Pair Creation Constrains Superluminal Neutrino Propagation

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Pair Creation Constrains Superluminal Neutrino Propagation

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Abstract

The OPERA collaboration claims that muon neutrinos with mean energy of 17.5 GeV travel 730 km from CERN to the Gran Sasso at a speed exceeding that of light by about 7.5 km/s or 25 ppm. However, we show that superluminal neutrinos may lose energy rapidly via the bremsstrahlung of electron-positron pairs ($\nu \to \nu + e^- + e^+$). For the claimed superluminal velocity and at the stated mean energy, we find that most of the neutrinos would have suffered several pair emissions en route, causing the beam to be depleted of higher energy neutrinos. This presents a significant challenge to the superluminal interpretation of the OPERA data. Furthermore, we appeal to Super-Kamiokande and IceCube data to establish strong new limits on the superluminal propagation of high-energy neutrinos.

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I. INTRODUCTION AND CONCLUSIONS

The OPERA collaboration reports evidence of superluminal neutrino propagation[1]. The CNGS beam, consisting of pulses of muon neutrinos with mean energy 17.5 GeV and energy spread extending beyond 50 GeV, travels about 730 km from CERN to the OPERA detector in the Gran Sasso Laboratory. The group reports that the travel time of the ultrarelativistic neutrinos is about 60 ns less than expected. We phrase our discussion in terms of the parameter $\delta \equiv (v_\nu^2 - 1)$ wherein we take the speed of light *in vacua* to be unity. The OPERA claim is $\delta = 5 \times 10^{-5}$, with no indication of a significant dependence on the neutrino energy. Recognizing the potential impact of this result, the collaboration writes that it intends "to continue its studies to investigate possible…systematic effects that could explain the observed anomaly."

The OPERA claim (hereafter, the anomaly) is compatible with earlier studies of high-energy neutrinos such as MINOS[2], which yielded the result $\delta = 10.2 \pm 5.8 \times 10^{-5}$. However, observations of ~ 10 MeV neutrinos from supernova SN1987a provide the constraint[3–5] $\delta < 4 \times 10^{-9}$. Thus, the alleged anomaly must be energy dependent, decreasing rapidly from 10 GeV to 10 MeV. We note in passing that observations of neutrino oscillations allow one to deduce far more severe constraints on neutrino velocities at relevant energies[6, 7]. Lorentz-violating velocity differences as large as 10^{-20} between neutrinos of different species would have been readily detected and are excluded. Thus, the velocity anomaly, if correct, must pertain to the propagation of all three types of neutrino.

Suppose muon neutrinos with energies of order tens of GeV travel at superluminal velocities. As in all cases of superluminal propagation, certain otherwise forbidden processes may be kinematically permitted, even *in vacua*. In particular, we focus on the following analogs to Cherenkov radiation:

$$\nu_\mu \longrightarrow \begin{cases} \nu_\mu + \gamma & (a) \\ \nu_\mu + \nu + \overline{\nu} & (b) \\ \nu_\mu + e^+ + e^- & (c) \end{cases} \tag{1}$$

Superluminality of the neutrino is the relation $v_\nu = dE_\nu/dp_\nu > c_\gamma$ where in our convention $c_\gamma = 1$. Assuming the neutrino velocity to vary only slowly in the energy interval relevant to the OPERA experiment and neglecting the tiny neutrino mass we obtain the dispersion relation for the neutrino: $E = v_\nu p$. We assume *a priori* the usual linear conservation laws of energy and momentum (the generators of time and space translations)[1]. The above processes then cause superluminal neutrinos to lose energy as they propagate and, as we shall see, process (c) places a severe constraint upon superluminal neutrino velocities.

Because neutrinos of comparable energy but different flavor are known to travel at virtually the same velocity, energy-momentum conservation forbids emission of a neutrino-antineutrino pair carrying a significant fraction of the initial muon-neutrino energy. While the energy dependence of the neutrino velocity may allow emission of low energy neutrino-antineutrino pairs, this is not an efficient energy-loss mechanism. Hence we consider process (b) no further.

Process (a) is kinematically allowed for all neutrino energies for which $v_\nu > 1$. However it is induced by a W loop diagram and thus we find its effect on neutrino propagation is smaller by a factor of α/π than that of process (c) whenever the latter is allowed by energy-momentum conservation.

Process (c), which we refer to as pair bremsstrahlung, proceeds through the neutral current weak interaction. It is kinematically allowed when the initial neutrino energy $E_\nu > 2m_e/\sqrt{\delta - \delta_e}$ where $\delta_e \equiv v_e^2 - 1$ and m_e is the electron mass[2]. At energies significantly above those in the OPERA domain δ_e is known[9] to be less than 2×10^{-16}. A constraint directly applicable at OPERA energies, $\delta_e < 10^{-9}$, follows from the absence of spontaneous decay of photons of tens of GeV into e^+e^-, and is sufficient for us henceforth to ignore δ_e. Using the OPERA value for δ we obtain the condition $E_\nu > 2m_e/\sqrt{\delta} \sim 140$ MeV, ensuring that process (c) is allowed throughout the OPERA domain. It is process (c) that allows us to constrain the OPERA anomaly and place a strong limit on neutrino superluminality.

We have computed both Γ, the rate of pair emission by an energetic superluminal neutrino, and dE/dx, the rate at which it loses energy in the high energy limit where the electron

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[1] Other assumptions may be possible. See [8] and references therein.

[2] Note that m_e is the electron mass as usually defined because no large departure from the usual Lorentz invariant electron dispersion relation at or below the energies germane to the OPERA experiment is compatible with experimental data.

and neutrino masses may be neglected[3]:

$$\Gamma = k' \frac{G_F^2}{192\pi^3} E^5 \delta^3 \tag{2}$$

$$\frac{dE}{dx} = -k \frac{G_F^2}{192\pi^3} E^6 \delta^3 \tag{3}$$

where k and k' are numerical constants: $k = 25/448$, $k' = 1/14$. These expressions, aside from the numerical factors, follow from simple arguments. The factors of G_F arise from the low energy form of the weak interactions while those of energy follow from dimensional analysis. The power of δ can be related to the power of energy. The energy and momentum of the superluminal neutrino are related by the dispersion relation $E = v_\nu |\vec{p}|$, so that the quantity $E^2 - \vec{p}^2$ is positive and equal to δE^2. That is, the energy and momentum of the neutrino may be collected into a 4-vector which is time-like relative to the speed of light and has a length-squared of δE^2. Thus we may work in the "rest frame" of the neutrino whose effective "mass" is $\sqrt{\delta} E$. In this frame the powers of δ track the powers of E^2. The relativistic dilation factor needed to boost back to the original frame is the ratio of the original energy to the effective "mass", $\gamma = 1/\sqrt{\delta}$. The usual dilation factors applied to Γ and dE/dx give our results.

Note that the mean fractional energy loss due to a single pair emission is $E^{-1}(dE/dx)/\Gamma = k/k' \approx 0.78$: about three-quarters of the neutrino energy is lost in each emission.

We integrate dE/dx assuming δ not to vary significantly in the relevant energy interval. We find that neutrinos with initial energy E_0, after traveling a distance L, will have energy E as given by:

$$E^{-5} - E_0^{-5} = 5k\delta^3 \frac{G_F^2}{192\pi^3} L \equiv E_T^{-5} \tag{4}$$

The steeply falling (with energy) form of dE/dx means that neutrinos with initial energy greater than E_T rapidly approach a terminal energy, E_T, which is essentially independent of the initial neutrino energy. Using OPERA's baseline of 730 km and its result of $\delta = 5 \times 10^{-5}$, we find a terminal energy of about 12.5 GeV. Few, if any, neutrinos will reach the detector with energies exceeding 12.5 GeV. Thus the CNGS beam would be profoundly depleted and spectrally distorted upon its arrival at the Gran Sasso. From the above expression for Γ we may also establish that *any* superluminal neutrino with the velocity claimed by OPERA of

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[3] Our expressions are leading order in δ. We have also neglected the vector-current coupling of the electron, putting $c_V = 0$ and $c_A = -1/2$.

any specific initial energy much greater than 12.5 GeV has a negligible chance of arriving at the Gran Sasso without having lost most of its energy. OPERA's detection of neutrinos with energies exceeding 12.5 GeV is difficult to reconcile with its claimed superluminal neutrino velocity measurement.

Our analysis also yields strong new constraints on superluminal velocities of higher energy neutrinos. Super-Kamiokande has carefully studied atmospheric neutrinos that traverse the earth (upward-going in the detector) over an energy range extending from 1 GeV to 1 TeV[10–12]. Upward directed neutrinos that traverse a distance of 10,000 km would experience a depletion and spectral distortion as we have described above. The observation of such neutrinos with 1 TeV energy allows us to conservatively deduce that $\delta < 1.4 \times 10^{-8}$, similar to but slightly weaker than the lower energy neutrino velocity constraint deduced from SN1987a.

The IceCube collaboration has reported the observation of upward-going showers with reconstructed shower energies above 16 TeV[13]. Using a neutrino energy of 16 TeV and a minimum baseline of 500 km (appropriate for horizontal neutrinos) we obtain a more stringent limit, $\delta < 3.75 \times 10^{-10}$. Finally, IceCube has also reported events with energies in excess of 100 TeV. Observations of neutrinos with this energy and a baseline of at least 500 km imply a limit of $\delta < 1.7 \times 10^{-11}$, superior to the SN1987a constraint by more than two orders of magnitude. A more careful analysis of the path-lengths and energies of the highest energy events from Super-Kamiokande, IceCube and other neutrino telescopes are likely to yield even stronger constraints.

How might nature evade the energy loss mechanism we have described? The conventional assumptions of Lorentz invariance and conservation of energy and momentum would forbid the processes we consider. Any theory that respects both these properties does not undergo the energy loss mechanism we have discussed. Another evasion might be unconventional dispersion relations of neutrinos, electrons and photons such that, in the energy domain of the OPERA experiment, these particles travel with a common velocity. Thus neutrinos would not be superluminal with respect to photons of comparable energies, yet might be superluminal with respect to photons of significantly lower energies. In this case the OPERA neutrinos do not lose energy rapidly, but nonetheless travel at speeds greater than the speed of light at optical wavelengths. Any implementation of this notion must be reconciled with observations of energetic cosmic rays.

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