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Three-dimensional topological insulators have protected Dirac-cone surface states. In this paper we argue that gapped excitonic superfluids with spontaneous coherence between top and bottom surfaces can occur in the TI-thin-film quantum-Hall regime. We find that the large dielectric constants of TI materials increase the layer separation range over which coherence survives and decrease the superfluid sound velocity, but have little influence on the superfluid density or on the charge gap. The coherent state at total Landau-level filling factor $\nu_T = 0$ is predicted to be free of edge modes, qualitatively altering its transport phenomenology compared to the widely studied case of $\nu_T = 1$ in GaAs double quantum wells.

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Introduction—Two-dimensional electron systems in which Landau levels that are localized in different layers approach degeneracy tend, when only one of the two levels is occupied[1, 2], to form broken symmetry states with spontaneous interlayer coherence[3]. These states support counterflow superfluidity and are sometimes referred to as quantum Hall superfluids[4], although they can also be viewed as exciton condensates or as pseudospin ferromagnets[1, 2, 5–8]. When the two layers are contacted independently their order gives rise to a number of subtle and interesting anomalies in transport[9–18] that have motivated considerable theoretical activity[19–37]. In this Letter we predict that quantum Hall superfluid states will also occur in topological insulator (TI) thin films and highlight important differences between the double well and TI-thin-film cases.

In TI thin films spontaneous coherence occurs between Landau levels localized on top and bottom surfaces. Since the quantum Hall superfluid state [9, 14, 17, 38, 39] appears at two dimensional (2D) layer separations d less than $\sim 3\ell$ where $\ell \sim 25\text{nm}\,(B[\text{Tesla}])^{-1/2}$ is the magnetic length, TI quantum Hall superfluid behavior is a possibility only in samples thinner than about 30nm, and therefore only in materials prepared by a thin film growth technique. Because topological insulators [40, 41] have protected surface states, spatially separated Landau levels are always present when the Fermi level lies in the bulk gap. Just as in double quantum wells, electron-electron interactions are responsible for the occurrence of interlayer spontaneous coherence between these surface states. Moreover, because the surfaces have massless Dirac electronic structure, their $N = 0$ Dirac-point Landau levels will be widely separated from other levels even at weak magnetic fields, making it easier to overcome disorder and reach the quantum Hall regime. Importantly quantum Hall superfluids in TI thin films, unlike those studied in quantum wells, can occur at total Landau level filling factor $\nu_T = 0$, thus sidestepping the dissipationless chiral-edge-state transport channel which can obscure [33] counterflow superfluidity in the $\nu_T \neq 0$ case.

Most properties of ideal quantum Hall superfluids are determined by a small set of parameters: the superfluid density ρ which relates dissipationless counterflow currents to condensate phase gradients, the charged-quasiparticle energy gap Δ which determines the temperature below which bulk quasiparticle transport currents are supressed, the single-particle tunneling gap Δ_{SAS} which limits the current which can be carried coherently between layers, and the phonon velocity v which sets an important energy scale when in-plane magnetic fields are present. Δ_{SAS} depends exponentially on the thickness of the film and is also sensitive to the relative position of the Fermi level on the two surfaces, whereas ρ, Δ, and v depend only on the electron-electron interaction strength and are less sensitve to film thickness. We focus here on the physics which controls the values of these parameters, and in particular on the consequences of the fact that known topological insulators tend to have small bulk energy gaps, and therefore large dielectric constants. This property is likely to persist as new materials are found, making dielectric screening an important issue. We find that although the phonon velocity of a TI quantum Hall superfluid is strongly suppressed by dielectric screening, the superfluid density and the energy gap are weakly altered. Surprisingly we find that the Kosterlitz-Thouless transition temperatures for these superfluids can be larger than in semiconductor quantum well systems. In the following we explain the basis for these estimates, point out some experimentally relevant differences between TI and semiconductor bilayer quantum Hall superfluids, and discuss the relationship between quantum Hall superfluids and zero-magnetic-field exciton condensates.

TI-Thin-Film Quantum Hall Superfluid Energy Scales—

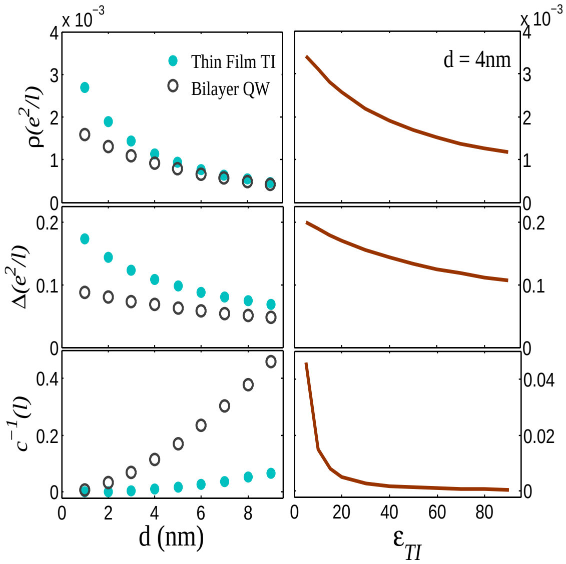

FIG. 1: Left panel: Superfluid density ρ, charge gap Δ, and reciprocal capacitance c^{-1} as a function of TI thickness d. The solid dots were calculated using typical topological insulator parameters ($\epsilon_{TI} = 40$, $\epsilon_S = 10$, and $B = 10$T), and the empty dots with GaAs double quantum well parameters. ρ and Δ are larger in the TI thin-film case than in the double quantum well case, in spite of the large value of ϵ_{TI}. The large TI dielectric constant directly in the capacitance, however, and therefore in the superfluid sound velocity. Right panel: ρ, Δ and c^{-1} as a function of ϵ_{TI} for $d = 4$nm. ρ and Δ are much more weakly dependent on ϵ_{TI} than is c^{-1}. The results in the right panel refer to the right vertical axis.

When Landau level mixing is neglected, the interaction dependent parameters can be estimated from the following mean-field-theory expressions [7]:

$$\begin{aligned} \rho &= \frac{\ell^2}{32\pi^2} \int dq\, q^3\, V_{tb}(q) F_{tb}(q)\, \exp(-q^2\ell^2/2), \\ \Delta &= \frac{1}{2\pi} \int dq\, q\, V_{tb}(q) F_{tb}(q)\, \exp(-q^2\ell^2/2), \\ v &= \frac{1}{2\hbar}\sqrt{\frac{\rho\, e^2}{c}}, \end{aligned} \tag{1}$$

where

$$c = \Big[\frac{4\ell^2}{e^2} \int dq\, q \big[V_\sigma(0) - V_\sigma(q)\big]\, \exp(-q^2\ell^2/2)\Big]^{-1} \tag{2}$$

is the bilayer capacitance per unit area, and $V_\sigma(q) = (V_{tt}(q)F_{tt}(q) + V_{bb}(q)F_{bb}(q) - 2V_{tb}(q)F_{tb}(q))/4$. In these equations V_{tt}, V_{bb} and V_{tb} are respectively the interactions between two top surface electrons, two bottom surface electrons, and a top and a bottom surface electron. V_σ therefore is the potential difference between surfaces generated by charge transfers. The $F_{ab}(q)$ are form factors [42] which capture the influence of Landau-index dependent changes in single-particle wavefunction shape.

It is important for TI quantum Hall superfluid properties to understand how the values of these characteristic parameters are influenced by the TI bulk dielectric-constant (ϵ_{TI}) which is expected to be large, and also by the dielectric constant ϵ_S of the substrate on which the film sits. An elementary calculation [43] yields the following results for electron-electron interactions that are screened by this dielectric environment, assuming the top surface is exposed to vacuum.

$$V_{tt}(q) = \frac{4\pi e^2}{qD(q)}\, \big[(\epsilon_{TI} + \epsilon_S)\, e^{qd} + (\epsilon_{TI} - \epsilon_S)\; e^{-qd}\big], \tag{3}$$

$$V_{tb}(q) = \frac{8\pi e^2}{qD(q)}\, \epsilon_{TI}, \tag{4}$$

and

$$V_{bb}(q) = \frac{4\pi e^2}{qD(q)}\, \big[(\epsilon_{TI} - 1)\, e^{qd} + (\epsilon_{TI} + 1)\; e^{-qd}\big], \tag{5}$$

where $D(q) = (\epsilon_{TI} + \epsilon_S)(\epsilon_{TI} + 1)\, e^{qd} + (\epsilon_{TI} - \epsilon_S)(1 - \epsilon_{TI})\, e^{-qd}$. Quantum Hall superfluid behavior is expected only when d/ℓ is not large. It follows qd will generally be small when the integrands in Eqs.(1) are large. Setting $qd \to 0$ we find that V_{tb}, which appears in the expressions for the superfluid density and charge gap goes to $4\pi e^2/q(\epsilon_S + 1)$, independent of ϵ_{TI}. For ρ and Δ, this leads to a weak dependence on ϵ_{TI} illustrated in the right panel of Fig. 1. In semiconductor quantum wells, for comparison, $V_{tb} \to 2\pi e^2/q\epsilon_{SC}$ where $\epsilon_{SC} \sim 13$ is the dielectric constant of the surrounding semiconductor. V_{tb} will tend to be, if anything, larger for TI thin films than for typical double quantum wells assuming that $\epsilon_S \approx \epsilon_{SC}$. On the other hand $V_\sigma \to \pi e^2 d/\epsilon_{TI}$ for $qd \to 0$ and is therefore strongly reduced in TI's. This is reflected in the suppressed value of c^{-1} in the lower panel of Fig. 1 and also in the phonon velocity v.

TI Quantum Hall Superfluid Spectra—For Dirac two-dimensional electron systems the $i = N$ Landau level is half-filled at filling factor $\nu = N$. In Fig. 2 we plot the film thickness dependence of the charge gaps that appear in TI thin films for several total ν, top (ν_t) and bottom (ν_b) layer filling factors ($\nu = (\nu_t) + (\nu_b)$). The gaps plotted in Fig. 2 were evaluated by solving mean-field equations including Landau level mixing [44]. For the special case of total filling factor $\nu_T = 0$ ($\nu_t = -\nu_b$) single-particle tunneling between surfaces contributes to the gap [45], but this effect vanishes exponentially with thickness becoming unimportant for thicknesses exceeding a few nanometers. We see that the values of the interaction induced gaps are not strongly dependent on the orbital character of the valence Landau level. The nature of the order responsible for the gap is, however, sensitive to orbital character as revealed by examining its dependence on an external potential applied across the thin film.

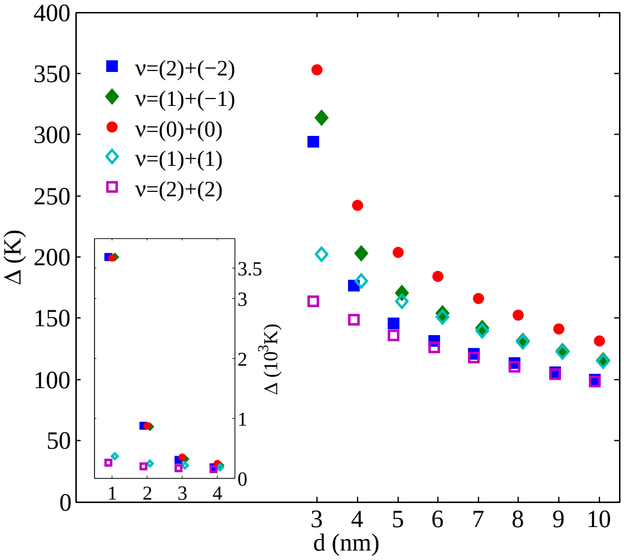

FIG. 2: Charge gap in Kelvin units as a function of TI film thickness for degenerate half-filled Landau levels. The gap values are larger than in the corresponding GaAs double-quantum well case. These results were obtained with dielectric constant parameters $\epsilon_{TI} = 40$ and $\epsilon_S = 10$ and magnetic field $B = 10$T. The labels specify the filling factor contributions ν_t, ν_b from top and bottom surfaces (or equivalently the orbital label of the partially filled level). Inset: For samples thinner than 3nm single-particle tunneling grows rapidly and dominates the gap when $\nu_t = -\nu_b$. Note that the energy scale for the inset is larger and specified on the right axis.

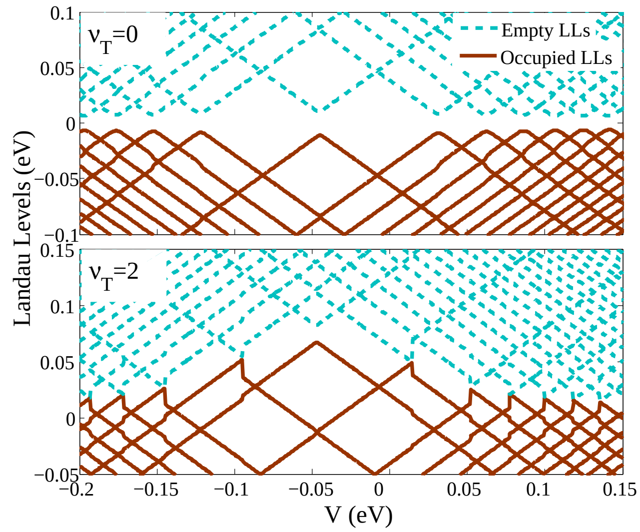

FIG. 3: Coulomb interaction modified Landau Levels for a TI thin film with $\epsilon_{TI} = 40$ and $\epsilon_S = 10$ and thickness $d = 4$nm at $B = 10$T as a function of inter-layer external potential V. Gaps open at the Fermi level due to interaction effects at crossings between top and bottom layer Landau levels. At this thickness, single-particle tunneling is small compared to interaction induced gaps. In the top panel, corresponding to total filling factor $\nu_T = 0$, gaps associated with inter-surface coherence occur over a broad range of V values. In the bottom panel $\nu_t + \nu_b \neq 0$ so that the crossing Landau levels have different orbital character for $V \neq 0$. The states at the $V \neq 0$ crossing points have Ising pseudospin which leads to jumps in layer polarization.

In Fig. 3 we illustrate the dependence of the $d = 4$nm full Landau-level spectra at $\nu_T = 0$ and $\nu_T = 2$ on the potential difference between layers V. The $(\nu_t, \nu_b) = (N, -N)$ gaps decrease slowly in size with increasing N. This slow decrease can be traced to partial cancellation between a decrease in exchange integral magnitude with increasing N because of form factor effects, and an increase in supportive Landau-level mixing effects that are enhanced by decreasing Landau level spacing. Although the dependence illustrated here is *vs.* Landau level index tuned by potential V at fixed field, a similar robustness in the n-dependence can be expected when indices are increased at fixed V by reducing the magnetic field strength. In the limit of zero field the inter-layer coherence effect illustrated here reduces to zero-field bilayer exciton condensation [46–48]. For $\nu_T = 2$ ($\nu_t \neq -\nu_b$), on the other hand, the gaps tend to be smaller. In this case mean-field theory predicts states with Ising-like order in which charge is shifted between layers rather than states with inter-layer coherence. Ising like order is expected [2] because of the difference between the form factors of crossing Landau levels with different kinetic energy (different $|N|$) and therefore different semiclassical cyclotron orbit radii. For Ising order the inter-layer potential acts like an external field which couples to the order parameter. The jumps in the Landau level spectrum in the $\nu_T = 2$ panel are associated with pseudospin reversal (sudden jumps in layer polarization) near Landau level crossing points. The mean-field solutions in this case are hysteretic and the plotted spectra were calculated for one V sweep direction.

Discussion—There is at present great interest in the electronic properties of TI thin films with Fermi levels close to their surface state Dirac points. When electron-electron interactions are neglected and there is no electric potential drop across its bulk, a TI thin film has only odd integer quantum Hall plateaus obtained by summing the half-quantized Hall effects on both surfaces. In this Letter we have demonstrated that even integer quantum Hall plateaus are also expected due to electron-electron interaction effects which lead to the formation of gapped quantum Hall superfluid states. We know from experience with semiconductor quantum well bilayers that spontaneous coherence states are easily destroyed by disorder, particularly disorder which allows Landau level mixing, so these states should be most easily realized when the $N = 0$ Dirac Landau levels (which are widely separated from $N \neq 0$ levels) on both top and bottom surfaces are half-filled, *i.e.* at total Landau level filling factor $\nu_T = 0$.

The nature of the charged excitations that contribute to low-temperature transport in spontaneous coherence states depends on the strength of interlayer tunneling [20]. It follows from the single particle physics of TI thin films that interlayer tunneling produces gaps only at $\nu_T = 0$. For $\nu_T \neq 0$, or at $\nu_T = 0$ for samples thicker than a few nanometers, the charged excitations are topological meron-antimeron pairs with layer polarization near their cores. Because of the larger superfluid densities ρ in TI quantum Hall superfluids, we predict that the Kosterlitz-Thouless phase transitions that occur when interlayer tunneling is negligible will take place at higher temperatures than in quantum wells. As in the semiconductor bilayer case, disorder is expected to induce merons in the bilayer ground state, complicating all coherent state properties. In thinner TI's, the meron-antimeron pairs at $\nu_T = 0$ are transformed by tunneling into domain wall pseudospin textures and the sensitivity to disorder should be reduced.

The transport phenomenology of $\nu_T = 0$ quantum Hall systems, which do not occur in semiconductor bilayers, is like that of magnetic field $B = 0$ bilayer exciton condensates [33] because the system no longer supports non-dissipative chiral edge conduction channels. The presence of a $\nu = 0$ gap is readily revealed by low-temperature insulating behavior in standard transport measurements. If standard transport measurements revealed the formation of a $\nu = 0$ gap, separate-contact measurements which are more challenging experimentally, would be necessary to confirm the quantum Hall superfluid character of the ground state.

One important property of quantum Hall superfluids is that they tolerate potential differences between 2D surfaces which shift the condensate charge more to one layer than to the other. Spontaneous coherence and the associated condensation energy can be maintained while satisfying electrostatic requirements on charge density distributions. This property might explain some of the quantum Hall effects already observed in strained HgTe systems [49] which cannot be understood in terms of independent particle physics. Strained HgTe is a three-dimensional TI with a small gap and the MBE grown thin films studied experimentally should therefore support two Dirac-like 2D electron gases. Because the quantum well thickness in these experiments exceeded the magnetic length by a factor of ~ 10, the surprising quantum Hall effects discovered in these systems appear at first to occur at 2DEG separations that exceed the coherence limit. The TI surface states in this material are however spatially extended and could be peaked quite far from the wide quantum well edges, reducing the effective 2D layer separation. In addition the same dielectric screening effects discussed here which increase the capacitance without substantially decreasing the charge gaps, also increase the layer separation at which coherence survives. For Bi_2X_3 TI's (X=Se, Te) we have estimated that the maximum d/ℓ ratio is increased by half compared to the GaAs case [38, 39]. Future work which accounts quantitatively for the orbital character of the topologically protected surface states in strained HgTe wide well TI's will be needed to explore the role of inter-surface coherence in this TI more fully.

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