Hangup Kicks: Still Larger Recoils by Partial Spin-Orbit Alignment of Black-Hole Binaries

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We revisit the scenario of the gravitational radiation recoil acquired by the final remnant of a black-hole-binary merger by studying a set of configurations that have components of the spin both aligned with the orbital angular momentum and in the orbital plane. We perform a series of 41 new full numerical simulations for equal-mass and equal-spin-magnitude binaries, but with different spin orientations. We extend previous recoil fitting formulas to include nonlinear terms in the spins and successfully include both the new and known results. For this new formula the predicted maximum velocity approaches 5000km/s. More importantly, from the astrophysical point of view, it reaches this maximum for spins partially aligned with the orbital angular momentum. The optimal configuration is near an equipartition of the hangup and superkick contributions. This newly discovered contribution to the recoil leads to an important increase of the probabilities of large recoils in generic astrophysical mergers. We measure these probabilities for the case of accretion-aligned spins and find non-negligible probabilities for supermassive black hole encounters leading to recoil velocities of several thousand km/s.

Introduction: With the breakthroughs of 2005 in the numerical techniques to evolve black hole binaries (BHBs) [1–3], Numerical Relativity (NR) became a very important tool to explore highly-dynamical and nonlinear predictions of General Relativity. In the last few years we have gained notable insight into the modeling of gravitational radiation to assist laser interferometer detectors [4]. There are also numerous examples of explorations in the realm of Mathematical Relativity and in the astrophysical scenarios for supermassive black hole mergers and retention of black holes in galaxies and globular clusters [5, 6].

Some of the most striking recent discoveries are related to effects due to the intrinsic spin of the individual BHs during the final merger stage of BHBs. In general, the spins and orbital plane precess during the inspiral and the spin direction of the remnant is misaligned with the individual BH spins [7] prior to merger (spin flips). Spins can also have a dramatic effect on the inspiral rate. When the BH spins are partially aligned with the orbital angular momentum, the merger is delayed, while when they are anti-aligned, the merger happens much more quickly. This "hangup" effect [8] is due to an unexpectedly strong spin-orbit coupling. Perhaps even more surprisingly are the very large recoils [5] acquired by the remnant of the merger of comparable-masses, highly-spinning, BH in a particular configuration with spins lying along the orbital plane (equal magnitude, but opposite in direction) [5, 9, 10]. Initial studies, which indicated that these BHBs recoil at up to ~ 4000 km s^{-1} [9, 11], prompted astronomers to search for possible recoil candidates. To date, a few interesting cases of galaxies with cores dis-

playing differential radial velocities of several thousands km/s [12–14] have been found. More systematic recent studies produced tens of potential candidates [15, 16].

The discovery of large recoils also triggered theoretical statistical studies of BHB dry mergers [17] and Monte Carlo simulations of wet pre-mergers to study the effect of accretion on spin distributions [18–20]. Accretion tends to align spins with the orbital angular momentum [21, 22]; resulting in a notable reduction in the probabilities of observing large recoils, either directly, or through their influence on the galactic cores, for small and medium sized galaxies [6]. For BHs with masses larger than $10^8 M_\odot$, alignment by accretion is less effective. In this letter we revisit the scenarios for the generation of recoils by studying a set of configurations that combine two of the largest spin effects observed in BHBs, the hangup effect and superkicks. The combined effect appears to be a dramatic change in the probability distribution for large recoils, as we will show below.

Full Numerical Simulations: We evolved a set of 41 equal-mass, spinning, quasi-circular configurations using the LazEv [23] implementation of the moving puncture formalism [2, 3], with the conformal factor $W = \sqrt{\chi} = \exp(-2\phi)$ suggested by [24] as a dynamical variable. For the runs presented here we use centered, eighth-order finite differencing in space [25] and a fourth-order Runge-Kutta time integrator. The LazEv code used the Cactus/EinsteinToolkit [26, 27] numerical infrastructure along with the Carpet [28] mesh refinement driver. We use the TwoPunctures [29] thorn to calculate the initial data. We use AHFinderDirect [30] to locate apparent horizons. We measure the magnitude of the hori-
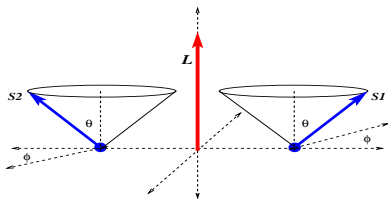
FIG. 1: Black-hole-binary configuration for hangup recoils.

zon spin using the Isolated Horizon algorithm detailed in [31].

Our configurations have the property that the in-plane components of the spins of the two BHs have the same magnitude, but opposite signs, while the out-of-plane components have the same magnitude and sign (see Fig. 1). They thus combine the hang-up [8] and super-kick [5, 9] effects. Additionally, the orbital plane does not precess, but rather moves up and down along the direction of orbital angular momentum as the binary evolves.

We performed a set of 30 simulations with individual BH spins of magnitude $\alpha = 1/\sqrt{2}$ and 11 simulations with BH spin magnitudes of $\alpha = 0.91$, where $\vec{\alpha}$ is the normalized spin of the BH. The 30 $\alpha = 1/\sqrt{2}$ configurations were split into five sets of 6, where the runs in each individual set had the same initial angle θ between the spin direction and orbital angular momentum direction (here we chose $\theta = 22.5°$, $45°$, $60°$, $120°$, $135°$). In each set with a given θ, we chose the initial orientation ϕ_i between the in-plane spin and linear momentum to be $0°$, $30°$, $90°$, $130°$, $210°$, and $315°$. For the $\alpha = 0.91$ runs, we had the same initial 6 ϕ_i configurations for $\theta = 60°$, while for $\theta = 15°$, we used 5 ϕ_i configurations (all but $\phi_=315°$; due to limited resources). We combine these results with the simulations of [11] (which have $\theta = 90°$) in order to perform our analysis below.

We set up the initial separations, such that each binary completed 5-6 orbits, prior to merger (to reduce eccentricity). The initial separations varied between 10.16M and 8.2M (depending on the magnitude of the hangup effect).

Results and Analysis: In a previous study [11], we found that the superkick recoil (where the two BHs have equal mass, equal intrinsic spin magnitudes α, and spins lying in the orbital plane in opposite directions) has the following dependence on spin α and orientation ϕ (the angle between the in-plane spin vector and the infall direction near merger),

$$\begin{aligned} V &= V_1 \cos(\phi - \phi_1) + V_3 \cos(3\phi - 3\phi_3), \\ V_1 &= V_{1,1}\alpha + V_{1,3}\alpha^3, \\ V_3 &= V_{3,1}\alpha + V_{3,3}\alpha^3, \end{aligned} \qquad (1)$$

where $V_{1,3} = (-15.46 \pm 2.66)$ km s^{-1}, $V_{3,1} = (15.65 \pm 3.01)$ km s^{-1}, and $V_{3,3} = (105.90 \pm 4.50)$ km s^{-1}, while $V_{1,1} = (3681.77 \pm 2.66)$ km s^{-1}. From that study, it was

clear that in the superkick configuration, the dominant contribution, even at large α, is linear in α and proportional to $\cos(\phi)$. Note that because of the small contributions of V_3 and $V_{1,3}$, we neglect these terms in the statistical studies below (where we take a uniform distribution in $\phi - \phi_1$).

Our initial motivation for the current study was to determine if the hangup effect [8], which amplifies the amount of radiation emitted by the BHB, also affects the maximum recoil. To this end, we looked at configurations that combined both the hangup and superkick effects. Based on the superkick formula (1), we expected that the recoil would have the form

$$\begin{aligned} V_1 = V_{1,1}\alpha \sin\theta + A\alpha^2 \sin\theta \cos\theta + \\ B\alpha^3 \sin\theta \cos^2\theta + C\alpha^4 \sin\theta \cos^3\theta, \end{aligned} \qquad (2)$$

where V_1 is the component of the recoil proportional to $\cos\phi$, $V_{1,1}$ arises from the superkick formula, and the remaining terms are proportional to linear, quadratic, and higher orders in $S_z/m^2 = \alpha\cos\theta$ (the spin component in the direction of the orbital angular momentum). Here, we do not consider terms higher-order in the in-plane component of $\vec{\Delta} \propto \vec{\alpha}_2 - q\vec{\alpha}_1$ denoted by $\Delta^\perp$ ($\Delta^\perp \propto \alpha\sin\theta$ here), where $q = m_1/m_2$ is the mass ratio, because our previous studies showed that these terms were small at $\theta = 90°$. A fit to this ansatz (2) showed that the coefficients $V_{1,1} = (3677.76 \pm 15.17)$ km s^{-1}, $A = (2481.21 \pm 67.09)$ km s^{-1}, $B = (1792.45 \pm 92.98)$ km s^{-1}, $C = (1506.52 \pm 286.61)$ km s^{-1} converge very slowly and have relatively large uncertainties. In addition, we propose the modification

$$V_1 = D\alpha\sin\theta \left(\frac{1 + E\alpha\cos\theta}{1 + F\alpha\cos\theta} \right), \qquad (3)$$

which can be thought of as a resummation of Eq. (2) with an additional term $E\alpha\cos\theta$, and fit to D, E, F (where we used the prediction of [11] to model the V_1 for $\theta = 90°$) and find $D = (3684.73 \pm 5.67)$ km s^{-1}, $E = 0.0705 \pm 0.0127$, and $F = -0.6238 \pm 0.0098$. Note that E is approximately $1/10$ of F, indicating that corrections to this formula converge quickly. The two formulas (2) and (3) give very similar results for a broad range of α. We then use Eq. (3) to predict the recoil for higher spin $\alpha = 0.91$ and test this formula for two angles $\theta = 90°$ and $\theta = 60°$, with very good agreement (see Fig. 2). In actuality, both Eq. (3) and Eq. (2) provide accurate predictions for our measured recoils at $\alpha = 0.91$. The results are startling. The recoil is not maximized at $\theta = 90°$, as was previously assumed based on linear spin-orbit PN expressions, but rather at smaller angles (see Table I). Additionally, the maximum recoil is closer to 5000km s^{-1}. Note that we measured recoils from actual simulations as large as 4171km s^{-1} for $\alpha = 0.91$, which is larger than the previously predicted maximum possible recoil of 3681km s^{-1} for quasi-circular binaries. This can

TABLE I: The angle θ that gives the largest recoil along with the magnitude of this recoil for different values of the individual BH spin α. The columns to the left are from Eq. (3), while the columns to the right are for Eq. (2)

α	$\theta_{\max}$	$V_{\max}$	$\theta_{\max}$	$V_{\max}$
0.1	$86.02°$	369.36	$86.13°$	368.62
0.5	$70.16°$	1961.38	$69.99°$	1955.51
$1/\sqrt{2}$	$61.90°$	2968.71	$61.33°$	2967.85
0.91	$53.55°$	4224.93	$53.92°$	4231.93
1	$49.67°$	4925.94	$51.22°$	4915.22

TABLE II: Probability that the recoil velocity will be in a given range P, and the probability that the recoil will be a in a given range along the line of sight P_{obs} (to relate to possible redshift measurements in galaxies) for the new prediction (LEFT) and the old prediction (RIGHT)

range	P	P_{obs}	P old	P_{obs} old
0-500	79.027%	92.641%	94.888%	98.914%
500-1000	15.399%	6.177%	4.921%	1.067%
1000-2000	5.384%	1.164%	0.191%	0.019%
2000-3000	0.189%	0.018%	0	0
3000-4000	0.001%	0.0001%	0	0

FIG. 2: A fit of the recoil (V_1) to the form Eq. (3) for the $\alpha = 1/\sqrt{2}$ configurations, and predictions (based on this fitting) for the $\alpha = 0.91$ recoils. Note how well the $\alpha = 0.91$ curve matches the three measured values. For reference, curves corresponding to the original empirical formula prediction (which only had terms linear in Δ) for $\alpha = 1/\sqrt{2}$ and the new formula for $\alpha = 1$ are also included. Note the skew in the velocity profile compared to the linear predictions.
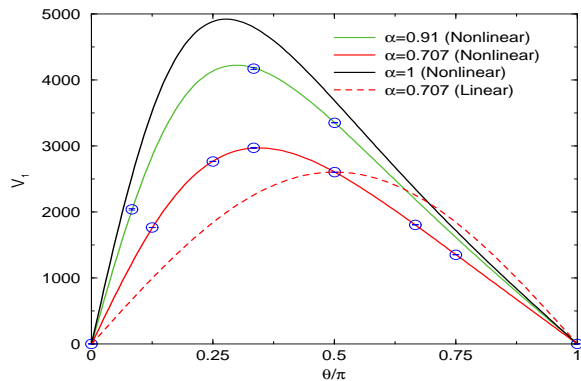
have profound astrophysical implications because partial alignment of the binary (e.g. due to accretion), rather than inhibiting large recoils, can actually amplify them, leading to much larger probabilities for observing high recoils.

Using the same post-Newtonian analysis [32] as in [33], we can extend formulas (2) and (3) to less symmetric configurations by replacing $\alpha \sin\theta$ by $[\alpha_2^\perp - q\alpha_1^\perp]/(1+q)$ and $\alpha\cos\theta$ by $2[\alpha_2^z + q^2\alpha_1^z]/(1+q)^2$. Importantly, we are assuming that terms proportional to $[\alpha_2^\perp - q\alpha_1^\perp]^n$ (for $n > 1$) are negligible. This can be verified by confirming that formulas (2) and (3) are accurate for all θ and α (a subject of our ongoing analysis that will be reported in a forthcoming paper). We emphasize that the proposed extension is an ansatz, that while reasonable as a starting point for the modeling, needs to be thoroughly tested and refined. This modeling will be the subject of an upcoming paper. In this letter we will use Eq. (2) when generalizing to unequal masses and arbitrary spin orientations. A generalization of the resummation (3) to the generic mass ratio q case is non-trivial, and will be discussed in an upcoming paper. Our ansatz for the generic recoil (for

brevity, we only display the term in the direction of the orbital angular momentum proportional to $\Delta_\perp$) then is

$$v_\parallel = 16\frac{\eta^2}{(1+q)}\left|\alpha_2^\perp - q\alpha_1^\perp\right|\left[V_{1,1} + A\tilde{S} + B\tilde{S}^2 + C\tilde{S}^3\right] \times \cos(\phi_\Delta - \phi_1), \quad (4)$$

where $\tilde{S} = 2(\alpha_2^z + q^2\alpha_1^z)/(1+q)^2$, ϕ_Δ is angle between $\vec{\Delta}_\perp \propto \vec{\alpha}_2^\perp - q\vec{\alpha}_1^\perp$ and the infall direction (evaluated at a fiducial point around merger), ϕ_1 is a constant, and $\eta = q/(1+q)^2$ is the symmetric mass ratio. See Ref. [17] for the remaining components of the recoil velocity.

Astrophysical Implications: To test the effect of our new empirical formula on the predicted recoil rates, we consider a model distribution of BHBs with mass ratio distribution $P(q) \propto q^{-0.3}(1-q)$ [34–36], spin distribution $P(\alpha) \propto (1-\alpha)^{(b-1)}\alpha^{(a-1)}$, with parameters $a = 4.8808$ and $b = 1.72879$ (which models the distribution kindly provided by M. Volonteri [37] resulting from spin-up effects due to accretion), spin-direction distribution $P(\theta) \propto (1-\theta)^{(b-1)}\theta^{(a-1)}$, with parameters $a = 2.5$ and $b = 7$ (which approximates the spin-direction distribution provided by M. Volonteri [37], which is a simplified model for the spin distribution for hot accretion [22]), and $P(\phi)$ uniform in $0 \leq \phi \leq 2\pi$. We model 10^7 BHBs consistent with these distributions and measure the predicted recoil using Eq. (4), as well as our original empirical formula, and compare the probabilities for observing a recoil in a given velocity range. Our results are summarized in Fig. 3 and Table II. The dramatic effect of the new recoil prediction is apparent. Recoils in the range $1000\,\mathrm{km\,s^{-1}} - 3000\mathrm{km\,s^{-1}}$ are significantly enhanced, leading to a realistic chance for observing large recoils. Details on how the statistical studies were performed can be found in [17].

Conclusions and Discussion: We revisited the scenario for the generation of large gravitational radiation recoils acquired by the remnant BH after the merger of BHBs and found that configurations with spins partially aligned with the orbital angular momentum produce larger recoils (up to 1200 km/s more) than those with spins lying in the orbital plane (aka superkicks). The new configuration maximizes the total momentum radiated by optimiz-

FIG. 3: The recoil probability distribution using the new and old empirical formulas for the recoil, starting from a distribution of BHB configurations consistent with recent models for accreting binaries [37]. The new formula predicts a significantly larger probability for high recoils.
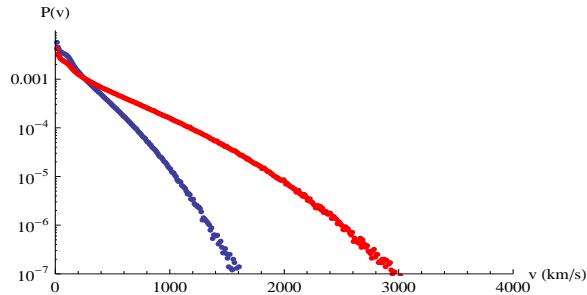
ing over the competing requirements of maximizing the total power radiated (which occurs for the hangup configuration) and the skew in the power distribution (which is maximized in the superkick configuration). Our results imply a nonlinear coupling among components of the spins that can be and expressed in the simple form given by Eq. (4). Based on these new terms in the empirical formula for recoils, we recalculate the probabilities for large recoils to occur in astrophysical scenarios of BHB encounters when accretion effects are included. At small angles of the spins with the orbital angular momentum, the new term magnifies recoil velocities by up to a factor 2.8 with respect to the previous formula, and we find non-negligible probabilities of observing black holes recoiling at several thousand km/s, as reported in Table II. Our results indicate that there is a need for additional theoretical searches for large recoils in other regions of the parameter space, and lend support for additional observational searches for high-velocity black holes.

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