Mirror Mode Expansion in Planetary Magnetosheaths: Bohm-like Diffusion

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Phys. Rev. Lett. **107**, 245005 — Published 7 December 2011

DOI: 10.1103/PhysRevLett.107.245005

A Theoretical Model of Mirror Mode Expansion in Planetary Magnetosheaths: Bohm-Like Diffusion

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Observed scale sizes of mirror modes in planetary magnetosheaths tend to be equal or larger than those that correspond to the maximum growth rate of the mirror instability: 9 ρ_p (proton gyroradius). These phenomena can be accounted for by introducing a diffusion process (Bohm) that shifts the spectra to lower wavenumbers as the mode convects away from the source to the observation point. The theory is applied to data obtained in the magnetosheaths of Earth, Jupiter, Saturn and the heliosheath, and shown to provide reasonable agreement to past spacecraft observations. Further observational tests of the theory are suggested.

Mirror mode (MM) instability [1,2] has been detected in planetary magnetosheaths [3-12] and in the heliosheath [13, 14]. The predicted scale size of the nonoscillatory structures based on the maximum growth rate of kinetic theory [15,-17] is ~9 ρ_p, where a ρ_p is a proton gyroradius. Figure 1 shows mirror mode structures in the magnetosheath of Saturn. The scale sizes appear to increase as the spacecraft goes from the bow shock (BS) to the magnetopause (MP). Table 1 shows the measured typical scale sizes in the various planetary magnetosheaths and the heliosheath. It is noted from the Table that the measured sizes are all equal to or larger than the theoretical prediction. It also can be noted that in some magnetosheaths (Earth, Jupiter and Saturn), different measurements gave different scale sizes. Some of this variability is due to the fact that the measurements were taken under different solar wind conditions (velocity, density and magnetic field orientation), and different regions within the magnetosheaths (the relative distance between the shock and the magnetopause). Most of the measurements were not made exactly along the Sun-Earth stagnation line and that difference also may cause variation from one result to the next.

Magnetosheath environment	MM scale, ρ_p	Source
Earth	20	Tsurutani et al., 1982
	20	Lucek et al., 2001
	10	Narita et al., 2006;
	10	Horbury and Lucek, 2009
Jupiter	20	Tsurutani et al., 1982
	20	Erdos and Balogh, 1996
	10- 100	Bavassano et al. 1998
Saturn	40	Tsurutani et al., 1982
	10	Violante et al., 1995
Heliosheath	80	Burlaga et al., 2006
	57	Tsurutani et al., 2010

Table 1. The scale sizes of mirror mode structures in various planetary magnetosheaths determined from various studies (the references are given). Most studies did not say where in the magnetosheaths the measurements were made nor what uncertainties of the measurements were.

The purpose of this paper is to present a scenario for MM structural evolution based on Bohm diffusion. As the structures convect away from the source region, they will be diffused and the scale size will increase by selective decay of the high wavenumber portion of the spectra, similar to the process observed in an expanding ring of smoke. Specific solar wind and magnetosheath parameters will be applied to the expressions of the derived diffusion to compare to the measured MM scale sizes. Additionally the concept of MM evolution by diffusion will be used to make predictions which can be tested by future measurements.

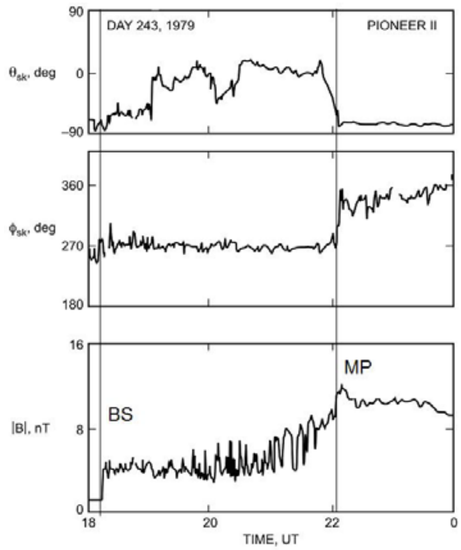

FIG. 1. The NASA Pioneer 11 spacecraft crossing of the magnetosheath of Saturn. The three panels are the two polar angles in a Saturn-centric coordinate system and the magnetic field magnitude (bottom panel). The bow shock (BS) and magnetopause (MP) are indicated by vertical lines. The mirror mode structures, given by the magnetic field magnitude variations in the bottom panel, appear to become larger as the MP is approached. The figure is reproduced from [18].

Let $N(x, t)$ be the structure of the mirror mode that deforms due to plasma diffusion and D be the diffusion coefficient. The diffusion equation gives:

$$\frac{\partial N(x,t)}{\partial t} = D\frac{\partial^2 N(x,t)}{\partial x^2} \qquad (1)$$

To see the spectral evolution of D we do a Fourier transform:

$$N_k(k,t) = \int_{-\infty}^{\infty} N(x,t)\exp(ikx)dx \qquad (2)$$

where $N_k(k, t)$ evolves according to:

$$N_k(k,t) = N_k(k,t=0)\exp\left(-k^2 D t\right) \quad (3)$$

If we take a shifted-Gaussian for the initial spectrum of the mirror mode (MM) structures, where the center of the spectrum is designated as k_0 and the spectral width a, one gets:

$$N_k(k,0) = \frac{1}{\sqrt{2\pi}\,a}\exp\left[-2\left(k-k_0\right)^2/a^2\right] (4)$$

The spectrum at a later time t, is obtained from the Fourier transform of the diffusion equation (1):

$$N_k(k,t) = \frac{1}{\sqrt{2\pi}a}\exp\left[-2(k-k_0)^2/a^2\right]\exp\left(-k^2 Dt\right) \ (5)$$

This expression simply shows that both the width of the spectrum and the center of the spectrum changes in time according to:

$$a(t)=\frac{a}{\sqrt{1+a^2Dt/2}},\quad k_0(t)=\frac{k_0}{\sqrt{1+a^2Dt/2}} \quad (6)$$

Therefore the peak in the spectrum shifts to smaller values with time and thus the structure will have a scale larger than the ion gyroradius, ρ_p. The same is true for the spectral width as well.

We estimate the wavelength at a distance L from the source, λ (L). We note that the wave number k at the source is given by the value at maximum growth rate, $k = k_0 = 0.7/\rho_p$. This gives the wavelength at the source, $\lambda_0 = 2\pi\rho_p/0.7 = 9\rho_p$. We also assume that the convection speed is given by *u* and the effective ion-ion collision rate is given by ν.

From the above expression for diffusion, we now get:

$$\lambda(L)=9\rho_p\left(1+a^2DL/2u\right)^{1/2} \quad (7)$$

We need to know the spectral width *a* and the diffusion rate *D*. We may take the spectral width at the source to be comparable to inverse of the proton gyroradius, while appropriate diffusion coefficient may be Bohm type because plasma in magnetosheaths is strongly turbulent, thus:

$$D=\frac{\omega_{ci}}{16}\rho_p^2 \quad (8)$$

(since the relevant diffusion is in the direction perpendicular to the magnetic field). We note here that Bohm diffusion, when compared with the classical diffusion coefficient, is equivalent to have anomalous (Bohm) ion-ion collision rate given by $\omega_{ci}/16$.

Then the scale size of the mirror mode at a distance L may be expressed as:

$$\lambda(L)=9\rho_p\left(1+\frac{\omega_{ci}L}{32u}\right)^{1/2} \quad (9)$$

This expression may be used to obtain the observed scale size of the mirror mode wavelength.

The theoretical results in (9) can be applied and compared with space measurements given in Table 1. To obtain the estimate of the mirror mode size, we first use the example of the Earth's magnetosheath. The physical size of the Earth's magnetosheath is taken from measurements to be ~3 R_E = 1.9 x 10^4 km. The convection speed of the plasma within the magnetosheath is taken to be ~150 km/s. The mirror mode structures are assumed to be generated in the magnetosheath just downstream of the bow shock and diffusively spread in scale size and width distribution as they are convected towards the magnetopause or downtail. To get the estimate size of the observed mirror mode, we assume that the measurement is taken 1/3 of the distance from the bow shock to the magnetopause, or ~6,300 km. We assume that the spectral width, a, is initially ρ_p^{-1}. The magnetosheath flow speed is assumed to be ~150 km/s. The proton cyclotron frequency for 20 nT is 2/s. Based on these numbers let us calculate the mirror mode size using Eq. (9):

$$\lambda(L) = 9\rho_p(1+\frac{2\times 6300}{32\times 150})^{1/2} = 17\rho_p \quad (10)$$

This value agrees the mirror mode scale size in the Earth's magnetosheath as shown in Table 1 is ~10-20 ρ_p.

For other planets, we use the following information for our calculations. For Jupiter at ~ 5 AU, [19] obtained a value of ~1 x 10^7 km for the Jovian magnetosheath (see [20] for a larger estimate). For Saturn at ~10 AU, [4] obtained a magnetosheath size of ~3.3 x 10^6 km. We assume the convection speed of ~150 km/s for both the Jupiter and Saturn magnetosheaths.

Because Jupiter and Saturn are much further from the Sun than Earth, the solar wind ion densities are lower due to the radial expansion of the solar wind. The solar magnetic field falls off as ~ $r^{-1.7}$ [21] and thus the Bohm collision rate will decrease as $r^{-1.7}$. The proton cyclotron frequencies may be reduced to $2/5^{1.7}$=0.13/s for Jupiter and to $2/10^{1.7}$=0.04/s for Saturn. With these numbers we may obtain the scale size of the mirror mode at Jupiter to be 86 ρ_p (at a magnetosheath distance 1/3 along the Sun-Jupiter line). For Saturn, we obtain a scale size of ~29 ρ_p. These are comparable to the measured values in Table 1. We note that Bohm diffusion is an upper limit in anomalous diffusion. Thus using it will give an overestimate of the observed scale size.

What can we say about the scale size and spectral widths of mirror modes in the heliosheath? We unfortunately do not have any idea of the spacecraft location relative to the bow shock because the latter is highly dynamic. However one can work backwards by using the measured mirror mode scale sizes and give a distance to the origin of the MM structures.

The heliospheric shock was detected at ~95 AU with an upstream magnetic field strength of ~0.05 nT [13]. The proton cyclotron frequency in the upstream region becomes 5 x10^{-3} s^{-1}. We assume a solar wind speed of 400 km s^{-1}. Since the

MM structures are ~60 ρ_p in scale size, the distance to the source L is ~1.2 x 10^8 km upstream of the shock.

By taking Bohm diffusion into account, the scale sizes of mirror modes in the magnetosheaths of Jupiter and Saturn are predicted to be ~86 ρ_p and 29 ρ_p, at a distance ~1/3 from the bow shock downstream to the magnetopause, respectively. This is with the assumption of MM generation at the theoretical scale sizes near the bow shocks and convection and diffusion thereafter. These values are in good agreement with past measurements (Table 1).

The results of these calculations and comparison to observations can explain the variable scale sizes of mirror mode structures in planetary magnetosheaths. Diffusion can explain why the observed sizes are always bigger than the theoretical size predicted [15,17]. Larger magnetosheaths (e.g., Jupiter) allow for longer diffusion times and thus larger mirror mode sizes observed in situ.

Have past observations given any indications that this diffusion model may be correct? As mentioned previously, most observations were focused on identifying the correct wave mode, rather than details of changes in MM scale sizes. However [6] did record the MM scale sizes at various distances as the Voyager spacecraft passed through the magnetosheath of Jupiter. [6] found scale sizes of 10-30 ρ_p in a region close to the bow shock and then ~100 ρ_p deeper in the magnetosheath. These results (see also Figure 1) are in excellent agreement with the model presented here. However [6] also noted that at distances close to the magnetopause the MM scale size became reduced, ~10-20 ρ_p . A possible explanation for the latter is that Voyager entered the low β plasma depletion layer [22] where enlarged MM structures were dissipated. Then under certain conditions, growth of new MM structures becomes possible?

If diffusion is indeed the explanation for the large scale sizes of MM structures, do the MM amplitudes decrease at the same time? From Figure 1, it appears that the answer is no, they do not, at least for this one case. However one mechanism that has not been taken into account in this model is that continuous free energy for mirror instability is put into the system as the draped magnetic field lines squeeze plasma out the ends of the lines of force [23, 24]. Thus mirror instability will continue throughout the magnetosheath from the shock to the magnetopause, enhancing the size of the MM structures.

These calculations also offer

predictions for further observational testing. Our conclusions indicate that MM scales sizes in all magnetosheaths should be ~ 9 ρ_p at the location of generation. The width of the distribution should be ~ ρ_p^{-1} there. As the MM structures get convected towards the magnetopause, both the mean size and spectral widths should increase by diffusion. Experimental physicists can test these ideas with the expressions and numbers given in this paper.

This theory is a simple first effort. In actuality, there are multiple sources for mirror mode initiation: pickup ions, shock compression and magnetic field line draping (see [18] for a detailed discussion and references). Thus in reality, planetary magnetosheaths are populated with growth centers throughout the magnetosheaths. To understand the full scope of MM development in planetary magnetosheaths and the heliosheath should be the next step for detailed calculations.

The "visualization" of diffusion is similar to an expanding "smoke ring". At the source, the scale is small. With increasing distance from the source, both the radius and the width increase. Since most spacecraft observations are made away from the source, the present theory is in general applicable for deduction of the original structural profile from the observed data. Planetary magnetosheaths are generally highly turbulent, justifying the use of the Bohm diffusion for these cases. However, even in a quiescent collisionless plasma, dispersive trajectories of plasma particles due to the thermal effect can also contribute to effective diffusion of the structure with a value similar to that of the Bohm rate. Thus we consider that the present concept may be applicable universally in space observations.

Portions of this work were performed at the Jet Propulsion Laboratory, California Institute of Technology under contract with NASA. A. Hasegawa thanks Profs. Zhihong Lin and Liu Chen of the Department of Physics and Astrophysics of the University of California, Irvine for their stimulating discussions and hospitality, during which this paper was completed.

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