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# Single Phase Slip Limited Switching Current in 1-Dimensional Superconducting Al Nanowires

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An Aluminum nanowire switches from superconducting to normal as the current is increased in an upswing. The switching current ( $I_s$ ) averaged over upsweeps approximately follows the depairing critical current ( $I_c$ ) but falls below it. Fluctuations in  $I_s$  exhibit three distinct regions of behaviors and are non-monotonic in temperature: saturation well below the critical temperature  $T_c$ , an increase as  $T^{2/3}$  at intermediate temperatures, and a rapid decrease close to  $T_c$ . Heat dissipation analysis indicates that a single phase slip is able to trigger switching at low and intermediate temperatures, whereby the  $T^{2/3}$  dependence arises from the thermal activation of a phase slip, while saturation at low temperatures provides striking evidence that the phase slips by macroscopic quantum tunneling.

One of the fundamental questions in one-dimensional (1D) superconductivity is the nature of the current-induced transition from the superconducting to the normal state. Ideally, the maximum current (called the critical current  $I_c$ ) is set by the depairing mechanism, where the Cooper pairs are destroyed by the electron velocity. Experimentally, however, the depairing limit is difficult to achieve. The maximum current is often limited by either the self-generated field in 3D samples or the motion of magnetic flux in 2D thin film (2D) [1]. It was believed depairing  $I_c$  may be achievable in a narrow superconducting wire. Nevertheless, fluctuation effects, specifically the spatio-temporal fluctuations of the order parameter known as phase slips (explained below), can induce premature switching [2, 3]. A recent study reported phase-slip-induced switching [4]. Unexpectedly, it was found that the switching current ( $I_s$ ) fluctuations increased *monotonically* with decreasing temperature. To date, clear evidence is lacking as to whether a single phase slip is capable of inducing switching. Establishing the relationship between individual phase slips and switching provides a tool to study the phase slip, to help establish whether they are caused by thermal fluctuations or by macroscopic quantum tunneling [5–12]. Such studies not only elucidate the fundamental physics, e.g. a superconductor-insulator transition caused by QPS [9, 13, 14], but also provide the basis for applications from a new current standard to quantum qubits [15, 16].

The phase slip is a topological event, during which the superconducting order parameter phase between two adjacent regions of the superconductor changes by  $2\pi$  over a spatial distance of the order of the coherence length. To understand the destruction of supercurrent in a nanowire, one may gain valuable insight from phase slips in a Josephson junction [1]. In both systems, the motion of the phase is described by a tilted washboard potential (Fig. 1a). Josephson junctions are classified within a Resistively and Capacitively Shunted Junction (RCSJ) model as either under- or over-damped, depending on whether the quality factor,  $Q = \sqrt{2eI_c C / \hbar R}$ , is greater or less than 1. An under-damped junction is

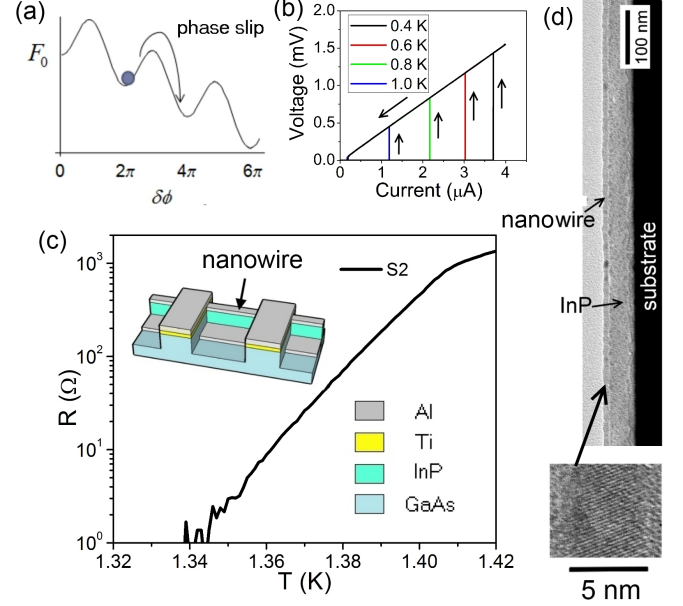


FIG. 1. (a) Schematic of a washboard potential. (b) Hysteretic IV curves and (c) resistive transition versus temperature for nanowire S2. (d) Transmission Electron Microscope images of a typical nanowire. The long and uniform nanowire sits on an InP ridge.

readily driven normal by a single phase slip event; the phase keeps running downhill subsequent to overcoming the free-energy barrier, as damping is insufficient to re-trap into a local minimum. Thus, the junction exhibits zero resistance up to  $I_s$  and the voltage is hysteretic in an current-voltage (IV) measurement. In an over-damped junction, the phase moves diffusively between minima; the IV is nonlinear and hysteresis is often not present. The different situations thus lead to differing characteristics.

Due to an extremely small capacitance, a nanowire is believed to be heavily over-damped ( $Q \ll 1$ ). Recently, experimental evidence has accumulated indicating that heating can also lead to hysteretic behavior in overdamped Superconducting-Normal-Superconducting

TABLE I. Specifics of the samples.

Sample	S1	S2	S3	S4	S5
Length ( $\mu\text{m}$ )	1.5	10	10	10	3
Width (nm)	10.0	9.3	8.4	7.0	5.4
R/L <sup>a</sup> (K $\Omega/\mu\text{m}$ )	0.33	0.38	0.50	0.82	1.17
$I_m$ <sup>b</sup> ( $\mu\text{A}$ )	4.7	4.1	3.3	2.3	1.4
$T_s$ <sup>c</sup> (K)	1.36	1.33	1.33	1.25	1.03

<sup>a</sup> Resistance per unit length of the nanowires in normal state.

<sup>b</sup> Maximum switching current has measured

<sup>c</sup> Transition temperature, below which the nanowires show zero measured resistance

bridges[17]. Here, we report on measuring the fluctuations of  $I_s$  in Al superconducting nanowires. In stark contrast to the previously reported monotonic increase with decreasing temperature [4], the fluctuations in our Al nanowires is non-monotonic with three distinct regions of behaviors. Below  $\sim 0.3 T_c$ , a clear saturation of the fluctuations is observed indicating the switching to be caused by a quantum-phase-slip (QPS). At intermediate temperatures  $\sim 0.3 - 0.6 T_c$ , the fluctuations increase as  $T^{2/3}$ , signifying that switching is caused by a thermal phase slip (TAPS). At high temperatures above  $\sim 0.6 T_c$ , the rapid decrease of fluctuations points to multiple TAPSs triggering the switching. Although in appearance, this behavior is reminiscent of an under-damped Josephson junction, for  $T < 0.6 T_c$ , quantitative estimation demonstrates that heat generated by a single phase slip likely causes a thermal runaway, triggering switching [3]. Because a single QPS or TAPS is sufficient to trigger switching close to  $I_s$ , the resistance of the SC state at current levels below  $I_s$  can remain zero much of the time, with occasional jumps as a rare single phase slip event occurs.

Five nanowires were studied (TABLE I). Each end of a nanowire is connected to a large 2D superconducting pad rather than to a normal metal pad [12]. S3 was obtained by further oxidizing the surface of S2. The fabrication was described previously [19]. The superconducting coherence length at base temperature  $\xi_0 \sim 100\text{nm}$ . The length of the wires range from  $15\xi_0$  to  $100\xi_0$ , while the width is roughly  $1/10\xi_0$ . The resistivity ( $4.5 \mu\Omega\text{cm}$ ; same as co-evaporated films), along with the inverse proportionality to 20% accuracy between normal resistance per-unit-length and  $I_s$  at base temperature, indicates that there are no resistive tunneling barriers. These nanowires are in the fully metallic limit, ( $k_F l \sim 60 \gg 1$ , where  $k_F$  is the Fermi wavenumber, and  $l$  the electron mean-free-path), in contrast to those studied by Sahu *et al.* [4] which appear to be grainy with  $k_F l$  is much closer to 1, and for which Coulomb effects may be important.

S1 and S2 were measured in a  $^3\text{He}$  system and S3, S4 and S5 in a dilution refrigerator. To fully remove interference from unwanted noise, each electrical line is

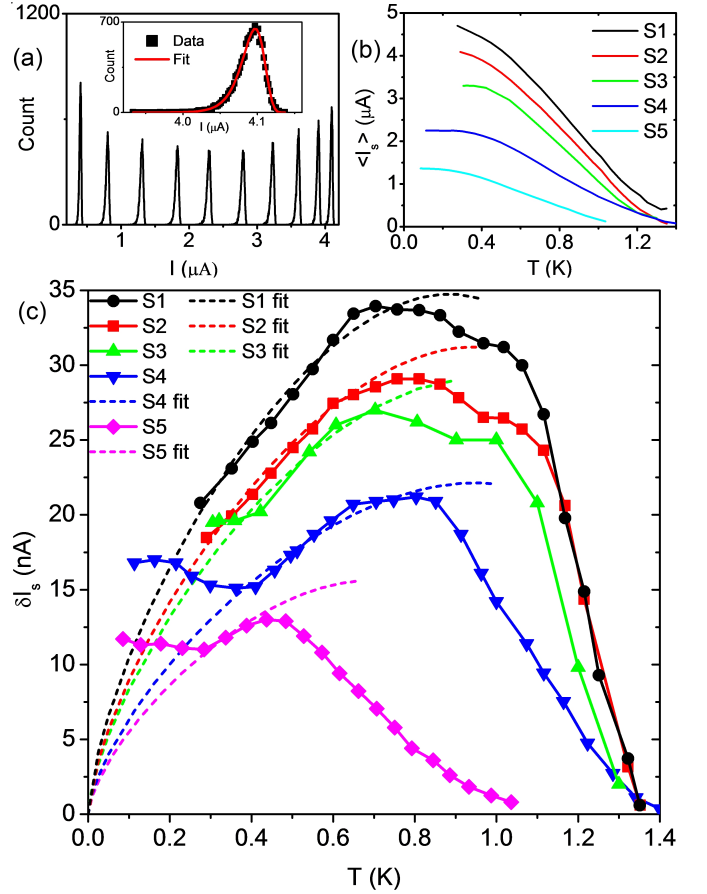


FIG. 2. (a)  $I_s$  distribution for S2 at different temperatures—right to left: 0.3 K to 1.2 K in 0.1 K increments. Inset shows the 0.3K distribution, fitted by the Gumbel distribution [18]. (b)  $\langle I_s \rangle$  versus temperature. (c) Symbols— $\delta I_s$  versus temperature. Dashed lines—fits in the single TAPS regime using Eq. 2. An additional scale factor of 1.25, 1.11, 1.14, 0.98 and 1.0, for S1 - S5, respectively (average  $1.1 \pm 0.1$ ), is multiplied to match the data. Alternatively, a  $\sim 6\%$  adjustment in the exponent fits the data without the scale factor.

equipped with RL filters (1 MHz cutoff) at room temperature, Thermocoax cables (1 GHz cutoff) extending to the mixing chamber of the dilution refrigerator, and RC filters (34 KHz cutoff) at the mixing chamber. For the current sweep a sawtooth waveform was used at a repetition of 10 Hz. The upsweep ramp rate was  $50 \mu\text{A/s}$  for S1, S2 and S3 and  $25 \mu\text{A/s}$  for S4 and S5. Decreasing the rate by a factor of 10 yielded nearly identical results. Immediately after a voltage jump, the current was turned off, reducing the resistive heating time in the normal state to less than  $100 \mu\text{s}$ , and ensuring adequate time to re-thermalize the sample ( $\sim 10^{-7}\text{s}$ ) before the next cycle.

At vanishing current, the nanowires become superconducting below the switching temperature  $T_s(I \rightarrow 0)$ . In Fig. 1(c), the resistive transition is broadened due to TAPSs. To measure  $I_s$  fluctuations, we performed  $\sim$

10,000 I-V sweeps at each temperature, recorded the up-sweep  $I_s$  and plotted the histogram (Fig. 2(a)). The probability density function  $P(I)$  obeys the expression (suppressing the subscript in  $I_s$ ):

$$P(I) = \Gamma(I) (dI/dt)^{-1} \left( 1 - \int_0^I P(u) du \right) \quad (1)$$

where  $dI/dt$  is the ramping rate, and  $\Gamma(I)$  is the switching rate at current  $I$  [20]. If a single phase slip triggers switching, the switching and the phase slip rates are identical, enabling to extract the phase slip rate from the distribution. Using the single TAPS rate  $\Gamma(I) \sim \exp(F(T, I)/k_B T)$ , where  $F(T, I)$  is free energy barrier, and linearizing the current dependence of  $F(T, I)$ , the solution for  $P(I)$  is in the form of a Gumbel distribution [18]. An example of the fitting to this functional form to data is shown in the inset to Fig. 2(a). For each distribution, we deduce the mean value  $\langle I_s \rangle$  and the standard deviation  $\delta I_s$ , as shown in Figs. 2(b) and (c), respectively.

In the single TAPS regime, the fluctuation in  $I_s$  is approximately proportional to

$$\delta I_s \sim (k_B T / \phi_0)^{2/3} I_c(T)^{1/3} \quad (2)$$

where  $I_c(T)$  is the depairing  $I_c$  at temperature  $T$ . The  $T^{2/3}$  is from the exponent in the current dependence of  $F(T, I)$ ,  $F(T, I) = F(T)(1 - I/I_c)^{3/2}$  [21]. In all samples,  $\delta I_s$  in the intermediate temperature range ( $\sim 0.3T_c$  -  $0.6T_c$ ) can be fitted by Eq. 2 very well, shown in Fig. 2(c). The good agreement between data and theoretical fittings indicates that switching is induced by a single TAPS.

When the nanowire has narrower width, it becomes more probable for the phase to undergo a macroscopic quantum tunneling process through the barrier. The QPS rate is proportional to  $\exp(-\alpha F(T, I)/\Delta)$ , where  $\alpha$  is constant of order unity, and different possibilities for  $\Delta$  have been proposed. These include  $\hbar/\tau_{GL}$ , with  $\tau_{GL}$  the Ginzburg-Landau time, and the superconducting gap [4, 8, 22]. In S4 and S5, we find a slight increase of the fluctuations with decreasing temperature in the QPS regime (below  $\sim 0.3T_c$ ), consistent with  $\Delta$  scaling as the superconducting gap.

To check the approximate linearized expressions for  $\delta I_s$  and  $\langle I_s \rangle$  in the single TAPS regime, a full numerical simulation is performed to solve Eq. 1, as shown in Fig. 3. The TAPS rate is given by:

$$\Gamma_{TAPS}(T, I) = \Omega_{TAPS} \exp \left( \frac{-F(T, I)}{k_B T} \right) \quad (3)$$

where  $\Omega_{TAPS} = L/\xi_T \sqrt{F(T, I)/k_B T}/\tau_{GL}$  is the attempt frequency,  $\tau_{GL} = \pi\hbar/(8k(T_c - T))$ ,  $\xi_T$  is the superconducting coherence length at temperature  $T$ , and

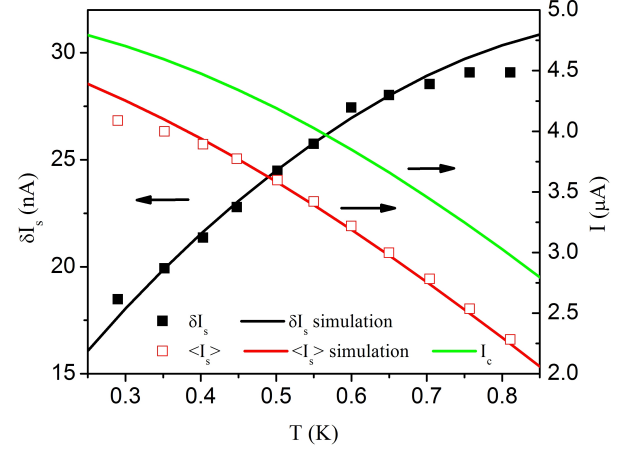


FIG. 3.  $\delta I_s$  and  $\langle I_s \rangle$  fitted by TAPS model: Squares—experimental data; solid lines—simulation.  $I_c(0)=5 \mu\text{A}$  and  $T_c=1.5 \text{ K}$  are two fitting parameters. A constant scaling factor  $\sim 1.15$  is multiplied to the calculated  $\delta I_s$  to match the data.

$F(T, I) = \sqrt{6}\hbar/(2e)I_c(T)(1 - I/I_c(T))^{3/2}$  is the free energy barrier [4]. The zero current free energy barrier,  $F(T) = \sqrt{6}\hbar I_c(T)/2e$ , bears similarity to the Josephson energy  $E_J = \hbar I_c/2e$  [2]. To extend to the entire temperature range, a phenomenological  $I_c(T) = I_c(0)(1 - (T/T_c)^2)^{3/2}$  was employed [23]. Good agreement is achieved up to 0.8 K ( $\sim 0.6T_c$ ) in S2 as shown in Fig. 3. Other samples exhibit a similar agreement.

Above 0.8 K,  $\delta I_s$  falls below the simulated value, at first decreasing gradually, then rapidly. This behavior is associated with the need for more-than-one phase slips to heat up the wire as the current drops [3]. A similar decrease is familiar in Josephson junctions within the phase diffusion regime, where multiple phase slips are required to induce switching [24, 25].

To achieve a consistent picture, it must be demonstrated that heat generated by a single phase slip is sufficient to raise the local temperature and trigger switching. A single phase slip deposits an energy  $\phi_0 I$  in a time  $\phi_0 I/(I^2 R_{core}) \sim 50 \text{ ps}$ , where  $\phi_0 = h/2e$  is the flux quantum, and  $R_{core}$  is the normal state resistance of the phase slip core. Due to the low  $R_{core}$  ( $\sim 100 \Omega$ ), this energy is deposited predominantly in the normal core rather than removed via plasmon emission [14]. Heat loss through the InP ridge is also ineffective. Immediately after the phase slip, the hot normal electrons are decoupled from the superconducting electrons for a duration the charge-imbalance time  $\tau_{imb}$ . Within this time, heat diffuses out primarily within the normal electron component to a charge imbalance distance,  $\Lambda_{imb} \sim \sqrt{D\tau_{imb}}$ , where  $D$  is diffusion coefficient, before it can be transferred to the superconducting electrons to raise their effective temperature to  $T_f$ . [3] Following transfer, this entire region of size  $2 \times \Lambda_{imb}$  either becomes normal or returns to su-

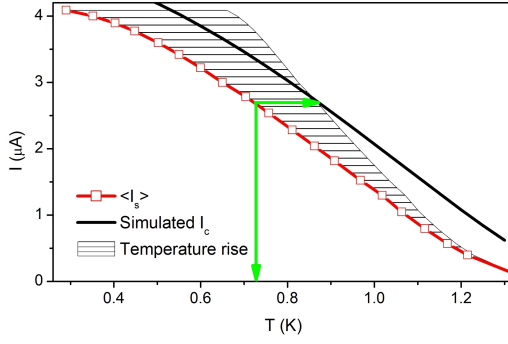


FIG. 4. Temperature boundary between single- and multiple-phase-slip regimes. The red curve is measured  $\langle I_s \rangle$ . The thick black curve is simulated depairing  $I_c$ . Following the black shaded region toward the right, the thin black curve shows the electronic temperature rises due to a single phase slip event at current  $I$  according to Eq. 4. Where the thick black curve exits the shaded area marks a change from single- to multiple-phase-slip regimes. The corresponding ambient temperature boundary 0.7 K can be found by moving horizontally to the left to  $\langle I_s \rangle$  (shown by green arrows).

perconducting depending on whether  $I_c$  at the elevated electronic  $T_f$  is exceeded or not [28]. If exceeded, this region becomes normal; its resistance contributes further to heating, causing a thermal runaway.  $T_f$  can be estimated as

$$T_f = \sqrt{T_0^2 + \phi_0 I / (\gamma A \Lambda_{imb})} \quad (4)$$

where  $\gamma = C_v/T = 135 \text{ J}/(\text{m}^3 \text{ K}^2)$ ,  $C_v$  is the specific heat of normal Al [26],  $A$  is cross section area, and  $T_0$  is the ambient temperature. Setting  $\Lambda \sim 0.8 \mu\text{m}$ , Fig. 4 shows that a boundary between the single TAPS regime and the multiple TAPS regime occurs around 0.7 K ( $\sim 0.3T_c$ ), consistent with our experimental result. This value for  $\Lambda_{imb}$  is close to the findings in recent experiments on Al wires of sub- $\mu\text{m}$  diameter [27], and is expected to be temperature independent [29]. The same analysis yields a boundary of 0.7 K, 0.6 K, 0.6 K and 0.45 K for S1, S3, S4, and S5, respectively, consistent with the behavior in Fig. 2(c).

In summary, we demonstrate that 1D Al superconducting nanowires can be switched into the normal state by a single phase slip, over a sizable temperature range. At low  $T$ , QPS-induced switching was found in the narrower wires. In the single TAPS regime,  $I_s$  fluctuations are proportional to  $T^{2/3}$ . The fluctuations decrease at higher temperature, where multiple phase slips are needed to trigger switching. Heating by phase slips appears to play a major role in the switching process. The behavior found is likely relevant to nanowires of different materials, due to the commonality of restricted geometry.

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