Proposal for Entangling Remote Micromechanical Oscillators via Optical Measurements

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Proposal for entangling remote micromechanical oscillators via optical measurements

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We propose an experiment to create and verify entanglement between remote mechanical objects by use of an optomechanical interferometer. Two optical cavities, each coupled to a separate mechanical oscillator, are coherently driven such that the oscillators are laser cooled to the quantum regime. The entanglement is induced by optical measurement and comes about by combining the output from the two cavities to erase which-path information. It can be verified through measurements of degrees of second-order coherence of the optical output field. The experiment is feasible in the regime of weak optomechanical coupling. Realistic parameters for the membrane-in-the-middle geometry suggest entangled state lifetimes on the order of milliseconds.

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Despite the tremendeous success of quantum mechanics at explaining the behaviour of the microscopic world, many people have found it uncomfortable that macroscopic objects should also obey the laws of quantum mechanics. This goes back to the founders of quantum mechanics, and is the origin of the famous Schrödinger cat thought experiment [1]. However, if all time evolution is unitary according to the Schrödinger equation, as in Everett's relative-state interpretation [2], there is nothing that forbids counterintuitive phenomena such as superpositions of macroscopically distinct states [3]. It should therefore be possible to observe quantum effects on arbitrarily large scales if the system is adequately shielded from environmentally induced decoherence. Conversely, experiments might reveal the existence of unknown sources of decoherence [4, 5], which limit quantum mechanics to small scales by causing an objective wavefunction collapse and creating a quantum-classical boundary.

Cavity optomechanics, where mechanical oscillators are coupled to light or microwaves [6–8], is a promising field for experimental tests of quantum mechanics at large scales. The motion of a micromechanical object can be cooled via radiation pressure forces [9–16]. One group has already reported to have reached the quantum regime by resolved sideband cooling [17], and others are expected to soon follow. An interesting future direction for experiments would be to detect quantum entanglement [18] between a micromechanical oscillator and another system, such as an optical cavity field mode [19] or a second mechanical oscillator [20–23].

In this Letter, we propose a new and promising route to create and verify entanglement between two remote mechanical oscillators. Our idea is based on an optomechanical interferometer where two optical cavities, each coupled to a separate mechanical oscillator, are coherently driven in parallel. The drive frequency is chosen such that the oscillators are cooled close to the motional ground state. In short, entanglement is achieved by combining the optical output from the two cavities to erase which-path information. The mechanical objects in our setup have no direct interaction, but are projected onto an entangled state through optical measurements, in contrast to various earlier proposals [20–22]. A scheme related to ours involving laser pulses was discussed in Ref. [23]. Similar approaches have been successfully applied to entangle remote atomic ensembles [24, 25] as well as individual trapped ions [26]. We also present a new and feasible way of detecting the entanglement. The experiment we propose should be realizable with present day technology in the regime of weak optomechanical coupling. For the membrane-in-the-middle geometry [16], we estimate that the entangled states can have decoherence times on the order of milliseconds. This could have relevance to quantum information and communication technologies [27, 28].

Consider the standard optomechanical system where the position of a mechanical oscillator modulates the frequency of an optical cavity mode. We discuss a single cavity first and move on to a system with two cavities later. We will have the membrane-in-the-middle setup [16] in mind, but our discussion can apply to many other realizations. The photon and phonon annihilation operators will be denoted by $\hat{a}$ and $\hat{c}$, respectively. The interaction Hamiltonian is $H_{\rm int} = \hbar g \hat{x} \hat{a}^\dagger \hat{a}$ where g is a coupling constant. The mechanical position operator is $\hat{x} = x_{\rm zpf}(\hat{c} + \hat{c}^\dagger)$, with $x_{\rm zpf}$ being the size of the zero point fluctuations. Input-output theory [29, 30] leads to the quantum Langevin equations

$$\begin{aligned} \dot{\hat{a}} &= -\left(\frac{\kappa}{2} + i\omega_{\rm C}\right)\hat{a} - ig\hat{x}\hat{a} + \sum_i \sqrt{\kappa_i}\,\hat{a}_{{\rm in},i} \ , \qquad (1) \\ \dot{\hat{c}} &= -\left(\frac{\gamma}{2} + i\omega_{\rm M}\right)\hat{c} - igx_{\rm zpf}\hat{a}^\dagger\hat{a} + \sqrt{\gamma}\,\hat{c}_{\rm in} \ . \end{aligned}$$

The bare mechanical and optical frequencies are $\omega_{\rm M}$ and $\omega_{\rm C}$. The mechanical oscillator is coupled to a thermal bath, resulting in a nonzero linewidth γ and a fluctuating force determined by the operator $\hat{c}_{\rm in}$. For systems where the mechanical quality factor $\omega_{\rm M}/\gamma$ is high, the Markov approximation gives $[\hat{c}_{\rm in}(t), \hat{c}^\dagger_{\rm in}(t')] = \delta(t-t')$ and $\langle \hat{c}^\dagger_{\rm in}(t)\hat{c}_{\rm in}(t')\rangle = n_{\rm th}\delta(t-t')$. Here, $n_{\rm th} \approx k_{\rm B}T/\hbar\omega_{\rm M}$, where T is the bath temperature. The optical linewidth is $\kappa = \sum_i \kappa_i$, where κ_i is the decay rate through decay channel i. We imagine a two-sided cavity with a left (L) and a right (R) input port, and assume that the cavity is driven from the left. The optical input modes then take the form $\hat{a}_{\rm in,L} = e^{-i\omega_{\rm D}t}\bar{a}_{\rm in} + \xi_{\rm L}(t)$ and $\hat{a}_{\rm in,R} = \xi_{\rm R}(t)$, where $\bar{a}_{\rm in}$ is a constant, $\omega_{\rm D}$ is the drive frequency, and the vacuum noise operators ξ_i obey $[\xi_i(t), \xi^\dagger_j(t')] = \langle \xi_i(t)\xi^\dagger_j(t')\rangle = \delta_{ij}\delta(t-t')$. We define the detuning between the drive and the mean cavity frequency as $\Delta = \omega_{\rm D} - \omega_{\rm C} - g\langle\hat{x}\rangle$.

The cavity mode operator can be written as a sum of a mean and a fluctuating part, $\hat{a}(t) = e^{-i\omega_{\rm D} t}(\bar{a} + \hat{d}(t))$, where $|\bar{a}|^2$ is the mean number of photons in the cavity. Here, we focus on the situation $\langle \hat{d}^\dagger \hat{d} \rangle \ll |\bar{a}|^2$. In that case, Eqs. (1) can be linearized and solved analytically. The effective coupling between the optical and mechanical fluctuations is given by $\alpha = g x_{\rm zpf} \bar{a}$. In the regime of weak coupling $|\alpha| \ll \kappa$ and for negative detuning Δ, the mechanical oscillator is approximately in a thermal state, but with renormalized parameters compared to the case of $g = 0$. The frequency $\tilde{\omega}_{\rm M}$ is shifted from its bare value due to the interaction with the optical field, often referred to as the optical spring effect [31]. The effective linewidth $\tilde{\gamma} = \gamma + \gamma_{\rm opt}$ is now a sum of the bare value and a contribution $\gamma_{\rm opt}$ due to the optomechanical coupling. The latter is positive when the detuning Δ is negative, leading to line broadening. The effective mean phonon number is the weighted sum $n_{\rm M} = (\gamma n_{\rm th} + \gamma_{\rm opt} n_{\rm opt})/\tilde{\gamma}$, where $n_{\rm opt}$ is a measure of the effective temperature of the radiation pressure shot noise [30, 32, 33]. Below, we will set the detuning to $\Delta = -\omega_{\rm M}$, which is optimal for cooling, and assume that $\gamma_{\rm opt} \gg \gamma$ and $n_{\rm opt} = \kappa^2/(4\omega_{\rm M})^2 < 1$. Note however that our results are also of interest when $n_{\rm opt} > 1$ (see Ref. [34]).

The optical output mode on the right-hand side of the cavity is $\hat{b}(t) = \sqrt{\kappa_{\rm R}}\hat{a}(t) - \hat{\xi}_{\rm R}(t)$. The optomechanical interaction leads to sidebands in the output which are displaced from the drive frequency $\omega_{\rm D}$ by the mechanical frequency $\omega_{\rm M}$. The sidebands originate from Raman scattering off the mechanical oscillator, where a photon gains (loses) energy by destroying (creating) one phonon. We will imagine that photons at the drive frequency can be completely filtered away and that the two sidebands can be measured independently. We will refer to the output at frequency $\omega_{\rm D} \pm \omega_{\rm M}$ as the blue (red) sideband, and denote the output mode filtered around this frequency by $\hat{b}_{\rm b(r)}(t)$. See the Supplementary Material [34] for details. The width of the sidebands are given by the mechanical linewidth $\tilde{\gamma}$, so we assume the filter bandwidth λ to obey $\tilde{\gamma} \ll \lambda \ll \omega_{\rm M}$. The ratio between the output fluxes of red and blue photons is $n_{\rm opt}(n_{\rm M}+1)/(n_{\rm opt}+1)n_{\rm M}$.

A filter that removes the carrier photons at $\omega_{\rm D}$ and splits the red and blue photons into different spatial modes might pose a technical challenge. One reason is that the mechanical frequency is typically in the kHz-MHz range. However, there are also experimental setups with $\omega_{\rm M}$ in the GHz range [35–37]. Another reason is that the ratio between the fluxes of blue and carrier photons, given by $4(g x_{\rm zpf}/\kappa)^2 n_{\rm M}$, is very small in the weak coupling limit. An alternative and more feasible way to achieve the filtering is through heterodyne photodetection, where the blue and red sidebands can easily be distinguished in the Fourier domain. This is discussed in detail in the Supplementary Material [34].

We define the degrees of second-order coherence [38]

$$g^{(2)}_{j|i}(\tau) = \frac{\langle \hat{b}_i^\dagger(t)\hat{b}_j^\dagger(t+\tau)\hat{b}_j(t+\tau)\hat{b}_i(t)\rangle}{\langle \hat{b}_i^\dagger(t)\hat{b}_i(t)\rangle\langle \hat{b}_j^\dagger(t)\hat{b}_j(t)\rangle}\,, \qquad (2)$$

where steady state has been assumed, $\tau > 0$, and the indices i and j denote either red (r) or blue (b). We find that $g^{(2)}_{\rm r|r}(\tau) = g^{(2)}_{\rm b|b}(\tau) = 1 + e^{-\tilde{\gamma}\tau}$. This is what one would expect for thermal radiation seen through a Lorentzian filter of width $\tilde{\gamma}$ [38]. The photon statistics of the red and blue sidebands is that of thermal radiation simply because the mechanical oscillator is approximately in a thermal state. More interestingly, the cross-correlators become

$$\begin{aligned} g^{(2)}_{\rm b|r}(\tau) &= 1 + \frac{n_{\rm M}+1}{n_{\rm M}} e^{-\tilde{\gamma}\tau}\,, \\ g^{(2)}_{\rm r|b}(\tau) &= 1 + \frac{n_{\rm M}}{n_{\rm M}+1} e^{-\tilde{\gamma}\tau}\,, \end{aligned} \qquad (3)$$

when restricting the delay time such that $\tau \gg \kappa^{-1},\ \lambda^{-1}$ [34]. The result (3) has a simple physical explanation. If the effective phonon number $n_{\rm M}$ is less than 1, the mechanical oscillator spends most of the time in the ground state $|0\rangle$. It can gain energy and reach the excited state $|1\rangle$ by the creation of a red photon. However, it is bound to return to the ground state quickly, through the creation of a blue photon. This means that conditioned on the detection of a red photon, the probability of detecting a blue photon is high, such that $g^{(2)}_{\rm b|r}(\tau)$ becomes large. The opposite is not the case. For $n_{\rm M} < 1$, once a blue photon is detected, it probably means that the oscillator is now in the ground state $|0\rangle$ and detection of a red photon is not particularly likely. In fact, in the limit $n_{\rm M} \to 0$, having detected a blue photon does not change the probability of detecting a red one, such that $g^{(2)}_{\rm r|b}(\tau) \to 1$. Note that for sufficiently small $n_{\rm M}$, the classical inequality $[g^{(2)}_{\rm b|r}(\tau)]^2 \le g^{(2)}_{\rm r|r}(0) g^{(2)}_{\rm b|b}(0)$ [38] can be violated. This happens when $|\langle \hat{b}_{\rm r}^\dagger(t)\hat{b}_{\rm b}^\dagger(t+\tau)\rangle|^2 > \langle \hat{b}_{\rm r}^\dagger(t)\hat{b}_{\rm r}(t)\rangle\langle \hat{b}_{\rm b}^\dagger(t)\hat{b}_{\rm b}(t)\rangle$ and means that the fields $\hat{b}_{\rm b}$ and $\hat{b}_{\rm r}$ are entangled. This is a necessary requirement for detecting entanglement between mechanical oscillators in the setup that we discuss next.

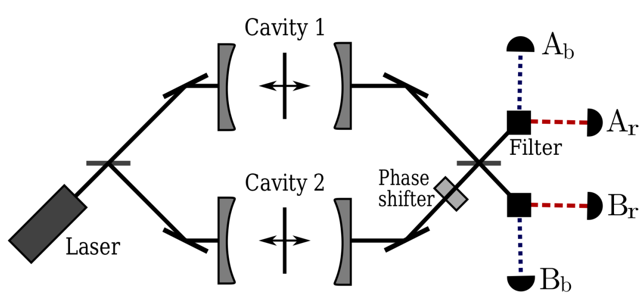

FIG. 1: (Color online) Schematic view of our proposed experimental setup. Combining the output from the two cavities on a beam splitter can create entanglement between the mechanical oscillators. This can be verified by measuring the photon statistics of the red (dashed) and the blue (dotted) sidebands. The sideband filtering can also be achieved through heterodyne photodetection.

We now move on to the main part of this Letter and study the setup presented in Fig. 1. We consider two optical cavities, 1 and 2, each coupled to a mechanical oscillator. For simplicity, the cavities are assumed to be identical, such that

$\omega_{\mathrm{C},1} = \omega_{\mathrm{C},2} \equiv \omega_{\mathrm{C}}$, $\kappa_1 = \kappa_2 \equiv \kappa$, etc. We imagine that $\kappa_{\mathrm{R}} \gg \kappa_{\mathrm{L}}$, such that most photons leave the cavity through the mirror on the right. Furthermore, we assume that the optomechanical coupling constants are equal. We do however allow for the two mechanical oscillator frequencies to differ, defining $\omega_{\mathrm{M},1} = \omega_{\mathrm{M}}$ and $\omega_{\mathrm{M},2} = \omega_{\mathrm{M}} + \delta$, but we restrict the difference to $|\delta| \ll \kappa, \omega_{\mathrm{M}}$. The cavities are driven in parallel by a laser optimally detuned to $\Delta = -\omega_{\mathrm{M}}$, such that cooling to the quantum limit is achieved for both oscillators. The right-hand side output from the two cavities are combined on a 50:50 beam splitter and then filtered into red and blue sidebands, as discussed above. The red photons are detected at either the photomultiplier $\mathrm{A_r}$ or $\mathrm{B_r}$, whereas the blue photons are detected at photomultiplier $\mathrm{A_b}$ or $\mathrm{B_b}$.

Let us first discuss how this setup can lead to entanglement between the two mechanical oscillators. Assume that the two oscillators are identical and in the state $|0,0\rangle$, i.e., zero phonons in both. The detection of a red sideband photon means that one mechanical excitation, a phonon, has been created. However, the information on which cavity the photon came from has been erased. This means that, conditioned on detecting a red photon at either $\mathrm{A_r}$ or $\mathrm{B_r}$, the wavefunction of the oscillators has now collapsed onto the superposition $(|1,0\rangle + e^{i\theta}|0,1\rangle)/\sqrt{2}$. This is an entangled state, even though it contains only one phonon [39, 40]. The phase θ depends on whether the photon was detected at $\mathrm{A_r}$ or $\mathrm{B_r}$, and other factors such as optical path length differences. While this simplified discussion provides insight on how entanglement is created, the thermal baths must of course be taken into account.

It is interesting to examine the degrees of second-order coherence defined in Eq. (2) for this setup. Now we need to specify not only photon "color", but also which detectors we refer to. Taking $g^{(2)}_{\mathrm{A_b}|\mathrm{A_r}}(\tau)$ as an example, it is instructive to express this as

$$g^{(2)}_{\mathrm{A_b}|\mathrm{A_r}}(\tau) = \frac{\langle \hat{b}^\dagger_{\mathrm{A_b}}(t')\hat{b}_{\mathrm{A_b}}(t')\rangle_{\mathrm{A_r}}}{\langle \hat{b}^\dagger_{\mathrm{A_b}}(t')\hat{b}_{\mathrm{A_b}}(t')\rangle} , \tag{4}$$

defining $t' = t + \tau$. Here, the expectation value in the denominator is the photon flux at detector $\mathrm{A_b}$ with respect to the state ρ_{ss}, which is the steady state density matrix in the absence of measurements. In the state ρ_{ss} there is obviously no entanglement between the mechanical oscillators. On the other hand, the expectation value in the numerator is defined by $\langle \hat{O}(t')\rangle_{\mathrm{A_r}} = \mathrm{Tr}(\hat{O}\tilde{\rho}(t'))$, where, formally, $\tilde{\rho}(t') = e^{\mathcal{L}(t'-t)}\hat{b}_{\mathrm{A_r}}\rho_{\mathrm{ss}}\hat{b}^\dagger_{\mathrm{A_r}}/\mathrm{Tr}(\hat{b}_{\mathrm{A_r}}\rho_{\mathrm{ss}}\hat{b}^\dagger_{\mathrm{A_r}})$. This is the time-dependent density matrix conditioned on the detection of a red photon at $\mathrm{A_r}$ at time t. The Liouvillian $\mathcal{L}$ is that of the free evolution of the total system. In the state $\tilde{\rho}(t')$, entanglement between the remote oscillators can occur.

With our assumptions about the cavity and oscillator parameters, the effective phonon number n_{M} and linewidth $\tilde{\gamma}$ of the two oscillators will be approximately equal, such that we can drop indices on these quantities. For $\tau \gg \kappa^{-1}, \lambda^{-1}$, we find that [34]

$$\begin{aligned} g^{(2)}_{\mathrm{A_b}|\mathrm{A_r}}(\tau) &= 1 + \frac{n_{\mathrm{M}}+1}{n_{\mathrm{M}}} e^{-\tilde{\gamma}\tau}\cos^2(\delta\tau/2 + \phi) , \\ g^{(2)}_{\mathrm{B_b}|\mathrm{A_r}}(\tau) &= 1 + \frac{n_{\mathrm{M}}+1}{n_{\mathrm{M}}} e^{-\tilde{\gamma}\tau}\sin^2(\delta\tau/2 + \phi) , \end{aligned} \tag{5}$$

where the phase ϕ depends on path length differences. One could make this phase adjustable by introducing a phase shift in one of the arms of the interferometer, as suggested in Fig. 1. The interference effect in Eq. (5) can be understood classically for large n_{M}. It simply means that the red light phase difference between the two cavity outputs at time t is related to the blue light phase difference at $t+\tau$ because the mechanical oscillators have a well defined phase difference for times $\tau \lesssim \tilde{\gamma}$. This phase difference becomes time dependent if the mechanical frequency difference δ is nonzero. From a quantum mechanical point of view, the detection of a red photon creates a mechanical superposition with a definite phase θ. The subsequent blue photon will be superposed between the upper and lower arms of the interferometer, with a relative phase determined by θ. This again determines the detection probability at $\mathrm{A_b}$ or $\mathrm{B_b}$. A nonzero δ means that the superposition switches back and forth from symmetric to antisymmetric as τ increases, which is observable if $\delta \gtrsim \tilde{\gamma}$. Note also that the ratio $g^{(2)}_{\mathrm{A_b}|\mathrm{A_r}}/(g^{(2)}_{\mathrm{A_b}|\mathrm{A_r}} + g^{(2)}_{\mathrm{B_b}|\mathrm{A_r}})$ can exceed its classical bound of $2/3$ and come close to unity for small mean phonon numbers n_{M}. This means that, in the limit $n_{\mathrm{M}}, \delta/\tilde{\gamma} \to 0$, one can e.g. arrange the phases in such a way that when a red photon is detected at $\mathrm{A_r}$, the next blue always arrives at $\mathrm{A_b}$.

We now discuss how entanglement between the two mechanical oscillators can be detected. We wish to verify that following a red photon detection, the subsequent blue photon is in a superposition state of originating from cavity 1 and cavity 2, i.e., that there is entanglement between the output modes $\hat{b}_{\mathrm{b},1}$ and $\hat{b}_{\mathrm{b},2}$. This entanglement must be smaller than or equal to the entanglement between the mechanical oscillators. Again, we let $t' = t + \tau$. If $\tilde{\rho}(t')$ is a separable state, it is straightforward to show that

$$R(\tau) \equiv \frac{\langle \hat{b}^\dagger_{\mathrm{b},1}(t')\hat{b}_{\mathrm{b},1}(t')\hat{b}^\dagger_{\mathrm{b},2}(t')\hat{b}_{\mathrm{b},2}(t')\rangle_{\mathrm{A_r}}}{\left|\langle \hat{b}^\dagger_{\mathrm{b},1}(t')\hat{b}_{\mathrm{b},2}(t')\rangle_{\mathrm{A_r}}\right|^2} \geq 1 . \tag{6}$$

This also holds for classical correlations between the two fields, which e.g. could originate from technical laser noise. Thus, we can use $R(\tau)$ as an entanglement witness [18, 34].

Expectation values with respect to the state $\tilde{\rho}(t')$ can be measured through degrees of higher-order coherence, as Eq. (4) suggests. We find that $R(\tau) \leq R_{\mathrm{m}}(\tau)$, where the measurable upper bound for the entanglement witness is [34]

$$R_{\mathrm{m}}(\tau) = 4\,\frac{g^{(2)}_{\mathrm{A_b}|\mathrm{A_r}}(\tau) + g^{(2)}_{\mathrm{B_b}|\mathrm{A_r}}(\tau) - 1}{\left(g^{(2)}_{\mathrm{A_b}|\mathrm{A_r}}(\tau) - g^{(2)}_{\mathrm{B_b}|\mathrm{A_r}}(\tau)\right)^2} , \tag{7}$$

for a symmetric setup where the output flux is the same from both cavities. Thus, a measurement of $R_{\mathrm{m}}(\tau) < 1$ is evidence of entanglement between the mechanical oscillators.

We now show that the separability criterion (6) is violated as the oscillators are cooled close to the ground state. Inserting the expressions in Eq. (5), we arrive at

$$R_{\rm m}(\tau) = \frac{4n_{\rm M}\left[n_{\rm M} + (n_{\rm M}+1)e^{-\tilde{\gamma}\tau}\right]}{(n_{\rm M}+1)^2 e^{-2\tilde{\gamma}\tau}\cos^2(\delta\tau + 2\phi)}\,, \qquad (8)$$

again for $\tau \gg \kappa^{-1}, \lambda^{-1}$. In Fig. 2 we plot $\max(1 - R_{\rm m}(\tau), 0)$ as a function of phonon number $n_{\rm M}$ and delay time τ in units of $\tilde{\gamma}^{-1}$, when assuming $\delta\tau + 2\phi$ to be an integer of π. We observe that entanglement can be verified through violation of the separability criterion for mean phonon numbers $n_{\rm M} < 0.26$. For $n_{\rm M} \ll 1$, the entanglement can be detected for times $\tau < \tilde{\gamma}^{-1}\ln((\sqrt{2}-1)/2n_{\rm M})$. It is also worth mentioning that before the first blue photon is emitted, the entanglement between the mechanical oscillators only decays on the timescale of the mechanical phase decoherence, which is likely to be much larger than $\tilde{\gamma}^{-1}$. This could be measurable by looking at the statistics of only the first blue photon following a red.

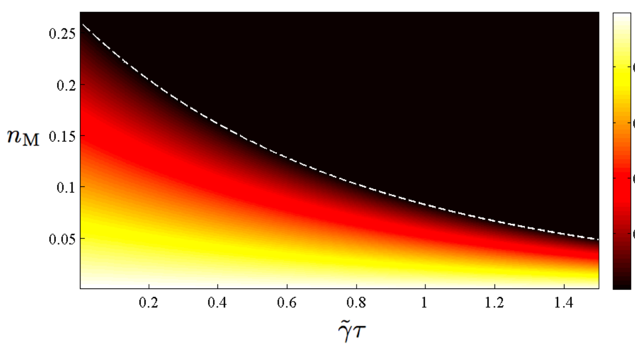

FIG. 2: (Color online) Density plot of $\max(1 - R_{\rm m}(\tau), 0)$ as a function of mean phonon number $n_{\rm M}$ and time delay τ. We set $\delta\tau + 2\phi = 0$. The separability criterion is violated below the white dashed curve, proving entanglement between the two mechanical oscillators. The entanglement can be verified for mean phonon numbers $n_{\rm M} < 0.26$ and time delays $\tau < \tilde{\gamma}^{-1}\ln((\sqrt{2}-1)(n_{\rm M}+1)/2n_{\rm M})$.

To show that our proposed experiment is within reach of present day experiments, we take the membrane-in-the-middle geometry [16] as an example. A set of realistic parameters is $\omega_{\rm M}/2\pi = 2$ MHz, $\omega_{\rm M}/\gamma = 2 \cdot 10^7$, $\kappa/2\pi = 1$ MHz, and $|\alpha|/2\pi = 10$ kHz. Assuming an initial temperature of 20 mK, this gives an effective mean phonon number $n_{\rm M} = 0.068$, with $n_{\rm opt} = 0.016$. The output flux of red (blue) photons would be 41 (172) photons per second. The separability criterion would be violated for times $\tau < 0.47$ milliseconds. Note that this is over 400 times longer than the entanglement lifetimes reported in a corresponding experiment with atomic ensembles by Chou *et al.* [24]. The long decoherence time is a result of high mechanical quality factors even when laser cooling to the quantum regime.

In conclusion, we have proposed an experiment for entangling remote mechanical oscillators. This would be an important milestone in the endeavour to explore quantum effects in macroscopic systems. The entanglement is induced by optical measurements and can be verified through second-order coherences of the optical field. Our proposal is relevant to present day experimental setups. We estimate entanglement storage lifetimes of milliseconds for the membrane-in-the-middle geometry, which could be of technological interest.

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