Cosmic Curvature from de Sitter Equilibrium Cosmology

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Cosmic curvature from de Sitter equilibrium cosmology

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I show that the de Sitter Equilibrium cosmology generically predicts observable levels of curvature in the Universe today. The predicted value of the curvature, Ω_k, depends only on the ratio of the density of non-relativistic matter to cosmological constant density, ρ_m^0/ρ_Λ, and the value of the curvature from the initial bubble that starts the inflation, Ω_k^B. The result is independent of the scale of inflation, the shape of the potential during inflation, and many other details of the cosmology. Future cosmological measurements of ρ_m^0/ρ_Λ and Ω_k will open up a window on the very beginning of our Universe and offer an opportunity to support or falsify the de Sitter Equilibrium cosmology.

The de Sitter Equilibrium ("dSE") cosmology is a framework for cosmology that pictures the Universe eternally fluctuating in an equilibrium state. In this picture phenomena similar to the cosmos we observe around us come about as fluctuations. The dSE framework assumes that the observed cosmic acceleration is driven by a true cosmological constant Λ which causes the Universe to approach a "de Sitter space" at late times when the cosmological constant dominates the cosmic evolution. The de Sitter space is the equilibrium state. It has an entropy given by $S_\Lambda = \pi(cH_\Lambda^{-1}/l_P)^2$ which is known to be maximal [1], a temperature given by $T_\Lambda = k_B\hbar H_\Lambda$ where $H_\Lambda = 8\pi G/3\rho_\Lambda \equiv \Lambda/3$ is the Hubble constant during the Λ dominated phase and l_P is the Planck length. Background on dSE cosmology, including how it evades the notorious "Boltzmann Brain" problem of equilibrium cosmologies may be found in [2, 3].

Cosmic inflation gives an established account of the very early history of the Universe. Inflation assumes that the early Universe was dominated by the potential energy of a scalar field, the "inflaton", which caused a period of accelerated cosmic expansion or "inflation" before decaying into ordinary matter through a process called reheating. A simple account of this inflationary epoch leads to a detailed set of predictions which so far have been born out by observations [4]. However, to understand inflation fully and make the predictions robust one must put cosmic inflation into a larger context that accounts for how inflation starts and assigns relative probabilities to different possible starts to inflation as well as other starts to the observed Universe that may not even involve inflation. dSE cosmology gives one way to do this.

All ideas for complete cosmological frameworks (including dSE and the popular "eternal inflation" picture) involve some ad hoc assumptions about how the underlying fundamental physics actually works [3]. Until we understand which assumptions about the fundamental physics are correct, the best any of these pictures can hope to provide is an opportunity for observational tests of one set of assumptions or another. This paper reports a prediction of the value of the cosmic curvature, Ω_k, from the dSE picture. The predicted value depends only on the ratio of the non-relativistic matter density today ρ_m^0 to ρ_Λ and is proportional to the initial curvature (Ω_k^B) provided by the bubble that started the period of cosmic inflation. The prediction is depicted in Fig. 1.

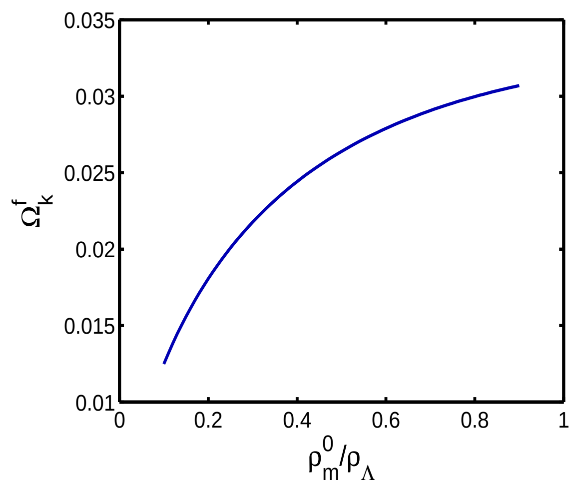

FIG. 1. The predicted value of Ω_k^f vs. ρ_m/ρ_Λ using the fiducial value $\Omega_k^B = 0.5$. Predications from other values of the bubble curvature are given by $\Omega_k = \Omega_k^f \times (\Omega_k^B/0.5)$

The prediction is interesting for a couple of reasons. Firstly, the result is only just consistent with current data [4], and uncertainties in Ω_k, ρ_m and ρ_Λ will reduce substantially in the foreseeable future [5]. Future data showing a positive value for Ω_k would offer strong support for the dSE picture. Data consistent with $\Omega_k = 0$ with very small uncertainties (the cosmic variance limit of $\Delta\Omega_k \approx 0.00001$ may be achievable) would rule out the dSE picture except for extremely small values of Ω_k^B.

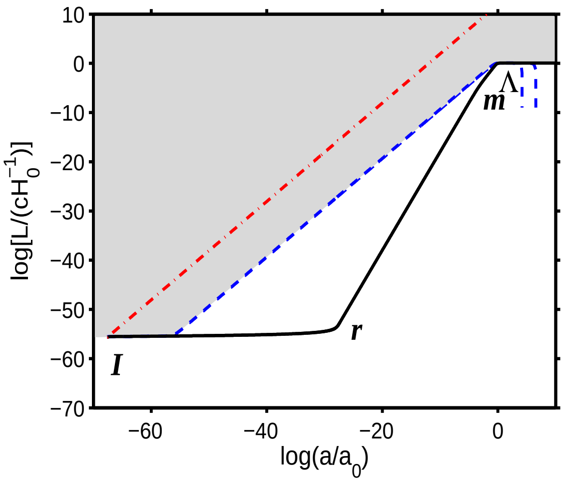

FIG. 2. *Solid*: The Hubble length vs. cosmic scale factor a (scaled by today's values, cH_0^{-1} and a_0 respectively). The letters mark the times when the density of the inflaton (I), relativistic matter (r), non-relativistic matter (m) and (Λ) in turn start to dominate Eqn. 1. *Dashed*: The past horizon of observations deep in the de Sitter era. The shaded region shows events that will never be seen by the observer no matter how late the observation is made. *Dot-Dashed*: Maximum length scale affected by inflation.

Further study of the initial bubbles could even completely rule out the small Ω_k^B dSE case, leading to the possibility of fully falsifying the dSE picture.

Secondly, the dSE prediction is interesting for its lack of dependence on many details of the cosmology. There is no dependence, for example, on the shape of the inflaton potential during inflation, the scale of inflation, the specifics and duration of the reheating and many other factors. As future data further constrain cosmological parameters, only Ω_k^B will be in play, and this quantity would be subject to the sort of pressures just discussed.

I now derive the result shown in Fig. 1. The Friedmann equation

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\rho_I + \rho_r + \rho_m + \rho_k + \rho_\Lambda\right) \tag{1}$$

relates the expansion rate H to the (effective) energy densities of, in order of appearance, the inflaton, relativistic matter, non-relativistic matter, curvature, and cosmological constant in a homogeneous and isotropic Universe. The scale factor a tracks the cosmic expansion. For the solutions I consider a is monotonic in time and I use it as a time variable in what follows. The "Hubble length" ($\equiv cH^{-1}$) is shown by the solid curve in Fig. 2 for the entire history of the Universe in a standard cosmological picture.

The dashed curve in Fig. 2 is the "past horizon"

$$h_p\left(a_1\right) \equiv a_1 \int_{a_1}^{a_\Lambda} \frac{da}{a^2 H} \tag{2}$$

of observations at a time deep in the de Sitter era given by a_Λ. Specifically, $h_p(a_1)$ is the physical distance at time a_1 between an observer at rest with the expansion and a photon that will just reach the observer when $a = a_\Lambda$. Curves are shown for two values of a_Λ (identifiable by the value of a where each curve drops toward zero). The two overlap except right near the respective values of a_Λ. This is due to the event horizon (of size $= cH^{-1}$) that forms in the de Sitter era. From right to left, the two past horizon curves run up against the event horizon and then "exit" out into the pre-de Sitter era together. The past horizon for any event deep in the de Sitter era will will do the same. The shaded region in Fig. 2 represents events that will never be seen by the observer even after waiting an infinitely long time.

Even though H appears in the integral defining the past horizon, the curve takes a simple form $h_p \propto a$ for much of the history of the Universe, independently of the (possibly complicated) behavior of $H(a)$. This is because the evolution of h_p is dominated by the cosmic expansion for $h_p \gg cH^{-1}$ (in this regime the cosmic expansion increases the distance h_p at a rate much faster than c). This simple behavior for $h_p(a)$, regardless of the behavior of $H(a)$ over much of the cosmic history, is central to how the prediction for Ω_k remains independent of many details of the cosmology [6].

In the dSE framework the equilibrium state has finite entropy S_Λ, and it has been argued that finite S_Λ implies the full physical system is finite, describable in a finite Hilbert space with dimension e^{S_Λ} [7]. For such a finite system, any field theoretic description is necessarily approximate and will only have a finite domain of validity (otherwise an infinite Hilbert space would be needed). Limitations on the validity of field theory will necessarily limit scalar field inflation. (When allowed an unlimited domain of inflaton validity, inflation typically leads to "eternal inflation" [8] which lasts forever, creates infinite entropy and volume, and incurs problematic measure issues as a result). The dSE picture reconciles inflation with the finite entropy by only allowing an amount of inflation sufficient to fill the horizon of the observer. This bound prevents the cosmology from producing more entropy than the maximum value S_Λ. Also, the dSE bound generically keeps inflation far from the "self reproduction" regime that leads to eternal inflation.

The dSE bound originates with the finiteness suggested by the de Sitter horizon from a truly constant Λ and in that sense is "holographic" [3]. Holographic bounds which do not incorporate Λ from today's acceleration (critical to my argument of finiteness) give much less restrictive results [9]. Interestingly, when re-expressed as a constraint on inflationary e-foldings the dSE bound looks similar to the bound found in [10] which does include Λ (although the methods in [10] appear different) [11].

By staying strictly finite, the dSE picture does not have the measure problems of the infinite inflationary

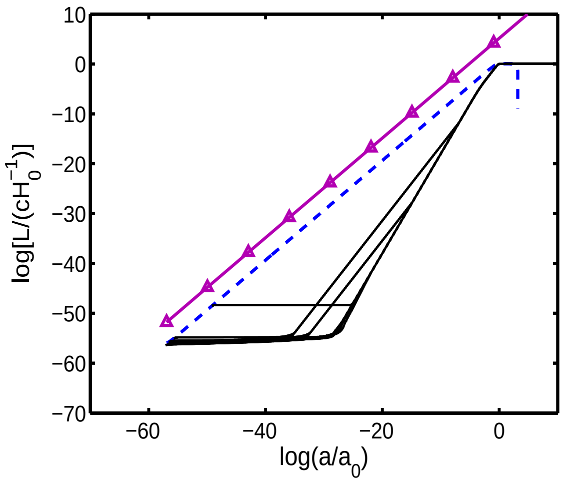

FIG. 3. *Solid*: The Hubble length for many different inflation models, all saturating the dSE bound by starting on the past horizon curve (*dashed*). The 47 different inflation models (not resolved on this plot) represent a wide range of inflation scales, inflaton potentials and reheating rates (but use the same values of ρ_m^0 and ρ_Λ). Also shown is the curvature length given by cH_k^{-1} (*triangles*). The fact that the past horizon (which defines the dSE bound) tracks cH_k^{-1} in such a simple manner over most of the history of the Universe leads to the simple predictions for Ω_k, independently of many details of the cosmology.

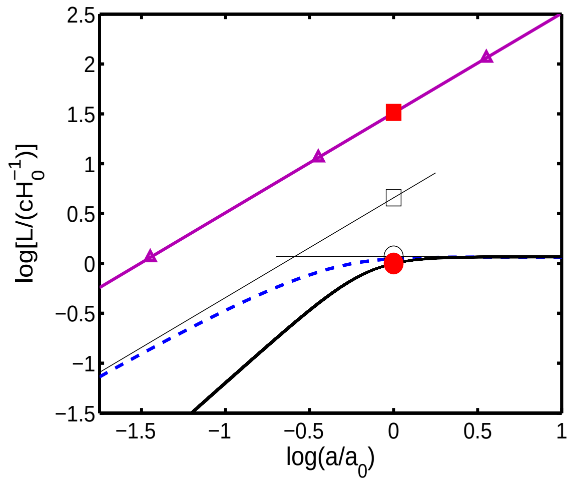

FIG. 4. This close-up of Fig 3 in the vicinity of $a = a_0$ shows the same curves, with additional curves (*thin solid lines*) showing the two asymptotes of the past horizon curve (*dashed*). Markers show quantities evaluated at $a = a_0$ relevant to calculating $\Omega_k \equiv (H_k/H)^2$ as discussed in the text. The value of cH_k^{-1} (*filled square*) is related to the *open square* by Ω_k^B while H^{-1} (*filled circle*) is related to the horizontal asymptote (partly hidden *open circle* at cH_Λ^{-1}) by Eqn. 1. The shape of the past horizon curve (Eqn. 2) quantitatively links the two asymptotes, giving Eqn. 3

scenarios. According to the dSE bound the early part of inflation adjacent to the shaded region of Fig. 2 is excluded by the breakdown of the field theory description. The earliest that inflation, and thus the $cH^{-1}(a)$ curve, is allowed to start is right on the past horizon (dashed) curve. Since other factors exponentially favor inflation starting as early as possible [3] I take the dSE bound to be saturated in what follows: I start all inflation scenarios "on the past horizon", giving $c(H^i)^{-1} = h_p(a^i)$ where i superscripts designate the start of inflation [12].

Figure 3 is similar to Fig. 2 except that a multitude of different inflation scenarios are shown on the same plot. There are actually 47 different solid curves (unresolved on the plot, and discussed in detail in [13]) which correspond to changing the inflation potential, the scale of inflation, and the rate of reheating (in the slowest cases, the reheating only completes at around $T = 10^{10}K$, just in time for Big Bang Nucleosynthesis). Each scenario starts right on the past horizon (dashed line) thus saturating the past horizon bound.

Figure 3 also shows the curvature radius cH_k^{-1} ($H_k^2 \equiv 8\pi G/3\rho_k$), displaying information about ρ_k on the plot and helping to illustrate how the simple dSE prediction comes about. The initial value H_k^i is related to the bubble curvature and the initial Hubble parameter H^i by $\Omega_k^B \equiv (H_k^i/H^i)^2$. Because by definition $\rho_k \propto 1/a^2$, $H_k^{-1} \propto a$, so cH_k^{-1} runs parallel to the past horizon in Fig. 3 for most of the history of the Universe. This parallel behavior is crucial to my result. According to the dSE picture, the curve cH^{-1} must start on the past horizon curve. The cH_k^{-1} and h_p curves evolve linearly together until near the current epoch (shown in Fig. 4), allowing a simple relationship to be established between Ω_k and Ω_k^B.

Figure 4 illustrates how the crucial ingredients needed to compute $\Omega_k \equiv (H_k/H)^2$ are all contained in the curves and asymptotic behaviors discussed above. To the extent that we know ρ_m^0, ρ_k and ρ_Λ we know the shape of $H(a)$ around today, since ρ_m and ρ_Λ (and to a much lesser extent ρ_k) completely dominant Eqn. 1 during the current era [14]. The quantity $H(a)$ appears in Ω_k as well the expression for $h_p(a)$ (Eqn. 2). Since $h_p(a)$ only deviates from its asymptotic values around the current era, only the ρ's listed here are needed to determine this curve as well. The quantitative expressions for these various ingredients (all given above) can be combined to produce the main result:

$$\Omega_k = \frac{1}{g^2} \frac{\Omega_k^B}{\left(\frac{\rho_m^0}{\rho_\Lambda} + \frac{\rho_k^0}{\rho_\Lambda} + 1\right)} \tag{3}$$

where

$$g\left(\frac{\rho_m^0}{\rho_\Lambda}, \frac{\rho_k^0}{\rho_\Lambda}\right) \equiv \int_0^\infty \frac{dx}{x^2\sqrt{x^{-3}\frac{\rho_m^0}{\rho_\Lambda} + x^{-2}\frac{\rho_k^0}{\rho_\Lambda} + 1}} \tag{4}$$

Due to the appearance of ρ_k^0 on the right hand side, Eqn. 3 give an implicit equation for $\Omega_k\left(\rho_m^0/\rho_\Lambda, \Omega_k^B\right)$ which can easily be solved numerically to give Fig. 1.

Attempts such as dSE to construct a complete theoretical framework for cosmology are in a primitive state. There are a number of ad hoc assumptions that go into the dSE framework (spelled out in [3]). The prediction I report here should be understood in that context. Probably the most popular cosmological framework is eternal inflation. That picture has its own particular assumptions, including the validity of the semiclassical inflaton field theory coupled to Einstein gravity over an infinite time and infinite volume. These infinities are critical to the mechanisms believed to cause eternal inflation to dominate the cosmos and any breakdown of these assumptions (such as replacing either of these infinities with arbitrarily large but finite values) would undermine much of the current thinking on this subject. The measure problem of eternal inflation that has so far undermined the ability of eternal inflation to actually make predictions is related to these infinities, but many are hopeful that this problem will eventually find a resolution without removing the infinities that are considered critical to the overall picture [8, 15]).

The dSE framework is a finite alternative to eternal inflation. The finiteness has its own intrinsic appeal (for example, dSE replaces assumptions about initial conditions with an equilibrium state for the Universe), and the finiteness prevents measures from being a problem. The dSE picture is based on the idea that physics operates in such a way that the physical world, at its most fundamental, is described by a finite Hilbert space of dimension e^{S_Λ}. One then has to view any field theoretic degrees of freedom such as the inflaton or those of Einstein gravity as approximate, since it would take an infinite Hilbert space to describe them fully. The dSE framework makes assumptions about when the field theoretic description is a good one and when and how it breaks down. These assumptions are chosen to give a workable cosmology. One can think of the dSE cosmology as an attempt to construct a realistic finite cosmology by exploiting uncertainties about the underlying fundamental physics.

The existence of such great uncertainties may not be satisfying, but it is the state of the art. Under these conditions one can hope that by demanding a realistic cosmology insights might be gained into the nature of the underlying physics. This project was conceived in this spirit and it is in this context that I find the result very interesting. Unlike other models that give nonzero Ω_k [16]), this result is independent of the shape and scale of the inflaton potential during inflation, the nature of reheating and many other details.

If future data reveal positive values of Ω_k close to the current bounds, that could be seen as support for the dSE picture. Such results could be interpreted as constraining the value of Ω_k^B, giving a direct window on the tunneling event that created the Universe we observe. Further work is needed to understand how low a value of Ω_k^B can be tolerated in this picture, but it seems unlikely that values much smaller than the current bound will make sense. If this claim is born out, the result presented here offers an opportunity to falsify the dSE picture.

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