Whistler Mode Based Explanation for the Fast Reconnection Rate Measured in the MIT Versatile Toroidal Facility

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Whistler Mode Based Explanation for the Fast Reconnection Rate Measured in MIT VTF Experiment

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Abstract

Despite the widely discussed role of whistler waves in mediating magnetic reconnection (MR), the direct connection between such waves and the MR hasn't been demonstrated by comparing the characteristic temporal and spatial features of the waves and the MR process. Using the whistler wave dispersion relation, we theoretically predict the experimentally measured rise time (τ_{rise}) of a few microseconds for the fast rising MR rate in the Versatile Toroidal Facility (VTF) at MIT. The rise time is closely given by the *inverse* of the frequency bandwidth of the whistler waves generated in the evolving current sheet (CS) during the course of the MR in the VTF. The wave frequencies lie much above the ion cyclotron frequency but they are limited to less than 0.1% of the electron cyclotron frequency in the Argon plasma. The maximum normalized MR rate R = 0.35 measured in the experiment is precisely predicted by the characteristic angular dispersion of the whistler waves.

1. Introduction

It has been suggested that the whistler waves facilitate fast magnetic reconnection (M) as seen in a series of simulation studies [1-4]. However, direct evidence of the whistler frequencies and associate time scale has never been presented from these simulations. Drake et al [5] argued that the MR in the simulations was facilitated by whistler waves, but the MR exhaust opened by the propagation of kinetic Alfven waves. The lack of clear signatures of the whistlers from the simulations led to an opposing view that the whistler waves might not facilitate fast MR [6].

However, there are evidences of whistler mode facilitating fast MR in laboratory experiments as reported by Ji et al. [7] and Egedal et al. [8].

Ji et al. [7] measured whistler waves within the electron diffusion region in the Princeton MRX plasma facility; they emphasized that the anomalous resistivity associated with the waves supported the fast MR. Egedal et al. [8] reported detailed information on MR time history measured in the Versatile Toroidal Facility (VTF) at MIT. They reported MR time scale of a few microseconds over which the spontaneous MR rate increased dramatically as the CS thinned in response to an applied magnetic stress. They found a maximum normalized MR rate R= 0.35. We find that these experimental measurements are of seminal importance as they enable us to demonstrate for the first time that the whistler waves do indeed facilitate fast MR in thin CSs. Using whistler mode dispersion relation, we demonstrate that the experimentally measured time scale or the rise time (τ_{rise}) of the MR rate is quite well determined by the inverse of the whistler wave frequencies (f) confined to the lower part of the whistler band, $f_{ci} << f << f_{ce}$, where f_{ci} and f_{ce} are the ion and electron cyclotron frequencies, respectively. The wave frequencies are in turn determined by the transit time of the reconnecting magnetic field into the diffusion region [9]. We further find that the maximum MR rate is R = tan (θ_{max}), where θ_{max} (=19.5°) is the maximum group velocity cone angle of the whistler waves [9, 10].

2. Results from MIT/VTF Experiments

The results from the MIT/VTF experiments are highlighted here. The experimental plasma parameters are as follows: plasma density $n_e \approx 1.5 \times 10^{18}$ m^{-3}, electron temperature $T_e \approx 25$ eV and reconnecting magnetic field $B_o = 4$ mT. Ions are Ar^+ with temperature $T_i \sim 1$ eV, and ion to electron mass ratio M/m = 73440, giving Alfven velocity $V_{Ao} = 10$ km/s with B_o. The VTF device has a guide magnetic field ($B_g \sim 36$ mT). The corresponding electron and ion cyclotron frequencies are f_{ce} = 1.1 GHz and f_{ci} = 15 kHz, respectively. The typical plasma scale lengths are the ion skin depth $d_i \approx 1$ m, electron skin depth $d_e \approx 0.37$ cm, and the ion-acoustic Larmor radius $\rho_s \sim 10$ cm $\sim 28\ d_e$.

The sequence of events leading to explosive MR occurs after applying magnetic stress by enhancing the reconnecting anti-parallel magnetic fields. The CS half width is w ~ 6.5 cm =18d_e at t = 45 μs after the stress application and it reduces to w ~ 2 cm = 5 d_e at t ≈ 80μs; the widths are estimated from Fig. 3 in Ref [8].

Figure 1 (Fig. 5 in Ref [8]) shows time history of the measured MR rate; for t < 70 μs the MR electric field is small (~ 2 V/m) and after t ~ 76 μs it dramatically increases to ~ 13 V/ m with a rise time $\tau_{rise} < 5$ μs. We find that the explosive MR waits until the CS thins to w ~ 5d_e (<< ρ_s ~28 d_e) at t ~76 μs. Since $\tau_{rise} < 5$ μs is nearly 15 times smaller than the ion cyclotron period, $\tau_{ci} \sim 74$ μs, the MR can not be attributed to the kinetic Alfven waves, which have time scales larger than τ_{ci}. We demonstrate here that the fast rise time (< 5 μs) and the large normalized MR rate R = 0.35 [8] are quite accurately determined by the whistler waves.

3. Whistler waves and MR

The MR produces time-dependent **e**lectro**m**agnetic **p**erturbations (EMP) confined within the CS in the diffusion region. The measured electric field shown in Fig. 1is an excellent example of such EMP. According to Maxwell theory such locally generated EMP must propagate away from the X-line with the group velocity vector ($\mathbf{V_g}$). It turns out that $\mathbf{V_g}$ for the whistler mode is confined within a wedge [9, 10]. Thus the propagation of the MR produced EMP is confined within the $\mathbf{V_g}$ wedge, which emanates from the X-line neighborhood and flares transverse to the plane containing the reconnecting and the guide fields. Using the conservation of magnetic flux from the inflow to the outflow region of the MR we relate the inflow to the outflow velocity via the group velocity cone angle.

The dispersive property of the whistler mode for wave frequencies between the ion (Ω_i) and electron (Ω_e) cyclotron frequencies is given by [1, 11]

$$\omega = [k^2/(k_e^2 + k^2)]\, \Omega_e \cos(\psi), \tag{1}$$

where $k_e = d_e^{-1} = \omega_{pe}/C$, C is the velocity of light, and k is the magnitude of the wave vector **k,** which makes an angle ψ from the ambient magnetic field **B,** the vector sum of the guide field $\mathbf{B_g}$ and the reconnecting components; Ω_e (Ω_i) and ω_{pe} (ω_{pi}) are the electron (ion) cyclotron and plasma frequencies, respectively. The cyclotron frequencies are based on **B.**

The directional behavior of $\mathbf{V_g}$ is readily determined by its components parallel and perpendicular to **B**, $V_{g\|} = (\partial\omega/\partial k_{\|})$ and $V_{g\perp} = (\partial\omega/\partial k_{\perp})$, giving $\tan(\theta_k) = V_{g\perp}/ V_{g\|} = -\partial k_{\|}/\partial k_{\perp}$, where θ_k is the angle between $\mathbf{V_g}$ and **B** for the wave vector **k** [9] and 'parallel' refer to **B**. The perpendicular direction is normal to the plane containing $\mathbf{B_g}$ and $\mathbf{B_o}$ vectors. Thus θ_k is the opening angle for a given $k_{\perp}$ and $k_{||}$, along which the corresponding Fourier component of the EMP propagates out of the diffusion region. Using (1), we have

$$\tan(\theta_k) = k_{\perp}k_{||}(k_e^2 - k_{\perp}^2)/[2k_e^2 k_{||}^2 + k_{\perp}^2(k_e^2 + k_{\perp}^2)], \tag{2}$$

So far, the meaning of frequency ω in connection with MR is not discussed. We take the approach that the frequencies and time scales in a physical process are inversely related. We relate the frequency (f) to the transit time (τ_r) of the inflow across the half width (w) of the diffusion region, namely, $f = 1/\tau_r$ and

$$\omega = 2\pi f = 2\pi/\tau_r = 2\pi V_{in} / w = 2\pi R V_{Ao}/w \tag{3}$$

where V_{in} is the inflow velocity into the diffusion region and the Alfven velocity V_{Ao} is determined by B_o, the reconnecting magnetic field. R is defined by $R = V_{in}/V_{Ao}$

We further assume that when MR occurs the inflowing magnetic flux into the diffusion region is converted in to the outflowing magnetic flux, that is $V_{in} B_x = V_{out} B_z$, where B_x is the reconnecting field and B_z connects the anti-parallel components after the MR. Thus we have $V_{in} = (B_z/B_x) V_{out}$.. If the MR exhaust half wedge angle in the outflow region is θ, we have $\tan(\theta) = B_z/B_x = V_{in}/V_{out}$. We assume that $V_{out} = V_{Ao}$ as found in the experiment [8]. In the Fourier domain, θ and V_{in} depend on **k** and we denote them by θ_k and $V_{in, k}$. Thus, the continuity of the magnetic flux yields,

$$V_{in, k} / V_{Ao} = \tan(\theta_k), \quad (4)$$

This also gives the normalized MR rate $R(k_\perp) = V_{in,k}/V_{Ao} = \tan(\theta_k)$ as a function of the wave number. Combining (2), (3) and (4), we find ω and equating it to the frequency given by the dispersion relation (1), we find the MR relation,

$$2\pi(m/M)^{1/2} [B_o/ (B_o^2 + B_g^2)^{1/2}] (d_e/w) (k_\perp / k)(k^2 + k_e^2)(k_e^2 - k^2) \times [2 k_e^2 k_{||}^2 + k_\perp^2 (k^2 + k_e^2)]^{-1} = 1. \quad (5)$$

The foregoing equation determines the spectrum of $k_{||}$ and $k_\perp$, for a CS half width w once the MR initiates. The k- spectrum determines the whistler wave frequencies and the time scale. The superposition of such waves yields spatial and temporal features of the MR structure. Note that the CS half width w, the reconnecting field B_o and guide field B_g appear in (5) as the main parameters, which control the $k_{||}$ and $k_\perp$ spectrum and hence the frequencies and the time scale of the MR process.

For given CS half widths w, and the values of B_o and B_g reported in the VTF, we solve for $k_{||}$ as a function of $k_\perp$ in Argon plasma. We find that (5) permits solutions only for $k_\perp << 1/w$. Using the solutions of (5) in (1), we determine the normalized wave frequency ω/Ω_i, where Ω_i is based on $B = (B_g^2 + B_o^2)^{1/2}$. Figure 2a shows the plots of ω/Ω_i versus $k_\perp$ for some selected values of w/d_e (labeled on the curves) over the entire $k_\perp$-spectrum allowed by (5); the widths are approximately the half widths of the evolving CS for t > 45 μs in the VTF. We see in Figure 2a that the thinner the CS the higher the wave frequency. For $w > 18d_e$ the whistler mode almost ceases to exist as $\omega/\Omega_i \rightarrow 1$. This simply implies that the CS has to be sufficiently thin for the transit time in (3) to be sufficiently small for the wave frequency $\omega >> \Omega_i$, as required for the whistler waves. We also note that the maximum frequency for $w/d_e = 2$ reaches the largest value $\omega/\Omega_i \sim 37$; which is only a tiny fraction ($\omega/\Omega_e \sim 0.05\%$) of the electron cyclotron frequency. *Thus the wave frequencies relevant to the MR are confined to the lower end of the whistler frequency range.*

Using the basic idea that the frequency and time are inversely related, we discuss here the time scale of the whistler-facilitated magnetic MR. Figure 2b shows estimates of MR time scale calculated by $\tau_{rec} = 2\pi/\omega$, as a function of $k_\perp$ for $w/d_e = 2, 3, 4, 5$ and 8. Note that τ_{rec} is simply the transit time introduced in (3). The widths considered in Fig. 2b are approximately those for t > 75 μs in the VTF experiment. The timescale curves in Fig. 2b show broad minima and the minimum MR timescale ranges from ~ 7 μs for $w = 8d_e$ to 2.5 μs for $w = 2d_e$. Thus our calculations suggest that as the CS thins, the time scale decreases and the rate of increase in the MR rate escalates.

From Fig. 3 in Ref. [8] we estimate that at t ~ 75 μs the CS half width is w ~ 3 cm = $8d_e$. For t > 75 μs the CS further thins and we estimate that at t ~ 80 μs the width is w ~ $5d_e$. The MR rate begins to increase at t ~ 72 μs (Fig. 1) but the increase is seen to escalate after t ~ 76 μs; The fastest increase occurs from about t =76 to 81μs as evidenced by the *increasing slope* of the MR rate curve in Fig. 1. This time scale of about 5 μs, over which the MR rate increases dramatically in the VTF, is comparable with the minimum time scales of the whistler waves shown for $3 < w/d_e < 5$ in Fig. 2b.

Figures 2a and 2b show the range of wave number ($k_{perp}= k_\perp$) spectrum, which could contribute to the whistler waves. However, the slower Fourier components ($k_\perp$) with smaller frequencies and larger time scales may not have the time to affect the MR process as the CS thins. Thus, only the Fourier components with the minimum time scale (τ_{rec}) could dominantly contribute to the MR in a thinning CS. Egedal et al [8] noted that the fastest time scale during the rise of the MR rate is about ~ 3μs. Fig. 2b shows that this time scale corresponds to the minimum value of τ_{rec} for w ~ $3d_e$. However, it is intuitive to think that the MR time scale over which the MR rate increases abruptly in Fig.1 is not associated with a single CS width w but with an evolving width, generating a range of whistler frequencies. When fast MR starts at t ~ 76 μs in the CS thinned to w =$5d_e$ and the thinning continues to w = $3d_e$ as the MR proceeds, Fig. 2a shows that whistler frequency range is $\Delta\omega \sim 20\ \Omega_i$, the difference in the frequencies shown by the horizontal broken lines. Corresponding whistler bandwidth is Δf = 300 kHz for f_{ci} = 15 kHz in the VTF. Thus, a good estimate for the MR rise time is $\tau_{rise} = 1/\Delta f = 3.3$ μs. *Our discussion here shows that whether we estimate the MR timescale from the minimum timescales seen in Fig. 2b for w < $5d_e$ or from the bandwidth consideration, the whistler time scales are surprisingly close to the measured MR time scale [8].*

Egedal et al. [8] report that the maximum MR rate is E_{rec} = 13 V/m (Fig.1). Since we have V_{Ao} 10 km/s with B_o = 4 mT and density n =1.5 ×10^{18} m^{-3}, the normalized MR rate is R = 13 / V_A B_o = 0.35. It is exactly the maximum MR rate, which could be realized by the MR mediated by whistler waves as shown in Ref. [9]. Figure 2c shows the MR rate $R(k_\perp) = V_{in,k}/ V_{Ao} = \tan(\theta_k)$ as a function of $k_\perp$. We find that irrespective of the CS width, maximum R is 0.35, corresponding to the maximum value of $\theta_k = 19.5^\circ$, which was first discovered by Storey [10] in connection with atmospheric whistlers. The range of $k_\perp$ over which R is near its maximum value (~0.35) in Fig. 2c corresponds to the range of minimum time scale in Fig. 2b. These observations on Figs. 2a and 2c are true only when the CS is sufficiently thin so that MR is triggered. In wide CSs above a threshold width, whistler may not contribute to the MR. In the VTF, the threshold width is about w ~ $5d_e$.

We suggest the following scenario for the time history of the MR rate in Fig. 1. For t < 70 μs the main feature of the CS evolution is its thinning as the magnetic flux is transported inward; for such times there is low MR rate, perhaps due to residual collisional resitivity. The most of the inward transported magnetic flux is used in increasing the magnetic field in close vicinity of the neutral line resulting into the thinning of the CS. For t > 76 μs, when the CS has thinned sufficiently and current density is sufficiently large to drive instabilities, anomalous resitivity sets in and leads to electron tearing instability involving the whistler waves resulting into MR. A similar scenario is mentioned in Ref. [8] and was seen in fully three-dimensional PIC simulations of thin CSs, which evolved from applied magnetic stress [12]; shear-assisted Buneman instability generated the anomalous resistivity leading to the explosive evolution of the tearing of the CS. In the VTF ions are cold and could give rise to the ion-acoustic turbulence. When MR is triggered, reconnected flux is transported outward in the outflow region bounded by the group- velocity half wedge angle of 19.5°. Electrons accelerated/heated by the MR process tend to flow out of the diffusion region, but they are tied to the ions by electrostatic coupling. Thus, both ions and electrons flow out together with the Alfven velocity [8] and carry the reconnected magnetic flux. Once the MR is triggered in a thin CS ($w < 5d_e$), both the MR rate and the time scale are determined by the whistler wave dynamics.

4. Conclusion and Discussion

Main aim of this paper is to demonstrate that time-dependent fast impulsive MR events can be explained in terms of whistler wave mode. This mode predicts the escalating decrease in the time scale of the increasing MR rate as well as its maximum value as measured in the explosive MR events in VTF [8]. The whistler waves with relatively short time scales, or equivalently with relatively high frequencies much above the ion cyclotron frequency, appear when the reconnecting CS thins to a half width (w) of a few electron skin depth. In the VTF experiment this happens when $w < 5d_e$ after $t \sim 75$ μs (Fig. 1). In such a thin CS tearing mode might initiate the MR. *Once the MR begins in the thin CS, the MR time scale and the rate are accurately predicted by the whistler wave dynamics.* The continued thinning of the CS causes the escalating decrease in the time scale of the MR. The agreements on the rise time and maximum MR rate found in our theory and the VTF are quantitatively close.

We point out that for the MR to take off, the time scale of the triggering instability (**TI**) must match with the rate of supply of magnetic flux into the diffusion region; otherwise, the MR will not be sustainable. The time scale of the flux supply depends on the inflow velocity V_{in} and the CS half width w. The synchronism between the **TI** and supply rates remains to be demonstrated by lab experiments and/or three dimensional simulations. In the VTF [8] for $w < 5d_e \sim 1.8$ cm, the time scale defined by (3) is $\tau_r < 5$ μs corresponding to $V_{in} \approx 0.35\ V_A \sim 3.5$ km/s, but the instability rate was not measured. In the MRX [7], the measured frequency shows a bandwidth of about ~ 1 MHz approximately revealing a time scale $\tau_r \sim 1$ μs. From Fig. 2a in [7] we infer that w ~ 1 cm at the time of MR onset, giving $V_{in} \sim w/\tau_r \sim 10$ km/s. Thus for the reconnecting magnetic field B ~ 10 mT, we estimate reconnection electric field E ~ 100 V/m as reported in [7]. This suggests that the time scale τ_r defined in (3) is a proxy for the instability rate.

Whistler waves are highly dispersive and involve range of parallel and perpendicular wave numbers and, therefore, a range of group velocity cone angles θ_k from 0 to 19.5°. In the VTF [8] the fast rising MR has the smallest time scale corresponding to $\theta_{max} \sim 19.5°$ and the associated wave numbers. In quasi-steady state MR occurring over a prolonged time, even the slower timescales corresponding to a broader range of wave numbers contribute to the MR. Thus, the MR rate averaged over the broad wave number spectrum is expected to be smaller than the maximum rate R ~ 0.35. The perpendicular wave number spectrum is determined by the localized structure of the EMP in the electron diffusion region.

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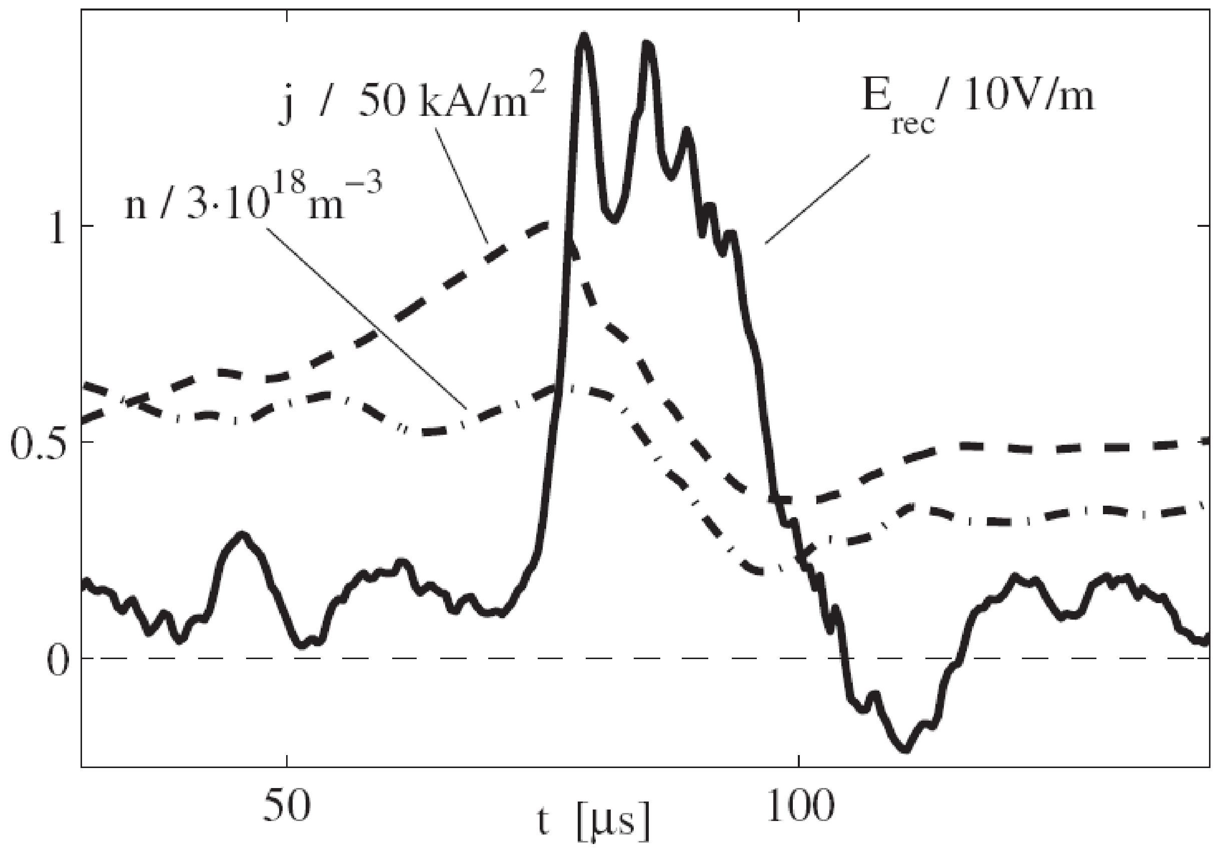

Figure 1. Temporal evolution of MR rate (E_{rec}) (solid line), density (dot-dash line) and current (dashed line) near the X-line measured in the MIT VTF device [8].

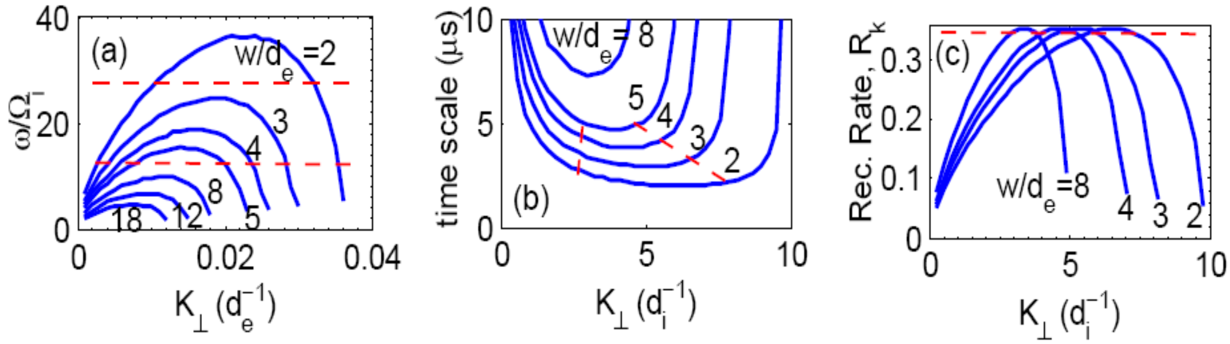

Figure 2a. (a) Whistler wave frequency ω, (b) MR Time scale $\tau_{rec} = 2\pi/\omega$ and (c) Normalized MR rate R_k as functions of $k_\perp$ for (w/d_e) as labeled on the curves. In (a) the horizontal axis is $k_\perp d_e$ while in (b) and (c) it is $k_\perp d_i$. The horizontal broken lines in (a) show the whistler frequency bandwidth when CS thins from w ~ 5 to w ~3 d_e. The red broken lines in (b) show the widening range of wave numbers with nearly the minimum time scale as the CS thins. Note the maximum value of $R_k = 0.35$ as shown by the red horizontal line in (c).