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Experimental Validation of the Largest Calculated Isospin-Symmetry-Breaking Effect in a Superallowed Fermi Decay

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A precision measurement of the γ yields following the β decay of ^{32}Cl has determined its isobaric analogue branch to be $(22.47^{+0.21}_{-0.18})\%$. Since it is an almost pure Fermi decay, we can also determine the amount of isospin-symmetry breaking in this superallowed transition. We find a very large value, $\delta_C = 5.3(9)\%$, in agreement with a shell-model calculation. This result sets a benchmark for isospin-symmetry-breaking calculations and lends support for similarly-calculated, yet smaller, corrections that are currently applied to $0^+ \rightarrow 0^+$ transitions for tests of the Standard Model.

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Precisely measured ft values of $J^\pi = 0^+ \rightarrow 0^+$ β decays of isospin $T = 1$ nuclei are used to set stringent limits on scalar and right-handed interactions, verify the conserved vector current (CVC) hypothesis at the $\sim 10^{-4}$ level, and provide the most precise measurement of V_{ud}, the up-down element of the Cabibbo-Kobayashi-Maskawa quark-mixing matrix [1, 2]. For these purposes, the vector coupling constant is extracted from the experimental ft values of these nuclei, after correcting for isospin-symmetry-breaking and radiative effects. To date, the ft values of 13 such nuclei have been experimentally determined to a precision of $\lesssim 0.3\%$ which sets one of the most demanding tests of the Standard Model [2]. These experimental advances have placed the theoretically calculated corrections under intense scrutiny in recent years. Emphasis has been placed on the nuclear-structure-dependent isospin symmetry breaking (ISB) corrections, denoted by δ_C [3, 4], and defined by the equation $|M_F|^2 = |M_0|^2(1-\delta_C)$. Here M_F is the Fermi matrix element for the transition and M_0 is its value in the limit of strict isospin symmetry, which is broken by Coulomb and charge-dependent nuclear forces. The 13 cases just mentioned are all $T = 1 \rightarrow 1$ transitions in $A = 4n + 2$ nuclei with corrections that are small and of order 1%.

A discriminating test of these calculations is realized by investigating cases where the correction is much larger. Up to now, though, there have been no nuclei studied where δ_C is larger than $\sim 2\%$ [2, 5, 6]. In this Letter, we focus on the β decay of $1^+, T = 1$ ^{32}Cl as a test of isospin-mixing calculations. Its Fermi decay branch feeds the analogue $1^+, T = 1$ state in ^{32}S, whose position in the spectrum at 7002-keV excitation is very close to a known $1^+, T = 0$ state at 7190 keV [7]. This greatly enhances the size of the isospin-breaking correction. Our calculation of the ISB effect for this $A = 4n$ nucleus is $\delta_C = 4.6(5)\%$, a value significantly larger than those found in any of the $A = 4n + 2$ nuclei. Thus the case of ^{32}Cl provides a unique opportunity to test isospin-symmetry-breaking calculations where the correction is relatively very large.

The experiment was performed at the Cyclotron Institute at Texas A&M University. Details of this experiment will appear in a separate paper [8]. Briefly, we produced ^{32}Cl via the inverse-kinematic transfer reaction ^{1}H(^{32}S, n)^{32}Cl using a LN_2-cooled, H_2 gas target with a 400 nA ^{32}S primary beam at 24.8 MeV/nucleon. The reaction products were spatially separated by the Momentum Achromatic Recoil Separator [9], resulting in a 91% pure, 20 MeV/nucleon ^{32}Cl beam with an intensity of $\sim 2 \times 10^5$ ions/s. The beam was implanted and collected in an aluminized-Mylar tape for 0.8 s before a fast tape-transport system moved the activity to a shielded counting station 90 cm away. Data for $\beta - \gamma$ coincident events were acquired using a 1.5 inch diameter, 1 mm-thick scintillator and a 70% HPGe detector. Count times were for 1, 2 and 4 sec (76%, 13% and 11% of the data respectively). The scintillator was placed 0.5 cm from the activity, detecting ≥ 40 keV positrons with $\approx 32\%$ efficiency. On the opposite side of the tape the HPGe was placed a large distance away (15.1 cm) to reduce the effects of coincidence summing of the γ rays. The cycle of collecting, transporting and measuring the ^{32}Cl activity was repeated continuously throughout the experiment.

Critical to the success of this experiment was the extremely precise photopeak efficiency calibration of our HPGe detector. As described in detail in Ref. [10], using a combination of measurements and Monte Carlo calculations with the code CYLTRAN [11], the efficiency is determined to $\pm 0.2\%$ from $E_\gamma = 50 - 1400$ keV and $\pm 0.4\%$ from $1.4 - 3.5$ MeV. The energy range of the HPGe in the present work, however, extends up to 7.35 MeV; so we extended the efficiency curve above 3.5 MeV using the same CYLTRAN code used in Ref. [10]. To estimate uncertainties in this extrapolation, we performed an independent calculation using the Monte Carlo code PENELOPE [12]. The difference between the two efficiency curves is shown

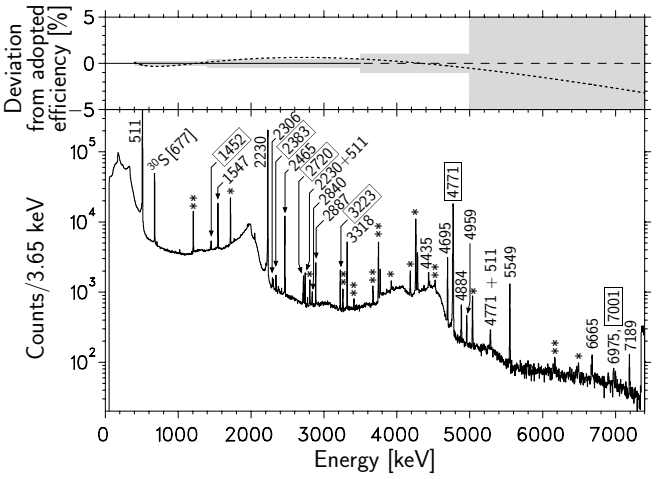

FIG. 1. Top: Percent differences in the absolute efficiency from Ref. [10] (solid line) and our adopted uncertainties (shaded region). This adopted efficiency curve and its CYLTRAN-calculated extrapolation (dashed line) are compared to a PENELOPE simulation (dotted line). Bottom: γ-energy spectrum of the 11.2×10^6 β-coincident events observed in this work. Aside from the isolated ^{30}S contaminant peak at 677 keV, labeled peaks are associated with the decay of ^{32}Cl (* and ** indicate their single- and double-escape peaks). Boxed values indicate transitions from the 7002-keV isobaric analogue state.

in the top panel of Fig. 1. Over the range of measured values, PENELOPE reproduces well the experimentally determined efficiency, mostly within the $\pm(0.2-0.4)\%$ uncertainty range from Ref. [10]; above 3.5 MeV, the difference between the two extrapolations is contained within our adopted uncertainties ranges of $\pm1\%$ from $3.5-5$ MeV and $\pm5\%$ above 5 MeV. The bottom panel of Fig. 1 shows a plot of the observed γ spectrum in coincidence with a β signal in the scintillator, where nearly every observed peak is associated with the decay of ^{32}Cl.

Once the peak areas were obtained, we used the precisely known efficiencies to convert them into relative yields of γ-rays. We then fit the β and γ branches to reproduce these yields. Our measurement has found 3 new β branches, 22 new γ lines, placed limits on 10 potential γ transitions, and improved the precision of the branches and yields reported previously [13] by about an order of magnitude [8]. The 12 β transitions we observe and the known ground state branch of $(1.0^{+0.2}_{-0.5})\%$ from Armini *et al.* [14] represent almost all of the β yield; however, there is potentially still a large number of weak β transitions which, though too weak to be seen individually, may sum up to a total β strength that is non-negligible. This "Pandemonium effect" [15] was recently raised [16] in the context of superallowed β decay in p, f-shell nuclei. Here we follow the approach advocated there: to compute these very weak, unobserved β branches using a shell-model calculation and include this predicted strength as a small correction in the analysis. We take the model space to be the full s, d shell and use the charge-independent effective interaction of Wildenthal `USD` [17], as well as the more recent `USDA` and `USDB` updates [18] of Brown and Richter [18].

Our analysis of the branches and yields includes a total of 51 excited states in ^{32}S. In addition to the 12 β branches observed, our shell model calculation identifies approximately 30 additional weak β transitions to states whose excitation energy lies between 7.485 and ≈ 11.8 MeV. Though none of these individually has a β-transition strength greater than 0.3%, the summed β strength of all of them is 0.60(10)%, where the uncertainty is a result of the different interactions used in the shell model. We include these weak β strengths and the de-excitation γ rays predicted by the shell model in our overall analysis to account for the small Pandemonium effect. We do not separately include the α-particle and proton-emitting states reported by Honkanen *et al.* [19] because their summed β strength of 0.080(5)% is significantly smaller than–and is no doubt already included in–the missing strength predicted by the shell model.

We find the β branch to the isobaric analogue state (IAS) at 7002 keV is $R = (22.47 \pm 0.13^{+0.16}_{-0.12})\%$. The first uncertainty is statistical and the second is dominated by two sources of systematic uncertainty: $^{+0.11}_{-0.05}\%$ from the $(1.0^{+0.2}_{-0.5})\%$ ground state branch reported by Armini *et al.*, and $\pm0.10\%$ from the photopeak efficiency of the HPGe detector.

To derive the experimental ft value, we obtain the partial half-life, t, from

$$t = \frac{t_{1/2}}{R}\,(1 + P_{\rm EC}) = 1.327(13)\ \text{s}, \qquad (1)$$

where the ^{32}Cl half-life is $t_{1/2} = 298(1)$ ms [14], R is the superallowed branching ratio quoted above, and the small electron-capture fraction is calculated to be $P_{\rm EC} = 0.071\%$. We use the shell model to compute the shape correction function $C(W)$ (as described in the appendix of Ref. [1]) when defining the statistical rate function

$$f = \int_1^{W_0} pW(W_0 - W)^2\, F(Z,W)\, C(W)\, dW, \qquad (2)$$

where $W = E_e/m_e$ is the total energy of the positron in electron rest-mass units, $p = (W^2-1)^{1/2}$ is its momentum, Z is the charge of the daughter nucleus, and $F(Z,W)$ is the Fermi function. The end-point energy, W_0, is determined using $-26015.535(2)$ keV for the mass excess of ^{32}S from Ref. [20], and we average Refs. [21–23] to get $-13334.60(57)$ keV for the mass excess of ^{32}Cl. Combined, the decay energy is $Q_{\rm EC} = 12680.9(6)$ keV. This gives $f = 2411.6 \pm 2.3 \pm 0.3$ for the phase-space factor, where the first uncertainty is from the $Q_{\rm EC}$ value and the second is from the shell-model calculation of $C(W)$. Thus the experimental ft value for decay to the IAS is $ft = 3200(30)$ s, where the precision is dominated by the $\pm0.9\%$ uncertainty in the branch to the IAS.

We now deduce an experimental value for the isospin-mixing parameter, δ_C, of the decay to the isobaric-analogue state from the measured ft value. Before this can be done, however, we need some way of separating out the Gamow-Teller component of this mixed $1^+ \to 1^+$ transition so that we can analyze the Fermi component alone. Fortunately, the `USD`, `USDA` and `USDB` shell-model calculations (described later) all predict that the Gamow-Teller matrix element is $\approx 0.1\%$ of the Fermi matrix element, and so is negligibly small for this transition. Thus we proceed to analyze this transition as if it were a pure-Fermi type via the equation

$$ft(1+\delta'_R)(1+\delta_{\rm NS}-\delta_C) = \frac{K\,/\,G_V^2(1+\Delta_R^V)}{B({\rm F})+B({\rm GT})}. \qquad (3)$$

Here $K/(\hbar c)^6 = 2\pi^3\hbar\ln 2/(m_e c^2)^5$ is a constant and G_V is the vector coupling constant characterizing the strength of the vector weak interaction. The numerator in Eq. (3) may be evaluated using the precision work on $0^+ \to 0^+$ superallowed transitions: $K/G_V^2(1+\Delta_R^V) = 2\langle \mathcal{F}t^{0^+\to 0^+}\rangle$, where the average of the 13 most precisely-measured cases yields $\langle \mathcal{F}t^{0^+\to 0^+}\rangle = 3071.81(83)$ s [2]. The radiative correction has been split into three pieces: (a) a nucleus-independent term, Δ_R^V, is included in $\langle \mathcal{F}t^{0^+\to 0^+}\rangle$; (b) a trivially nucleus-dependent term, δ'_R, is calculated to be 1.421(32)%; and (c) a second nuclear-structure-dependent term, $\delta_{\rm NS}$, is determined to be $-0.15(2)\%$ in a shell-model calculation following the procedures in Ref. [24]. Finally B(F) and B(GT) are the squares of the Fermi and Gamow-Teller matrix elements.

In the isospin-symmetry limit, $B({\rm F}) = |M_0|^2 = 2$ for $T = 1$ transitions. For B(GT), we take the three theoretical values from the shell model using the `USD`, `USDA` and `USDB` effective interactions, average them and assign an uncertainty which spans the three calculated values: $B({\rm GT}) = (1.8^{+2.3}_{-1.7}) \times 10^{-3}$. This is negligibly small compared to the dominant Fermi strength. On rearranging Eq. (3), we obtain

$$\delta_C^{\rm exp} = 1 + \delta_{\rm NS} - \frac{2\langle \mathcal{F}t^{0^+\to 0^+}\rangle}{ft(1+\delta'_R)\big[B({\rm F})+B({\rm GT})\big]} = 5.3(9)\%. \qquad (4)$$

This represents a very substantial isospin-symmetry-breaking term, the largest ever determined in a superallowed Fermi transition. As Fig. 2 shows, it is an order of magnitude larger than the typical correction applied to the $0^+ \to 0^+$ pure Fermi decays, and nearly $3\times$ larger than the biggest of these cases, ^{74}Rb. Thus this provides a strong benchmark with which to compare the variety of theoretical methods and models proposed to calculate ISB in nuclei [3, 4].

In what follows we present a shell-model calculation of δ_C to compare to the above result following the procedures developed by Towner and Hardy [3]. The technique

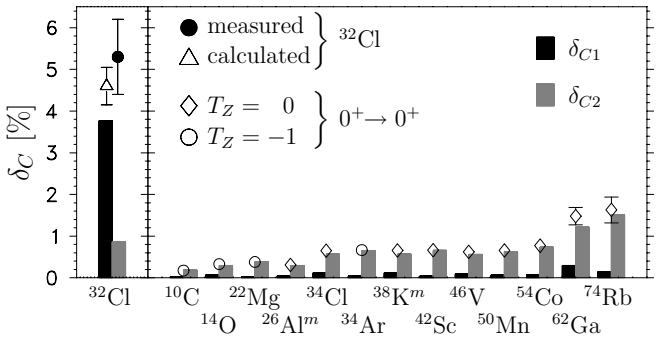

FIG. 2. Our determination of the isospin-breaking correction for ^{32}Cl (filled circle), and calculations for ^{32}Cl as well as other superallowed transitions (open points), with the δ_{C1} and δ_{C2} components shown separately. The measurement and prediction for ^{32}Cl, particularly the δ_{C1} component, is significantly larger than in any of the $0^+ \to 0^+$ transitions.

is to introduce Coulomb and other charge-dependent terms into the shell-model Hamiltonian. However, because the Coulomb force is long range, the shell-model space has to be very large indeed to include all the potential states that the Coulomb interaction might connect. Currently this is not a practical proposition. To proceed, Towner and Hardy divide δ_C into two parts: $\delta_C = \delta_{C1} + \delta_{C2}$, where δ_{C1} arises from configuration mixing between states of the same spin in a shell-model calculation using a restricted basis (in this case the full s,d shell), while δ_{C2} separately encompasses mixing beyond this model space.

Starting with δ_{C1}, we perform a shell-model calculation in the truncated $0\hbar\omega$ model space of the s,d-shell orbitals. Charge-dependent terms are added to the charge-independent Hamiltonians of `USD`, `USDA` and `USDB`. The strengths of these charge-dependent terms are adjusted to reproduce the $b = -5.4872(35)$ MeV and $c = 0.1953(37)$ MeV [25] coefficients of the isobaric multiplet mass equation as applied to the $1^+, T = 1$ triplet of states in $A = 32$, the states involved in the β-transition under study. As already mentioned, the bulk of the isospin mixing in the IAS occurs with the neighboring $1^+, T = 0$ state. In the limit of two-state mixing, perturbation theory implies that $\delta_{C1} \propto 1/(\Delta E)^2$, where ΔE is the energy separation of the analog and non-analog 1^+ states. Experimentally, it is known to be 188.2 ± 1.2 keV [7, 8] (compared to the much larger $2-4$ MeV of most $0^+ \to 0^+$ transitions [3]). The shell model calculates this separation to be 184 keV with `USD`, 248 keV with `USDA` and 387 keV with `USDB` interactions. We avoid the large uncertainties this would impose on our calculation by following the Towner-Hardy recommendation [3] of scaling the calculated δ_{C1} value by a factor of $(\Delta E)^2_{\rm theo}/(\Delta E)^2_{\rm exp}$, the ratio of the square of the energy separation of the 1^+ states in the model calculation to that known experimentally. Following this procedure,

the δ_{C1} values obtained in the three shell-model calculations are reasonably consistent: $\delta_{C1} = 3.73\%$ for USD, 3.32% for USDA, and 4.19% for USDB. We average these three results and assign an uncertainty equal to half the spread between them to arrive at $\delta_{C1} = 3.75(45)\%$. As Fig. 2 shows, this is over an order of magnitude larger than δ_{C1} calculated for any of the thirteen $0^+ \rightarrow 0^+$ transitions used to determine V_{ud}.

For the calculation of δ_{C2} we consider mixing with states outside the $0\hbar\omega$ shell-model space. The principal mixing is with states that have one more radial node. Such mixing effectively changes the radial function of the proton involved in the β decay relative to that of the neutron. The practical calculation, therefore, involves computing radial overlap integrals with modeled proton and neutron radial functions. Details of how this is done are given in Ref. [3]. The radial functions are taken to be eigenfunctions of a Saxon-Woods potential whose strength is adjusted so that the asymptotic form of the radial function has the correct dependence on the separation energy. The initial and final A-body states are expanded in a complete set of $(A-1)$-parent states. The separation energies are the energy differences between the A-body state and the $(A-1)$-body parent states. A shell-model calculation is required to give the spectrum of parent states and the spectroscopic amplitudes of the expansion. For the three USD interactions, we compute $\delta_{C2} = 0.827\%$ for USD and 0.865% for both USDA and USDB. Our adopted value is $\delta_{C2} = 0.85(3)\%$. The uncertainty, calculated in the same manner as described in Ref. [3], represents the range of results for the USD interactions, the different methodologies considered in adjusting the strength of the Saxon-Woods potential, and the uncertainty in the Saxon-Woods radius parameter as fitted to the experimental charge radius of ^{32}S.

Combining our adopted shell-model calculations, $\delta_{C1} = 3.75(45)\%$ and $\delta_{C2} = 0.85(3)\%$, we find $\delta_C^{\rm theor} = 4.6(5)\%$, which agrees with the experimentally determined 5.3(9)% of Eq. (4) within stated uncertainties. The agreement between theory and experiment in this case where δ_C is so large represents a very important validation of the theoretical procedures outlined here to calculate the ISB effects in nuclei. In particular, for (shell-model) calculations which separate configuration-mixing and radial-overlap components, this δ_{C1}-dominated result provides an especially sensitive benchmark for the approximations used when calculating configuration-mixing contributions to the total ISB effect in superallowed $0^+ \rightarrow 0^+$ decays.

In conclusion, we have measured relative γ-ray intensities and characterized the superallowed β branch for the decay of ^{32}Cl. The isospin-symmetry-breaking correction, δ_C, of this almost pure Fermi transition is considerably larger than the typical values found in other superallowed decays and can therefore be used as a stringent test of theoretical procedures to calculate these isospin-breaking effects. Agreement with a shell-model calculation validates the approach currently applied [2] to the 13 most precisely measured ft values of $0^+ \rightarrow 0^+$ transitions and adds confidence to the small corrections applied to them for Standard Model tests.

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