantum wells in which electrons occupy two electric subbands. The data reveal subtle and distinct evolutions as a function of density, magnetic field tilt-angle, or symmetry of the charge distribution. At intermediate tilt angles, for example, we observe a strengthening of the $\nu = 7/2$ FQHS. Moreover, in a well with asymmetric change distribution, there is a developing FQHS when the LL filling factor of the symmetric subband ν_S equals 5/2 while the antisymmetric subband has filling $1 < \nu_A < 2$.

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The fractional quantum Hall states (FQHSs) at the even-denominator Landau level (LL) filling factors [1] have recently come into the limelight thanks to the theoretical prediction that these states might be non-Abelian [2] and be useful for topological quantum computing [3]. This expectation has spawned a flurry of investigations, both experimental [4–10] and theoretical [11–13], into the origin and stability of the even-denominator states. Much of the attention has been focused on the $\nu = 5/2$ FQHS which is observed in very low disorder two-dimensional electron systems (2DESs) when the Fermi energy (E_F) lies in the spin-up, excited-state ($N = 1$), LL of the ground-state (symmetric, S) electric subband, namely in the S1$\uparrow$ level. Here we examine the stability of the FQHS at $\nu = 7/2$, another even-denominator FQHS, typically observed when E_F is in the S1$\downarrow$ level (Fig. 1(a)) [4, 7]. The $\nu = 7/2$ FQHS, being related to the 5/2 state through particle-hole symmetry, is also theoretically expected to be non-Abelian. Our study, motivated by theoretical proposals that the even-denominator FQHSs might be favored in 2DESs with "thick" wavefunctions [11–13], is focused on electrons confined to wide GaAs quantum wells (QWs). In a realistic, experimentally achievable wide QW, however, the electrons at $\nu = 7/2$ can occupy the second (antisymmetric, A) electric subband when the subband energy spacing (Δ) is comparable to the cyclotron energy $\hbar\omega_c$ (Figs. 1(b-d)). Here we experimentally probe the stability of the $\nu = 7/2$ FQHS in wide QW samples with tunable density in the vicinity of the crossings (at E_F) between the S1 and the A0 LLs.

Our samples were grown by molecular beam epitaxy, and each consist of a wide GaAs QW bounded on each side by undoped $Al_{0.24}Ga_{0.76}As$ spacer layers and Si δ-doped layers. We report here data, taken at $T \simeq 30$ mK, for three samples with QW widths of $W = 37$, 42, and 55 nm. The QW width and electron density (n) of each sample were designed so that its Δ is close to $\hbar\omega_c$ at the magnetic field position of $\nu = 7/2$. This enables us to make the S1 and the A0 LLs cross at E_F by tuning n or the charge distribution asymmetry, which we achieve by applying back- and front-gate biases [7, 14–16]. For each n, we measure the occupied subband electron densities n_S and n_A from the Fourier transforms of the low-field ($B \leq 0.5$ T) Shubnikov-de Haas oscillations [14, 15], and determine $\Delta = (\pi\hbar^2/m^*)(n_S - n_A)$, where $m^* = 0.067 m_e$ is the GaAs electron effective mass. At a fixed total density, Δ is smallest when the charge distribution is "balanced" (symmetric) and it increases as the QW is imbalanced. Our measured Δ agree well with the results of calculations that solve the Poisson and Schroedinger equations to obtain the potential energy and the charge distribution self-consistently (see, e.g., Figs. 1(a,d)).

Figure 1 shows a series of longitudinal (R_{xx}) and Hall (R_{xy}) resistance traces in the range $3 < \nu < 4$ for a 42 nm-wide QW sample, taken at different n from 2.13 to 2.96×10^{11} cm^{-2} while keeping the total charge distribution balanced. As n is increased in this range, Δ decreases from 64 to 54 K while $\hbar\omega_c$ at $\nu = 7/2$ increases from 50 K to 70 K, so we expect crossings between the S1 and A0 levels, as illustrated in Figs. 1(a-d). These crossings manifest themselves in a remarkable evolution of the FQHSs as seen in Fig. 1. At the lowest n, which corresponds to the LL diagram shown in Fig. 1(a), R_{xx} shows a reasonably deep minimum at $\nu = 7/2$, accompanied by a clear inflection point in R_{xy} at $7/2(h/e^2)$, and a weak minimum near $\nu = 10/3$. These features are characteristic of the FQHSs observed in high-quality, standard (single-subband) GaAs 2DESs, when E_F lies in the S1$\downarrow$ LL [4, 7]. As n is raised, we observe an R_{xx} spike near $\nu = 7/2$, signaling a crossing of S1$\downarrow$ and A0$\uparrow$. At $n = 2.51 \times 10^{11}$ cm^{-2}, these levels have crossed, and E_F is now in A0$\uparrow$ (Fig. 1(b)). There is no longer a minimum at $\nu = 7/2$ and instead, there are very strong minima at $\nu = 10/3$ and 11/3. Further increasing n causes a crossing of S1$\uparrow$ and A0$\uparrow$ and, at $n = 2.63 \times 10^{11}$ cm^{-2}, E_F at $\nu = 7/2$ lies in S1$\uparrow$ (Fig. 1(c)). Here the R_{xx} minimum and R_{xx} inflection point at $\nu = 7/2$ reappear, signaling the return of a FQHS. As we increase n even further, S1$\uparrow$ and A0$\downarrow$ cross and, at $n = 2.96 \times 10^{11}$ cm^{-2}, when E_F at $\nu = 7/2$ lies in A0$\downarrow$, there is again no $\nu = 7/2$ minimum but there are strong FQHSs at $\nu = 10/3$ and 11/3.

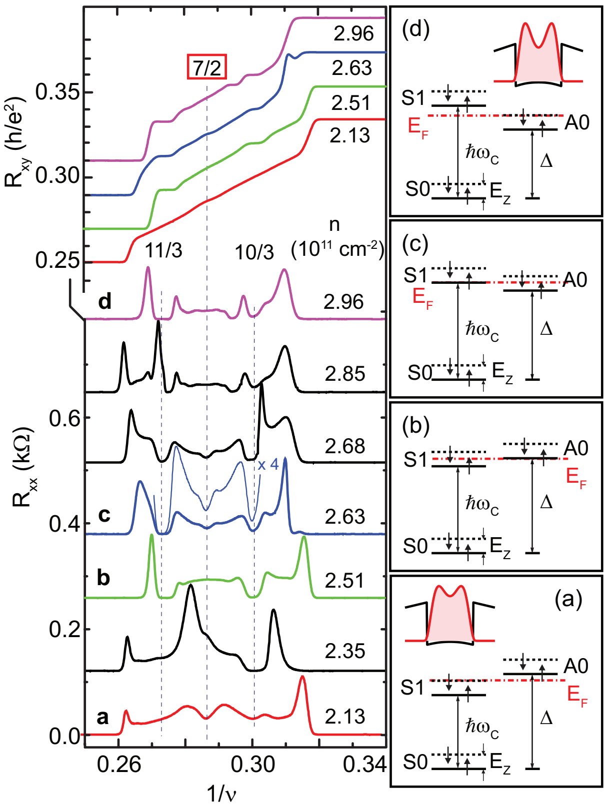

FIG. 1. (color online) Left panel: Waterfall plot of R_{xx} and R_{xy} traces at different densities for a 42-nm-wide GaAs QW. (**a-d**) Schematic LL diagrams at $\nu = 7/2$ for different densities corresponding to the traces marked a-d in the left panel. The subband separation, cyclotron, and Zeeman energies are marked as Δ, $\hbar\omega_c$, and E_Z, respectively. Self-consistently calculated charge distributions are shown in the insets to (a) and (d) for $n = 2.13$ and 2.96×10^{11} cm^{-2}.

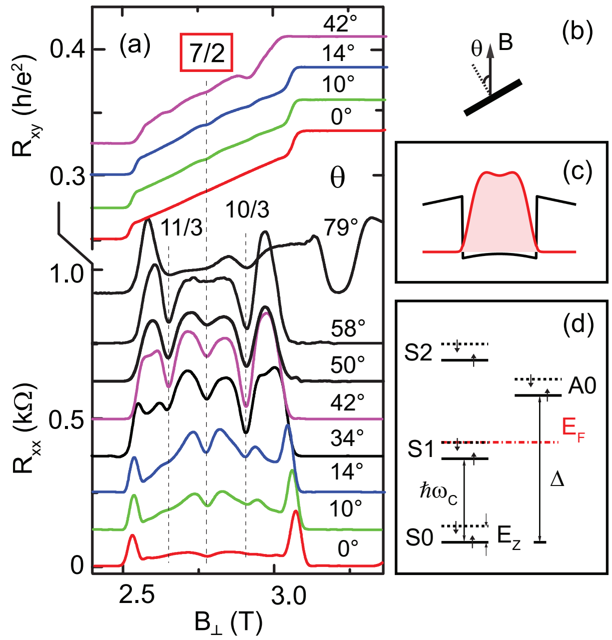

FIG. 2. (color online) (a) R_{xx} and R_{xy} traces for a 37-nm-wide GaAs QW at $n = 2.34 \times 10^{11}$ cm^{-2} at different tilt angles θ as depicted in (b). (c) Charge distribution calculated self-consistently at $B = 0$. (d) LL diagram at $\theta = 0$ at $\nu = 7/2$.

The above observations provide clear and direct evidence that the even-denominator $\nu = 7/2$ FQHS is stable when E_F is in an excited ($N = 1$) LL but not when E_F lies in a ground-state ($N = 0$) LL [7]. Examining traces taken at numerous other n, not shown in Fig. 1 for lack of space, reveal that the appearances and disappearances of the $\nu = 7/2$ FQHS are sharp, similar to the behavior of the 5/2 FQHS at a LL crossing [17]. It is noteworthy that when the two crossing levels have *antiparallel* spins, a "spike" in R_{xx} at the crossing completely destroys the FQHS at $\nu = 7/2$ and nearby fillings. At the crossing of two levels with *parallel* spins, on the other hand, there is no R_{xx} spike. These behaviors are reminiscent of easy-axis and easy-plane ferromagnetism for the antiparallel- and parallel-spin crossings, respectively [16, 18].

Next, we examine the evolution of the $\nu = 7/2$ FQHS in the presence of a parallel magnetic field component $B_{||}$, introduced by tilting the sample so that its normal makes an angle θ with the total field direction (Fig. 2(b)). Figure 2(a) captures this evolution for electrons confined to a symmetric, 37-nm-wide QW [19]. This QW is narrower so that, at $n = 2.34 \times 10^{11}$ cm^{-2}, its Δ ($= 82$ K) is well above $\hbar\omega_c$ ($= 55$ K). The $\theta = 0$ trace then corresponds to E_F lying in S1$\downarrow$, as shown in Fig. 2(d). As θ is increased, we observe only a gradual change in the strength of the $\nu = 7/2$ FQHS, until it disappears at large $\theta \gtrsim 55°$. This is not surprising since, in a two-subband system like ours, we expect a severe mixing of the LLs of the two subbands with increasing θ [20] rather than sharp LL crossings as manifested in Fig. 1 data.

We highlight three noteworthy features of Fig. 2 data. First, the $\nu = 7/2$ R_{xx} minimum persists up to relatively large θ (up to 50°), and it even appears that the R_{xy} plateau is better developed at finite θ (up to $\theta = 42°$) compared to $\theta = 0$, suggesting a strengthening of the 7/2 FQHS at intermediate angles. Second, deep R_{xx} minima develop with increasing θ at $\nu = 10/3$ and 11/3, implying the development of reasonably strong FQHSs at these fillings. This is consistent with the results of Xia *et al.* who report a similar strengthening of the 7/3 and 8/3 states - the equivalent FQHSs flanking the $\nu = 5/2$ state in the S1$\uparrow$ level - when a wide QW sample is tilted in field [9]. It is particularly remarkable that, at intermediate θ ($\simeq 40°$), there are well-developed FQHSs at $\nu = 10/3$ and 11/3 *as well as* at $\nu = 7/2$. Third, the large magnitude of $B_{||}$ at the highest angles appears to greatly suppress Δ, rendering the electron system essentially into a bilayer system [21]. This is evidenced by the dramatic decrease in the strength of the $\nu = 3$ QHS and the disappearance of the $\nu = 11/3$ R_{xx} minimum at $\theta = 79°$; note that a

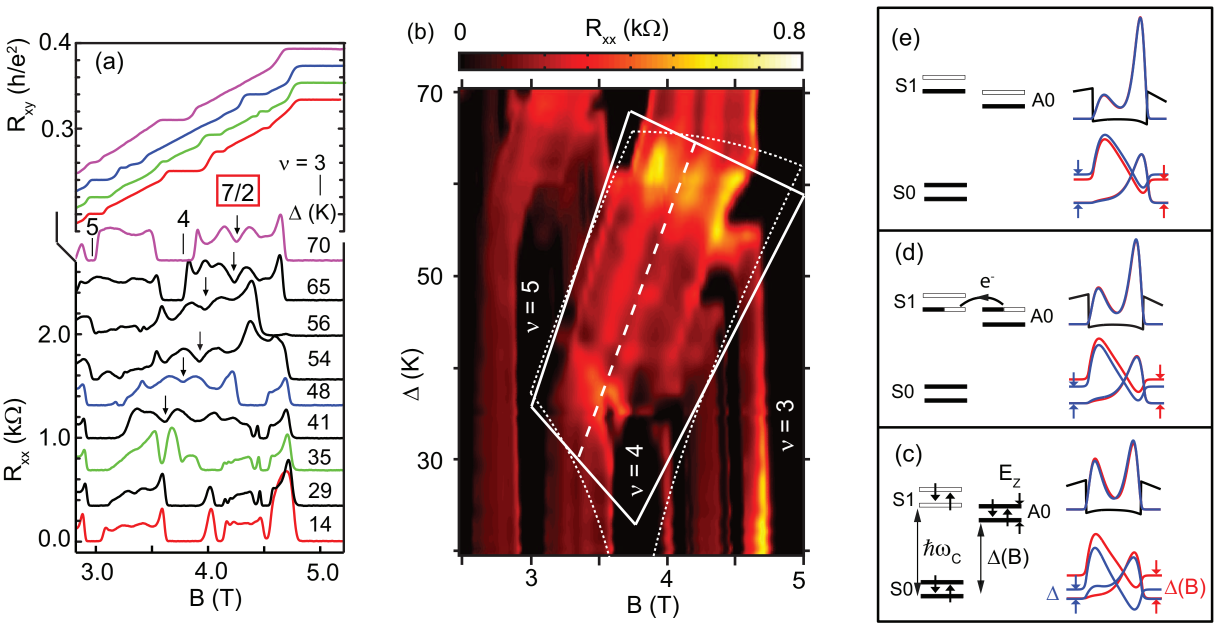

FIG. 3. (color online) (a) R_{xx} and R_{xy} traces for a 55-nm-wide GaAs QW, at a fixed $n = 3.62 \times 10^{11}$ cm^{-2}, as the charge distribution is made increasingly asymmetric. Values of Δ, measured from low-B Shubnikov-de Haas oscillations, are indicated for each trace. Vertical arrows mark the positions of observed anomalous R_{xx} minima. (b) A color-scale plot of data in (a). Solid and dotted lines are the calculated boundary within which the S1↑ and A0↑ levels are pinned together at E_F [24]. The dashed line represents the values of B at which, according to the calculations, the S1↑ level is half-filled; it tracks the positions of the observed R_{xx} minima marked by vertical arrows in (a). (c-e) Schematic LL diagrams (left), charge distributions and potentials (upper right), and wavefunctions ψ_S and ψ_A (lower right), self-consistently calculated at $B = 0$ (blue) and at $\nu = 4$ (red). In (c-e), the filling factor of the S1↑ level equals 0, 0.5, and 1, respectively. In each panel, the calculated wavefunctions are shifted vertically according to the calculated values of Δ and $\Delta(B)$.

FQHS should not exist at $\nu = 11/3$ in a bilayer system with two isolated 2DESs as such a state would correspond to 11/6 filling in each layer.

We now focus on data taken on a 55-nm-wide QW where we keep the total n fixed and change the charge distribution symmetry by applying back- and front-gate biases with opposite polarity. In Fig. 3(a) we show a set of R_{xx} traces, each taken at a different amount of asymmetry. The measured Δ is indicated for each trace and ranges from 14 K for the symmetric charge distribution to 70 K for a highly asymmetric distribution. In Fig. 3(b) we present a color-scale plot of R_{xx} with B and Δ as x and y axes, based on an interpolation of Fig. 3(a) data and many other traces taken at different values of Δ. When the charge distribution is symmetric or nearly symmetric in this QW, $\Delta \simeq 14$ K is much smaller than $\hbar\omega_c$ (= 85 K at $\nu = 7/2$) so that the LL diagram is qualitatively the one shown in Fig. 1(d). Consistent with this LL diagram, we observe a very strong $\nu = 4$ QHS. Also, since E_F lies in the A0↓ level at $\nu = 7/2$, there is no $\nu = 7/2$ FQHS and instead we observe strong FQHSs at $\nu = 10/3$ and 11/3. As Δ is increased, we expect a crossing of S1↑ and A0↓, leading to a destruction of the $\nu = 4$ QHS at the crossing. This is indeed seen in Figs. 3(a) and (b). What is striking, however, is that the $\nu = 4$ R_{xx} minimum disappears over a very large range of Δ, between 35 and 62 K. Even more remarkable are several anomalous R_{xx} minima in this range of Δ in the filling range $3 < \nu < 5$, particularly those marked by arrows in Fig. 3(a). These minima resemble what is observed in the top trace but are seen at lower fields.

These features betray a pinning together, at E_F, of the partially occupied S1↑ and A0↓ levels, and a charge transfer between them, in a finite range of B and gate bias. As pointed out in Ref. [22], when only a small number of quantized LLs belonging to two different subbands are occupied, the distribution of electrons between these levels does not necessarily match the $B = 0$ subband densities. This leads to a mismatch between the total electron charge density distributions at $B = 0$ and high B, which is given by:

$$\rho(B) = e(eB/h)[\nu_S \cdot |\psi_S(B)|^2 + \nu_A \cdot |\psi_A(B)|^2], \quad (1)$$

where ν_S and ν_A are the fillings of the S and A subbands and $\psi_S(B)$ and $\psi_A(B)$ are the in-field subband wavefunctions. The pinning and the inter-LL charge transfer help bring these distributions closer to each other [22, 23].

To demonstrate such a pinning quantitatively and determine the boundary inside which the S1↑ and A0↓ levels are pinned together, we performed self-consistent calculations of the potential energy and charge distribution at high B for different QW asymmetries [24]. The calculated boundary is shown by solid white lines in Fig. 3(b), and examples of the results of our calculations (at $\nu = 4$) are shown in Figs. 3(c-e). When we imbalance the QW at $\nu = 4$, at the lower boundary, as the A0$\downarrow$ level reaches the S1$\uparrow$ level from below, $\nu_S = \nu_A = 2$ (Fig. 3(c)). As we further imbalance the QW, electrons are transferred from A0$\downarrow$ to the S1$\uparrow$ level, while these two levels are pinned together and $\Delta(B) = \hbar\omega_c - E_Z$ remains unchanged. The charge transfer ends at the upper boundary when $\nu_S = 3$ and $\nu_A = 1$ (Fig. 3(e)). In our calculations, we use ($\nu_S = \nu_A = 2$) or ($\nu_S = 3$, $\nu_A = 1$) in Eq. (1) and find the zero-field subband spacings Δ for the two particular QW asymmetries which give an in-field subband spacing equal to $\Delta(B) = \hbar\omega_c - E_Z$ [24]. The boundary at other magnetic fields in the range $3 \leq \nu \leq 5$ is calculated in a similar fashion [24]. In Fig. 3(b), it is clear that the calculated boundary matches reasonably well the region (in Δ vs. B plane) in which we experimentally observe a disappearance of the $\nu = 4$ R_{xx} minimum and the appearance of R_{xx} minima at anomalous fillings. This matching is particularly remarkable considering that there are no adjustable parameters in our simulations, except for using a single value (7.3) for the enhanced g-factor [24].

In Fig. 3(b) we include a dashed line representing the values of B at which, according to our calculations, the S1$\uparrow$ level is exactly half-filled, i.e., $\nu_S = 5/2$ and $\nu_A = (\nu - 5/2)$. This dashed line tracks the positions of the observed R_{xx} minima marked by the vertical arrows in (a) very well, suggesting that these minima indeed correspond to $\nu_S = 5/2$ [25]. This is an astonishing observation, as *it implies a developing FQHS at 5/2 filling of the symmetric subband even when a partially filled A0$\downarrow$ level is pinned to the half-filled S1$\uparrow$ level at E_F!*

We remark that while LL pinning in two-subband systems is a general phenomenon [22], its manifestation is more pronounced in bilayer-like electron systems with asymmetric (imbalanced) charge distributions [23]. For example, we do not see signatures of LL pinning in the data of Fig. 1 which were taken on an electron system with a symmetric (balanced) charge distribution. This is because an inter-subband charge transfer barely changes the total charge distribution in this balanced QW. In wide ($W = 60$ and 80 nm) GaAs QWs with imbalanced charge distributions, on the other hand, in an independent study Nuebler *et al.* [26] observed a pining of the S1$\uparrow$ and A0$\uparrow$ levels, leading to the formation of a FQHS when $\nu_S = 5/2$ while the A0$\uparrow$ level is partially occupied.

In summary, our results reveal distinct metamorphoses of the ground-state of two-suband 2DESs at and near $\nu = 7/2$ as either the magnetic field is tilted, or the density or the charge distribution symmetry are varied. Most remarkably, we observe an apparent strenthening of the $\nu = 7/2$ FQHS at intermediate tilt angles, and a developing FQHS when a half-filled S1$\uparrow$ level is pinned to a partially-filled A0$\downarrow$ level.

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parallel field in our sample, if we use the distance between the peaks in the charge distribution (Fig. 2(c)) as d, we find that $\Delta \simeq \hbar\omega_c$ near $\sim 55°$, and $\Delta \lesssim 1$ K at $\theta = 80°$.

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