Absence of Spontaneous Magnetic Order of Lattice Spins Coupled to Itinerant Interacting Electrons in One and Two Dimensions

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Absence of spontaneous magnetic order of lattice spins coupled to itinerant interacting electrons in one and two dimensions

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We extend the Mermin-Wagner theorem to a system of lattice spins which are spin-coupled to itinerant and interacting charge carriers. We use the Bogoliubov inequality to rigorously prove that neither (anti-) ferromagnetic nor helical long-range order is possible in one and two dimensions at any finite temperature. Our proof applies to a wide class of models including any form of electron-electron and single-electron interactions that are independent of spin. In the presence of Rashba or Dresselhaus spin-orbit interactions (SOI) magnetic order is not excluded and intimately connected to equilibrium spin currents. However, in the special case when Rashba and Dresselhaus SOIs are tuned to be equal, magnetic order is excluded again. This opens up a new possibility to control magnetism electrically.

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Since the seminal work on phase transitions by Hohenberg [1] and Mermin and Wagner [2] it has become common knowledge that spontaneous order in low-dimensional systems is generically not possible at any finite temperature. In these studies, the use of the Bogoliubov inequality [3] was essential: Hohenberg used it to rule out superfuidity [1]and Mermin and Wagner to rule out magnetic order in Heisenberg spin systems [2] in dimensions $d < 3$. This approach is very powerful and was then applied to many different systems [4–10], including the Anderson and Kondo lattice models [11, 12].

For systems in the continuum, the weak coupling approximation is often applied leading to an effective exchange coupling between the localized spins which is of the RKKY-type [13]. RKKY interactions occur in many physical systems, prominent examples of present interest are heavy-fermion systems [14], diluted magnetic semiconductors [15–18], and nuclear spins in low-dimensional conducting nanostructures [19–21]. The latter system plays an important role as noise source for spin qubits in GaAs or InAs quantum dots [22–24], and much effort goes into understanding and controling the nuclear spin bath, with one possibility being to freeze out the nuclear noise by magnetic order [25, 26].

In contrast to the Heisenberg exchange, however, the RKKY interaction is long-ranged and thus is not covered by the original Mermin-Wagner theorem which requires the spin interactions to decay sufficiently fast with distance r (faster than $1/r^{2+d}$) [2]. Addressing precisely this issue, Bruno [10] was able to rule out in RKKY systems magnetic order in one dimension. A similar conclusion, however, for the two-dimensional counterpart appears still to be missing. Here we will fill this gap by rigorously proving the absence of order for a rather general class of systems which consist of lattice spins embedded in a continuum of itinerant electrons with which they interact by an isotropic on-site spin interaction. The allowed electron Hamiltonian H_e is very general and may include electron-electron interactions as well as any single-particle potential (such as lattice or disorder potential) that does not depend on spin. For this class of models we prove then that in the thermodynamic limit ferro- and antiferromagnetic, as well as helical, long-range order of the lattice spins is excluded at any finite temperature in dimensions one and two. We show that this conclusion remains valid when short-range Heisenberg interaction between lattice spins is included. Our result also applies to the RKKY case, since this regime is obtained from the full one by lowest order perturbation expansion in the on-site spin interaction [13] including the full H_e [26].

Moreover, we consider the effect of Rashba [28] and Dresselhaus [29] spin-orbit interactions (SOI) which explicitly break the spin symmetry. Our argument becomes then inconclusive and magnetic order cannot be excluded. While this finding is not unexpected it is remarkable that it is closely linked to the existence of equilibrium spin currents studied recently in spintronics [30–32]. Even more remarkably, we find that in the special case when Rashba (α) and Dresselhaus (β) SOIs become equal, magnetic order is excluded again. Since α can be electrically tuned to β [33–35], this opens up a new way to tune magnetism by electrical gates.

Finally, we note that the absence of spontaneous order proven here is valid only in the thermodynamic limit; thus, effective ordering in nanostructures of finite size at sufficiently low (but finite) temperatures is not in conflict with our findings.

Model. We consider a lattice $\{\mathbf{R}_j\}_{j=1}^{N_I}$ filled with N_I spins $\hat{\mathbf{I}}_j = (\hat{I}_j^x, \hat{I}_j^y, \hat{I}_j^z)$ located at the sites $\mathbf{R}_j$. The lattice is embedded into a volume Ω containing N_e itinerant electrons which couple to the lattice spins via on-site spin-spin interactions. The Hamiltonian for the entire system reads,

$$H = H_e + J\sum_{j=1}^{N_I}\hat{\mathbf{S}}_j\cdot\hat{\mathbf{I}}_j + h\sum_{j=1}^{N_I}(e^{-i\mathbf{Q}\cdot\mathbf{R}_j}\hat{I}_j^z + \text{h.c.}), \quad (1)$$

where $H_e = H_0 + V + U = \sum_{i=1}^{N_e}\hat{\mathbf{p}}_i^2/2m + \sum_{i<j}^{N_e} V_{ij} + \sum_{i=1}^{N_e} U(\hat{\mathbf{r}}_i)$ is the Hamiltonian describing the electron system. Here, m is the mass and $\hat{\mathbf{p}}_i$ the momentum operator of the i^{th} electron, $V_{ij} = V(\hat{\mathbf{r}}_i - \hat{\mathbf{r}}_j)$ the electron-electron interaction of electrons at positions $\hat{\mathbf{r}}_i$ and $\hat{\mathbf{r}}_j$, and $U(\hat{\mathbf{r}}_i)$ an arbitrary spin-independent single-electron potential. Typical examples for $U(\hat{\mathbf{r}}_i)$ are periodic lattice potentials, disorder potentials, electron-phonon interactions [36], etc. We remark that in contrast to previous work on lattice models [11, 12], we do not restrict the motion of the electrons to the sites of a lattice (tight binding limit) but allow them to move in the real space continuum. Further, J denotes the coupling strength of the isotropic spin interaction at lattice site $\mathbf{R}_j$, $H_J = J\sum_{j=1}^{N_I}\hat{\mathbf{S}}_j\cdot\hat{\mathbf{I}}_j$, where $\hat{\mathbf{S}}_j \equiv \hat{\mathbf{S}}(\mathbf{R}_j)$ is the electron spin density operator $\hat{\mathbf{S}}(\mathbf{r}) = \sum_{i=1}^{N_e}\hat{\mathbf{s}}_i\delta(\mathbf{r} - \hat{\mathbf{r}}_i)$, with $\hat{\mathbf{s}}_i = (\hat{s}_i^x, \hat{s}_i^y, \hat{s}_i^z)$ being the spin-1/2 of the i^{th} electron. The vector components of each spin, $\hat{s}_i^k$ and $\hat{I}_j^l$, satisfy standard spin commutation relations. Finally, to probe the order for the lattice spins $\hat{\mathbf{I}}_j$ we break the symmetry by an external (fictitious) field h pointing in, say, z direction, which we let then go to zero at the end. This leads to an additional Zeeman term $H_Z(\mathbf{Q}) = h\sum_{j=1}^{N_I} e^{-i\mathbf{Q}\cdot\mathbf{R}_j}\hat{I}_j^z$ + h.c. To rule out ferromagnetic order we will choose $\mathbf{Q} = 0$, whereas to exclude antiferromagnetic order we will choose $\mathbf{Q}$ such that $e^{-i\mathbf{Q}\cdot\mathbf{R}} = +1$, if $\mathbf{R}$ connects sites from the same sublattice, and $e^{-i\mathbf{Q}\cdot\mathbf{R}} = -1$, if $\mathbf{R}$ connects sites from different sublattices.

To prove the absence of spontaneous order for the lattice spins $\hat{\mathbf{I}}_j$ we follow Ref. [2] and make use of the Bogoliubov inequality [3], which is an exact relation between two operators A, C, and a Hamiltonian H,

$$\frac{1}{2}\langle\{A, A^\dagger\}\rangle\langle[[C, H], C^\dagger]\rangle \geq k_B T|\langle[C, A]\rangle|^2. \quad (2)$$

Here, $\langle A\rangle = Tre^{-H/k_BT}A/Tre^{-H/k_BT}$ denotes the expectation value in a canonical ensemble, T the temperature, k_B the Boltzmann constant, and $\{A, B\} = AB + BA$ the anticommutator and $[A, B] = AB - BA$ the commutator. It is assumed that all expectation values are well-defined and exist in the thermodynamic limit defined by $N_e, N_I, \Omega \to \infty$ with electron density $n_e = N_e/\Omega$ and density of lattice spins $n_I = N_I/\Omega$ finite.

Proof- The strategy of the proof consists of using the Bogoliubov inequality to derive an upper bound for the order parameter corresponding to the phase transition we want to discuss. If this bound turns out to be in contradiction with the presence of long-range magnetic order, then the absence of the corresponding phase transition is rigorously demonstrated. The success of the procedure depends crucially on the choice of the operators A and C in (2). As we shall see, the appropriate choice for our case is given by

$$C_{\mathbf{q}} = \hat{S}^-_{-\mathbf{q}} + \hat{I}^-_{-\mathbf{q}} + \hat{S}^+_{-\mathbf{q}} + \hat{I}^+_{-\mathbf{q}}\,,\ A_{\mathbf{q}} = \hat{I}^+_{\mathbf{q}+\mathbf{Q}} + \hat{I}^+_{\mathbf{q}-\mathbf{Q}}, \quad (3)$$

where the Fourier transforms are given by $\hat{\mathbf{S}}_{\mathbf{q}} = \sum_{i=1}^{N_e} e^{-i\mathbf{q}\cdot\hat{\mathbf{r}}_i}\hat{\mathbf{s}}_i$ and $\hat{\mathbf{I}}_{\mathbf{q}} = \sum_{j=1}^{N_I} e^{-i\mathbf{q}\cdot\mathbf{R}_j}\hat{\mathbf{I}}_j$ [37], and where $B^\pm \equiv B^x \pm iB^y$. Note that $C_{\mathbf{q}}$ and $A_{\mathbf{q}}$ are not hermitian in general. Since the Bogoliubov inequality (2) is valid for any wave vector $\mathbf{q}$, it can be generalized to

$$\frac{1}{2}\sum_{\mathbf{q}}\langle\{A_{\mathbf{q}}, A_{\mathbf{q}}^\dagger\}\rangle \geq k_B T\sum_{\mathbf{q}}\frac{|\langle[C_{\mathbf{q}}, A_{\mathbf{q}}]\rangle|^2}{\langle[[C_{\mathbf{q}}, H], C_{\mathbf{q}}^\dagger]\rangle}, \quad (4)$$

where the sum runs over all $\mathbf{q}$'s in the first Brillouin zone of the reciprocal lattice. We note that the above choice for $C_{\mathbf{q}}$ and $A_{\mathbf{q}}$ is essential also for the following reason. Besides the fact that $\sum_{\mathbf{q}}\langle[C_{\mathbf{q}}, A_{\mathbf{q}}]\rangle$ can be expressed in terms of the lattice spin magnetization, the generally complicated interaction terms V and U in H_e simply drop out of the calculation since they commute with $C_{\mathbf{q}}$,

$$[C_{\mathbf{q}}, H_e] = [\hat{S}^-_{-\mathbf{q}} + \hat{S}^+_{-\mathbf{q}}, H_0]. \quad (5)$$

This simplification is a crucial advantage of first over second quantization formalism since spin and position operators of the electrons trivially commute. [Note, however, that the expectation values still contain the full Hamiltonian including U and V.] Hence, our proof goes through for any form of the potentials V and U as long as they are spin independent.

We now focus on the various terms in Eq. (4) and find bounds for them. Here, we outline only the main steps of the calculations and defer details to the Appendix [38]. As a first step, let us evaluate the double commutator on the right-hand-side of inequality (4). By virtue of the commutation relation $[\hat{S}^\pm_{-\mathbf{q}}, H_0] = -\frac{\mathbf{q}}{2m}\sum_i \hat{s}_i^\pm\{\hat{\mathbf{p}}_i, e^{i\mathbf{q}\cdot\hat{\mathbf{r}}_i}\}$, we obtain that $[[C_{\mathbf{q}}, H_e], C_{\mathbf{q}}^\dagger] = \frac{1}{m}N_e q^2$. The part of the double commutator with H_J vanishes since $[C_{\mathbf{q}}, H_J] = 0$. Indeed, $[\hat{\mathbf{S}}^\pm_{-\mathbf{q}}, H_J] = i\sum_{i,j} e^{i\mathbf{q}\cdot\hat{\mathbf{r}}_i}\delta(\hat{\mathbf{r}}_i - \mathbf{R}_j)(\hat{\mathbf{I}}_j \times \hat{\mathbf{s}}_i)^\pm$, and thus $[\hat{\mathbf{S}}^\pm_{-\mathbf{q}}, H_J] = -[\hat{\mathbf{I}}^\pm_{-\mathbf{q}}, H_J]$. After some calculations (see [38]) we find that $[[C_{\mathbf{q}}, H_z(\mathbf{Q})], C_{\mathbf{q}}^\dagger] = -4h(\sum_j e^{-i\mathbf{Q}\cdot\mathbf{R}_j}\hat{I}_j^z + h.c.)$. Hence,

$$\langle[[C_{\mathbf{q}}, H], C_{\mathbf{q}}^\dagger]\rangle = N_e\left(\frac{1}{m}q^2 - 4h\frac{N_I}{N_e}m_I^z(\mathbf{Q})\right), \quad (6)$$

where the lattice spin magnetization appearing in Eq. (6), which we identify as the order parameter, is defined by $m_I^z(\mathbf{Q}) = \frac{1}{N_I}\langle\sum_j e^{-i\mathbf{Q}\cdot\mathbf{R}_j}\hat{I}_j^z + e^{i\mathbf{Q}\cdot\mathbf{R}_j}\hat{I}_j^z\rangle$. The commutator on the right-hand side of inequality (4) can also be expressed in terms of $m_I^z(\mathbf{Q})$,

$$\langle[C_{\mathbf{q}}, A_{\mathbf{q}}]\rangle = -2N_I m_I^z(\mathbf{Q}). \quad (7)$$

Finally, the sum on the left-hand side of Eq. (4) can be bounded as follows,

$$\sum_{\mathbf{q}}\langle\{A_{\mathbf{q}},A_{\mathbf{q}}^{\dagger}\}\rangle = 2N_I\sum_j\langle\{\hat{I}_j^+,\hat{I}_j^-\}(1+\cos(\mathbf{Q}\cdot\mathbf{R}_j))\rangle \leq 4N_I^2(2I)^2, \quad (8)$$

where we have used that $\sum_{\mathbf{q}} e^{i\mathbf{q}\cdot(\mathbf{R}_i-\mathbf{R}_j)} = N_I\delta_{\mathbf{R}_i,\mathbf{R}_j}$, and $\langle\{\hat{I}_j^+,\hat{I}_j^-\}\rangle \leq (2I)^2$. Using Eqs. (6), (7), and (8), we obtain from the Bogoliubov inequality (4)

$$4N_I^2(2I)^2/2 \geq k_BT\sum_{\mathbf{q}}\frac{4N_I^2 m_I^z(\mathbf{Q})^2}{\langle[[C_{\mathbf{q}},H],C_{\mathbf{q}}^{\dagger}]\rangle}. \quad (9)$$

Our goal is to rule out spontaneous magnetization in the lattice spin system, therefore we are interested in the behavior of the order parameter $m_I^z(\mathbf{Q})$ in the limit of vanishing external field, i.e., $h\to 0$, *after* we have taken the thermodynamic limit. We need to distinguish two cases: *i)* $m_I^z(\mathbf{Q})=0$, $\forall\, h$ around $h=0$; *ii)* $m_I^z(\mathbf{Q})\neq 0$, $\forall\, h$ around $h=0$. If *i)* is satisfied, there is no order and the proof is completed. If *ii)* is satisfied, we need to show that $\lim_{h\to 0} m_I^z(\mathbf{Q})=0$ follows from inequality (9) in the thermodynamic limit. In this limit, the sum can be replaced by an integral,

$$(2I)^2 \geq \frac{k_BTN_Iv}{N_e(2\pi)^d}\int\limits_{|\mathbf{q}|\leq|\mathbf{q}_c|} d^dq\,\frac{m_I^z(\mathbf{Q})^2}{\frac{q^2}{2m}+|\nu h m_I^z(\mathbf{Q})|}, \quad (10)$$

where $\nu = 2N_I/N_e$, $\mathbf{q}_c$ is an arbitrary cut-off vector lying in the first Brillouin zone, $v=\Omega/N_I$, and we have used that $\langle[[C_{\mathbf{q}},H],C_{\mathbf{q}}^{\dagger}]\rangle \leq N_e(q^2/m+|2\nu h m_I^z(\mathbf{Q})|)$. In the one-dimensional case ($d=1$), Eq. (10) gives

$$\frac{\lambda_1\sqrt{|h|}}{T}\left[\arctan\left(\frac{|\mathbf{q}_c|}{\sqrt{2m|\nu h m_I^z(\mathbf{Q})|}}\right)\right]^{-1} \geq \frac{m_I^z(\mathbf{Q})^2}{\sqrt{|m_I^z(\mathbf{Q})|}}, \quad (11)$$

where $\lambda_1 = \pi(2I)^2 n_e\sqrt{\nu}/(k_B\sqrt{2m})$. In the limit $h\to 0$, the left hand-side of inequality (11) vanishes and this implies that $\lim_{h\to 0} m_I^z(\mathbf{Q})=0$. The two-dimensional case can be treated in a similar way. For $d=2$, inequality (10) leads to the following relation

$$\frac{\lambda_2}{T}\left[\log\left(1+\frac{|\mathbf{q}_c|^2}{2m|\nu h m_I^z(\mathbf{Q})|}\right)\right]^{-1} \geq m_I^z(\mathbf{Q})^2, \quad (12)$$

where $\lambda_2 = 2\sqrt{2}\lambda_1/\sqrt{\nu m}$. It follows from inequality (12) that $\lim_{h\to 0} m_I^z(\mathbf{Q})=0$ here, too. Since our arguments were independent of the choice of $\mathbf{Q}$, we have proven that neither ferromagnetic nor antiferromagnetic long-range order of the lattice spins is possible at any finite temperature $T>0$ in one and two dimensions.

The absence of order can be traced back to the increased fluctuations in the lattice spin system in lower dimensions. These fluctuations, in turn, have their origin in the kinetic energy of the electrons, as one can explicitly see from Eq. (10) where the term $q^2/2m$ is responsible for the divergency in above q-integrals for $d=1$ and 2.

Next, we show that helical long-range order of the lattice spins is also excluded. The strategy of the proof remains the same and we shall be brief (for details see [38]). To study this type of order, we consider the symmetry breaking Zeeman term $\widetilde{H}_Z(\mathbf{Q}) = \sqrt{2/3}h\sum_j e^{-i\mathbf{Q}\cdot\mathbf{R}_j}\hat{I}_j^+ + h.c.$ and the magnetic order parameter $m_I^{\perp}(\mathbf{Q}) = \sqrt{2/3}\frac{1}{N_I}\langle\sum_j e^{-i\mathbf{Q}\cdot\mathbf{R}_j}\hat{I}_j^+ + h.c.\rangle$ which corresponds to a spin helix in the xy-plane. Note that the spin part of Hamiltonian (1) is isotropic and consequently all choices for the helix are equivalent. The operators $\widetilde{C}_{\mathbf{q}}$ and $\widetilde{A}_{\mathbf{q}}$ for the Bogoliubov inequality (4) are now chosen to be

$$\widetilde{C}_{\mathbf{q}} = \hat{S}^z_{-\mathbf{q}}+\hat{I}^z_{-\mathbf{q}} \quad\text{and}\quad \widetilde{A}_{\mathbf{q}} = \frac{1}{\sqrt{3}}\left(\hat{I}^+_{\mathbf{q}+\mathbf{Q}}-\hat{I}^-_{\mathbf{q}-\mathbf{Q}}\right). \quad (13)$$

The double commutator on the right-hand side of Eq. (4) becomes then $\langle[[\widetilde{C}_{\mathbf{q}},H],\widetilde{C}_{\mathbf{q}}^{\dagger}]\rangle = N_e\left(q^2/4m - \nu h m_I(\mathbf{Q})/2\right)$. Since $\langle[\widetilde{C}_{\mathbf{q}},\widetilde{A}_{\mathbf{q}}]\rangle = (N_I/\sqrt{2})m_I^{\perp}(\mathbf{Q})$ and $\sum_{\mathbf{q}}\langle\{\widetilde{A}_{\mathbf{q}},\widetilde{A}_{\mathbf{q}}^{\dagger}\}\rangle \leq 2N_I^2(2I)^2$, Eq. (4) takes in the thermodynamic limit exactly the same form as Eq. (10), where $m_I^z(\mathbf{Q})$ must be replaced by $m_I^{\perp}(\mathbf{Q})$. We thus conclude that $\lim_{h\to 0} m_I^{\perp}(\mathbf{Q})=0$ for any $\mathbf{Q}$ and hence long-range helical order is also excluded in one and two dimensions at any $T>0$ [27].

As a further generalization, short-range impurity-spin Heisenberg interaction $H_{\mathcal{I}} = \sum_{i,j}\mathcal{I}_{ij}\hat{\mathbf{I}}_i\cdot\hat{\mathbf{I}}_j$ is added to Hamiltonian (1). When the couplings $\mathcal{I}_{ij}$ satisfy $1/N_I\sum_{ij}|\mathcal{I}_{ij}|(\mathbf{R}_i-\mathbf{R}_j)^2 < \infty$, then both proofs to exclude (anti-) ferromagnetic and helical ordering remain valid and lead to Eq. (10) with renormalized mass $m^* = m/(1+8mI^2\frac{n_I}{n_e}\frac{1}{N_I}\sum_{ij}|\mathcal{I}_{ij}|(\mathbf{R}_i-\mathbf{R}_j)^2)$ [38].

Presence of spin orbit interaction. Next we investigate the question of magnetic order in a low-dimensional electron gas in the presence of Rashba [28] and/or Dresselhaus [29] spin orbit interaction which break the rotational spin symmetry of the Hamiltonian (1) explicitly. The spin-orbit Hamiltonian is given by $H_{\rm SO} = H_{\rm R}+H_{\rm D}$, with $H_{\rm R} = \alpha\sum_{i=1}^{N_e}(\hat{p}_i^y\hat{s}_i^x - \hat{p}_i^x\hat{s}_i^y)$, $H_{\rm D} = \beta\sum_{i=1}^{N_e}(\hat{p}_i^x\hat{s}_i^x - \hat{p}_i^y\hat{s}_i^y)$, where α (β) is the Rashba (Dresselhaus) coefficient. Using Eq. (3) for $C_{\mathbf{q}}$, we obtain $[[C_{\mathbf{q}},H_{\rm SO}],C_{\mathbf{q}}^{\dagger}] = 4m\alpha\hat{j}^y_{\mathbf{q=0},x} + 4m\beta\hat{j}^y_{\mathbf{q=0},y}$, where we have defined the spin-current density operator as $\hat{\mathbf{j}}^{\alpha}(\mathbf{r}) = \frac{1}{2m}\sum_{i=1}^{N_e}\hat{s}_i^{\alpha}\{\hat{\mathbf{p}}_i,\delta(\hat{\mathbf{r}}_i-\mathbf{r})\}$ and its corresponding Fourier component $\hat{\mathbf{j}}^{\alpha}_{\mathbf{q}} = \frac{1}{2m}\sum_i\hat{s}_i^{\alpha}\{\hat{\mathbf{p}}_i,e^{-i\mathbf{q}\cdot\hat{\mathbf{r}}_i}\}$. These spin currents may lead to an intrinsic cut-off for the fluctuations in q, and thus help to establish order. To see this, we evaluate now the spin currents perturbatively around the free electron limit, i.e. $U,V,J=0$, and at $T=0$ [39],

$$\langle j^y_{\mathbf{q}=0,x}\rangle_0 = \Omega\frac{mE_F}{4\pi}\alpha \quad (14)$$

$$\langle j^y_{\mathbf{q}=0,y}\rangle_0 = -\Omega\frac{mE_F}{4\pi}\beta\,, \quad (15)$$

where E_F is the Fermi energy and the results are valid in the regime $m\alpha^2, m\beta^2 \ll \hbar^2 E_F$ [40]. Performing now a perturbative expansion in the parameters V, U, J, T around above free case, we conclude that $\langle \hat{j}_y^y \rangle \neq 0$ and $\langle \hat{j}_x^y \rangle \neq 0$ [41]. (In passing we note that in the stationary and homogeneous limit, the spin-currents satisfy the relations $\langle \hat{j}_x^x \rangle = -\langle \hat{j}_y^y \rangle$ and $\langle \hat{j}_x^y \rangle = -\langle \hat{j}_y^x \rangle$ due to a generalized continuity equation, see [38].) As a consequence, the commutator $\langle [[C_{\mathbf{q}}, H_{\rm SO}], C_{\mathbf{q}}^\dagger] \rangle$ appearing in Eq. (9) does not vanish anymore and thus provides an intrinsic cut-off to the q-integral (cf. Eq. (10)). Hence, the bound for the order parameter we extract from inequality (10) is a constant which does not vanish in the limit $h \to 0$. Thus, our argument becomes inconclusive and we cannot rule out (anti-) ferromagnetic order in this case.

Similarly, for helical order our argument remains inconclusive, since $[[\widetilde{C}_{\mathbf{q}}, H_{SO}], \widetilde{C}_{\mathbf{q}}^\dagger] = m\alpha(\hat{j}^y_{\mathbf{q}=0,x} - \hat{j}^x_{\mathbf{q}=0,y}) + m\beta(\hat{j}^y_{\mathbf{q}=0,y} - \hat{j}^x_{\mathbf{q}=0,x})$, which, will not vanish in general.

Next, let us consider the special case $\alpha = \beta$ where new symmetries emerge [42]. Then, the leading terms, Eqs. (14), (15), cancel, indicating that the physics changes dramatically. Indeed, by making use of the 'gauge transformation' $U = e^{i\sum_k \hat{\mathbf{A}}_k \cdot \hat{\mathbf{r}}_k}$, where $\hat{\mathbf{A}}_k = -\alpha m(\hat{s}_k^x - \hat{s}_k^y)(1,1,0)$, to remove the SOI from the Hamiltonian, we can prove as before [38] that (anti-) ferromagnetic order in z-direction can now be excluded rigorously for any $T > 0$ and $d = 1, 2$. Similarly, we can rule out helical ordering described by the order parameter $m_I^{\perp'} = \frac{1}{N_I}\langle \sum_j e^{-i\mathbf{Q}\cdot\mathbf{R}_j} \hat{I}_j^{+'} + h.c. \rangle$ with $\mathbf{Q} = \sqrt{2}\alpha m(1,1,0)$ (for rotated coordinates $(x, y, z) \to (x', y', z') = (z, (x+y)/\sqrt{2}, (x-y)/\sqrt{2})$, see [38]).

Thus, quite remarkably, this spin orbit effect suggests the control of magnetism by electrical gates, namely by tuning the Rashba SOI (α) [33–35] from the regime $\alpha \neq \beta$ (ordering not excluded) to $\alpha = \beta$ (ordering excluded).

Conclusions. We proved an extension of the Mermin-Wagner theorem for lattice spins interacting with itinerant electrons, and showed that spontaneous order of the lattice spins is ruled out in one and two dimensions at finite temperature. In the presence of Rashba (α) and Dresselhaus (β) spin-orbit interactions, however, spontaneous order could not be excluded, unless for $\alpha = \beta$.

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[36] H also contains then a free-phonon part, which, however, is of no consequence since Eq. (6) remains valid.
[37] Here, $\mathbf{q}$ is restricted to the first Brillouin zone, and thus $\hat{\mathbf{S}}_\mathbf{q}$ is defined on the reciprocal lattice only, while $\hat{\mathbf{S}}(\mathbf{r})$ is a function in the real space continuum.
[38] See supplementary material.
[39] The leading contribution to the spin-currents are linear in α, β, in contrast to Refs. [30, 32]. This is due to the different definitions of spin-currents, see [38].
[40] Eqs. (14,15) are valid for $h \neq 0$ since the Zeeman term simply drops out when $J = 0$.
[41] Presumably, the spin currents can remain non-zero even beyond the perturbative regime.
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