New Formulation of γZ Box Corrections to the Weak Charge of the Proton

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New formulation of γZ box corrections to the weak charge of the proton

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We present a new formulation of one of the major radiative corrections to the weak charge of the proton – that arising from the axial-vector hadron part of the γZ box diagram, $\Re\mathrm{e}\,\Box^{\mathrm{A}}_{\gamma Z}$. This formulation, based on dispersion relations, relates the γZ contributions to moments of the $F_3^{\gamma Z}$ interference structure function. It has a clear connection to the pioneering work of Marciano and Sirlin, and enables a systematic approach to improved numerical precision. Using currently available data, the total correction from all intermediate states is $\Re\mathrm{e}\,\Box^{\mathrm{A}}_{\gamma Z} = 0.0044(4)$ at zero energy, which shifts the theoretical estimate of the proton weak charge from 0.0713(8) to 0.0705(8). The energy dependence of this result, which is vital for interpreting the $\mathrm{Q_{weak}}$ experiment, is also determined.

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As modern parity-violating (PV) experiments press to ever improving levels of precision, they remain a vital complement to direct tests of the Standard Model at the high energy frontier. The classic example of this, involving precise measurements of parity violation in atoms, led to a remarkably accurate determination of $\sin^2\theta_W$. A complementary PV electron-proton scattering measurement underway by the $\mathrm{Q_{weak}}$ Collaboration [1] at Jefferson Lab has the potential to increase the mass scale associated with new physics to 2 TeV or higher, provided that the critical radiative corrections are under control. In this Letter we present a new formulation of the important γZ radiative corrections which allows for their controlled, systematic evaluation.

Including electroweak radiative corrections, the proton weak charge is defined, at zero electron energy E and zero momentum transfer, as [2]

$$\begin{aligned} Q_W^p &= (1+\Delta\rho+\Delta_e)(1-4\sin^2\theta_W(0)+\Delta_e') \\ &\quad + \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z}(0)\,, \end{aligned} \tag{1}$$

where $\sin^2\theta_W(0)$ is the weak mixing angle at zero momentum, and the corrections $\Delta\rho$, Δ_e and Δ_e' are given in [2] and references therein. The contributions $\Box_{WW}$ and $\Box_{ZZ}$ arise from the WW and ZZ box and crossed-box diagrams, and can be computed perturbatively. They are expected to be energy independent for electron scattering in the GeV range. By contrast, the γZ interference correction $\Box_{\gamma Z}(E)$ depends on physics at both short and long distance scales.

In the classic work of Marciano and Sirlin (MS) [3], $\Box_{\gamma Z}(0)$ was evaluated in a quark model-inspired loop calculation using either a "perturbative" (p) or a "nonperturbative" (np) *ansatz*,

$$\Box_{\gamma Z}(0) = v_e(M_Z^2)\,\frac{5\alpha}{2\pi}\,B_{\mathrm{p(np)}}, \tag{2}$$

where $v_e(M_Z^2) = (1-4\hat{s}^2)$, and $\hat{s}^2 \equiv \sin^2\theta_W(M_Z^2) = 0.23116$ in the $\overline{\mathrm{MS}}$ scheme [4].

The perturbative *ansatz* [3]

$$B_{\mathrm{p}} = \ln\frac{M_Z^2}{m^2} + \frac{3}{2} \tag{3}$$

is the free quark model result, with m a hadronic mass scale, and shows the leading-log behavior. For the nonperturbative *ansatz*, $B_{\mathrm{np}} = K_m + L_m$ is the sum of a long-distance part, L_m, and a short-distance part, K_m, with

$$K_m = \int_{m^2}^{\infty} \frac{du}{u(1+u/M_Z^2)}\left(1-\frac{\alpha_s(u)}{\pi}\right). \tag{4}$$

Here m is a mass scale representing the onset of asymptotic behavior at large loop momenta, and the factor $(1-\alpha_s(u)/\pi)$ is the lowest-order correction induced by the strong interactions. In Ref. [3] L_m is taken to be the elastic nucleon (Born) contribution, which is evaluated to be 2.04 using the same dipole form factors for both the electromagnetic and axial-vector coupling. MS [3] originally adopted the value $K_m = 9.6\pm1$, based on calculations with m in the range 0.3–1.0 GeV. A more recent calculation by Bardin *et al.* [5] sets $0.5 \le m \le 0.6$ GeV, over which K_m varies from 9.20 to 9.17 using a 3-loop evaluation of α_s. Marciano [6] gives an updated value for B_{np} of 11.0 ± 1.0, but in view of the high momentum scales in Eq. (4), suggests replacing α by $\alpha(M_Z^2)$ in Eq. (2). This value for $\Box_{\gamma Z}$ is the one adopted in Ref. [2], and contributes almost half of the error in the theoretical estimate $Q_W^p = 0.0713(8)$.

To progress in a systematic way beyond the approach of MS [3], and to determine the dependence on energy E, we present a new formulation of the box diagram contribution in which the dominant part of the correction is expressed in terms of empirical moments of structure functions. At forward angles one can compute $\Box_{\gamma Z}(E)$ from its imaginary part using dispersion relations [7]. The imaginary part depends on the PV $ep \to eX$ cross section, which can be expressed in terms of the product of leptonic and hadronic tensors. Following standard conventions [4], the hadronic tensor can be written in terms of the interference electroweak structure functions as

$$MW^{\mu\nu}_{\gamma Z} = -g^{\mu\nu} F_1^{\gamma Z} + \frac{p^\mu p^\nu}{p\cdot q} F_2^{\gamma Z} - i\varepsilon^{\mu\nu\lambda\rho}\frac{p_\lambda q_\rho}{2p\cdot q} F_3^{\gamma Z}, \quad (5)$$

where p and q are the four-momenta of the proton and exchanged boson, respectively. The $F_{1,2}^{\gamma Z}$ contributions to $\Box_{\gamma Z}$ involve the vector hadron coupling of the Z, and were recently computed in Refs. [7–10].

Our focus here is on the $F_3^{\gamma Z}$ contribution involving the axial-vector hadron coupling of the Z. Following an analogous derivation in Ref. [8], we can write

$$\Im\mathrm{m}\,\Box^{\mathrm{A}}_{\gamma Z}(E) \;=\; \frac{1}{(2ME)^2}\int_{M^2}^{s} dW^2 \int_0^{Q^2_{\max}} dQ^2 \times \frac{v_e(Q^2)\,\alpha(Q^2)\,F_3^{\gamma Z}}{1+Q^2/M_Z^2}\left(\frac{2ME}{W^2-M^2+Q^2}-\frac{1}{2}\right), \quad (6)$$

with $s = M^2 + 2ME$ and $Q^2_{\max} = 2ME(1-W^2/s)$. The real part is determined from the dispersion relation

$$\Re\mathrm{e}\,\Box^{\mathrm{A}}_{\gamma Z}(E) = \frac{2}{\pi}\int_0^\infty dE'\,\frac{E'}{E'^2-E^2}\,\Im\mathrm{m}\,\Box^{\mathrm{A}}_{\gamma Z}(E'), \quad (7)$$

which accounts for both the box and crossed-box terms. Unlike the vector hadronic correction $\Re\mathrm{e}\,\Box^{\mathrm{V}}_{\gamma Z}(E)$, which vanishes at $E=0$, the axial-vector hadronic correction $\Re\mathrm{e}\,\Box^{\mathrm{A}}_{\gamma Z}(E)$ remains finite, and is dominant in atomic parity violation at very low electron energies [11].

We incorporate one further improvement over earlier calculations by allowing for the Q^2 dependence of $\alpha(Q^2)$ and $\sin^2\theta_W(Q^2) = \kappa(Q^2)\,\hat{s}^2$ in Eq. (6) due to boson self-energy contributions. Both quantities vary significantly over the range of Q^2 relevant to these integrals. The photon vacuum polarization expression is well-known, and expressions for the universal fermion and boson contributions to $\kappa(Q^2)$ are given in Ref. [12]. Following Ref. [3], we use effective quark masses to reproduce the hadronic contribution of $\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2) = 0.02786$ obtained from dispersion relations [4], yielding $\kappa(0) = 1.030$. This is sufficiently accurate for the purpose of calculating the box contributions. In the numerical results that follow, the effect of using $\alpha(Q^2)$ and $v_e(Q^2)$ reduces the total contribution to Eq. (7) by 17% relative to using α and $v_e(M_Z^2)$.

The imaginary part of $\Box^{\mathrm{A}}_{\gamma Z}$ can be split into three regions: (i) elastic (el) with $W^2 = M^2$; (ii) resonances (res) with $(M+m_\pi)^2 \le W^2 \lesssim 4$ GeV^2; and (iii) deep inelastic (DIS), with $W^2 > 4$ GeV^2. Contributions from region (i) can be written in terms of the elastic form factors as

$$F_3^{\gamma Z(\mathrm{el})}(Q^2) = -Q^2 G_M^p(Q^2) G_A^Z(Q^2)\delta(W^2-M^2). \quad (8)$$

For the proton magnetic form factor G_M^p we use the recent parametrization from Ref. [13] (the results are similar if one uses a dipole with mass 0.84 GeV), and take

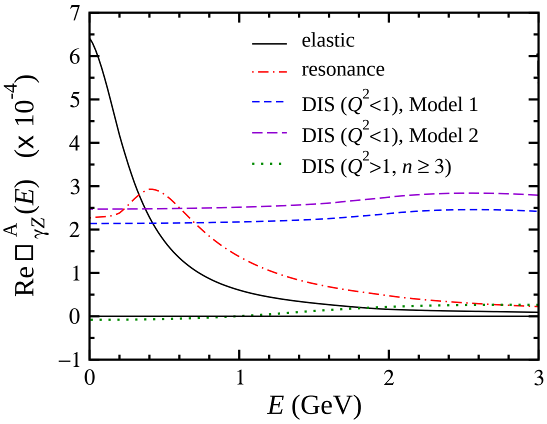

FIG. 1: Real part of $\Box^{\mathrm{A}}_{\gamma Z}(E)$ as a function of incident electron energy E. Shown are the elastic (solid) and resonance (dot-dashed) contributions. For the DIS part, the high-Q^2, $n \ge 3$ term (dotted) is negligibly small. The two $Q^2 < 1$ GeV^2 estimates (long and short dashes) show a very mild E dependence. Not shown is the dominant high-Q^2, $n = 1$ moment, which is 32.8×10^{-4}, and is independent of E.

the axial-vector form factor to be $G_A^Z(Q^2) = -1.267/(1+Q^2/M_A^2)^2$ with $M_A = 1.0$ GeV. A virtue of the dipole forms is that the integrals (6) and (7) can be performed analytically, which provides a useful cross-check.

To simplify notation in what follows, we denote $\Re\mathrm{e}\,\Box^{\mathrm{A}}_{\gamma Z}$ by $\Box^{\mathrm{A}}_{\gamma Z}$, since that is the quantity of interest in Eq. (1). The result for the elastic contribution $\Box^{\mathrm{A(el)}}_{\gamma Z}(E)$ is shown in Fig. 1. It agrees exactly with the direct loop calculations of $\Box^{\mathrm{A}}_{\gamma Z}$ in Refs. [14, 15], in which the intermediate nucleon is off-shell. It also agrees exactly at $E = 0$ with the value $L_m = 2.04$ if the parameters are adjusted to correspond to those of MS [3].

For the resonance contributions $\Box^{\mathrm{A(res)}}_{\gamma Z}$ from region (ii), we use the parametrizations of the transition form factors from Lalakulich *et al.* [16], but with modified isospin factors appropriate to γZ. These form factors have been fitted to the Jefferson Lab pion electroproduction data (vector part) and pion production data in ν and $\bar{\nu}$ scattering at ANL, BNL and Serpukhov (axial-vector part). The parametrizations include the lowest four spin-1/2 and 3/2 states in the first and second resonance regions, up to $Q^2 = 3.5$ GeV^2. At larger Q^2 the resonance contributions are suppressed by the Q^2 dependence of the transition form factors, which is stronger for the dominant $\Delta(1232)$ resonance than for the higher-mass resonances [16]. The resulting resonance contribution $\Box^{\mathrm{A(res)}}_{\gamma Z}(0)$ is smaller than the elastic term at $E = 0$, but decreases less rapidly with increasing energy. Varying the Q^2 dependence of the axial-vector form factors, which is not well determined, has a negligible effect on these results.

To compute the DIS contributions from region (iii) it is convenient to interchange the order of integration in (6) and (7), in which case the integral over energy can be performed analytically [9],

$$\Box_{\gamma Z}^{\rm A(DIS)}(E) = \frac{2}{\pi}\int_0^\infty dQ^2 \frac{v_e(Q^2)\alpha(Q^2)}{Q^2(1+Q^2/M_Z^2)} \times \int_0^{x_{\max}} dx\, F_3^{\gamma Z}(x,Q^2)\, f(r,t), \qquad (9)$$

$$f(r,t) = \frac{1}{t^2}\left[\log\left(1-t^2/r^2\right) + 2t\tanh^{-1}(t/r)\right],$$

with $r \equiv 1+\sqrt{1+4M^2x^2/Q^2}$, $t \equiv 4MEx/Q^2$, and $x_{\max} = Q^2/(W_{\min}^2 - M^2 + Q^2)$. For $t=0$, we find $f(r,0) = (2r-1)/r^2$. In the free quark model limit with $F_3^{\gamma Z} = (5/3)\,x\,\delta(1-x)$, Eq. (9) then gives exactly the perturbative result of Eq. (3) for $E=0$ (ignoring the Q^2 dependence of α and v_e).

To proceed, we divide the Q^2 integral of the full expression (9) into a low-Q^2 part, where the structure function $F_3^{\gamma Z}$ is relatively unknown, and a high-Q^2 part ($Q^2 > Q_0^2$), where at leading order (LO) the structure functions can be expressed in terms of valence quark distributions $q_v = q - \bar{q}$ [4],

$$F_3^{\gamma Z({\rm DIS})}(x,Q^2) = \sum_q 2\, e_q\, g_A^q\, q_v(x,Q^2). \qquad (10)$$

At high Q^2 and low E, the integrand in (9) can be expanded in powers of x^2/Q^2, yielding a series whose coefficients are structure function moments of increasing rank,

$$\Box_{\gamma Z}^{\rm A(DIS)}(E) = \frac{3}{2\pi}\int_{Q_0^2}^\infty dQ^2 \frac{v_e(Q^2)\alpha(Q^2)}{Q^2(1+Q^2/M_Z^2)} \times\left[M_3^{(1)}(Q^2) + \frac{2M^2}{9Q^4}(5E^2-3Q^2)M_3^{(3)}(Q^2) + \ldots\right]. \qquad (11a)$$

For completeness, we also quote the result for the vector hadronic correction,

$$\Box_{\gamma Z}^{\rm V(DIS)}(E) = \frac{2ME}{\pi}\int_{Q_0^2}^\infty dQ^2 \frac{\alpha(Q^2)}{Q^4(1+Q^2/M_Z^2)} \times\left[M_2^{(2)}(Q^2) + \frac{2}{3}M_1^{(2)}(Q^2) + \frac{2M^2}{3Q^4}(E^2-Q^2)M_2^{(4)}(Q^2) + \frac{2M^2}{5Q^4}(4E^2-5Q^2)M_1^{(4)}(Q^2) + \ldots\right]. \qquad (11b)$$

In Eqs. (11) the moments of the structure functions are defined as

$$M_i^{(n)}(Q^2) \equiv \int_0^1 dx\, x^{n-2}\mathcal{F}_i^{\gamma Z}(x,Q^2), \quad i=1,2,3, \qquad (12)$$

where $\mathcal{F}_i^{\gamma Z} = \left\{xF_1^{\gamma Z}, F_2^{\gamma Z}, xF_3^{\gamma Z}\right\}$. In approximating the upper limit $x_{\max}$ on the x-integrals in Eqs. (11) by 1, the resulting error is less than 10^{-4} for $Q^2 > 1$ GeV^2. The large-x contributions to $M_i^{(n)}(Q^2)$ become more important for large n; however, the higher moments are suppressed by increasing powers of $1/Q^2$. In practice, the integrals in Eqs. (11) are dominated by the lowest moments, with the $1/Q^2$ corrections being relatively small in DIS kinematics.

Equations (11) are major new results which provide a systematic framework within which to evaluate the radiative corrections. For the axial-vector hadron part, the lowest moment, $M_3^{(1)}(Q^2)$, is the γZ analog of the GLS sum rule [17] for νN DIS, which at LO counts the number of valence quarks in the nucleon. The corresponding quantity for γZ is $\sum_q 2\, e_q\, g_A^q = 5/3$, so that at next-to-leading order (NLO) in the $\overline{\rm MS}$ scheme

$$M_3^{(1)}(Q^2) = \frac{5}{3}\left(1 - \frac{\alpha_s(Q^2)}{\pi}\right), \qquad (13)$$

$$M_3^{(3)}(Q^2) = \frac{1}{3}\left(2\langle x^2\rangle_u + \langle x^2\rangle_d\right)\left(1 + \frac{5\alpha_s(Q^2)}{12\pi}\right),$$

where $\langle x^2\rangle_q = \int_0^1 dx\, x^2\, q_v(x,Q^2)$. Hence the lowest ($n=1$) moment contribution to Eq. (11a) is identical to the MS result [3] in Eq. (4). However, the parameter Q_0^2 in Eq. (11a) has a slightly different interpretation than the mass parameter m^2 of Eq. (4). Here Q_0 corresponds to the momentum above which a partonic representation of the non-resonant structure functions is valid, and above which the Q^2 evolution of parton distribution functions (PDFs) *via* the Q^2 evolution equations is applicable. We take $Q_0^2 = 1$ GeV^2, which coincides with the typical lower limit of recent sets of PDFs [18, 19]. The computation of the vector hadronic contribution to $\Box_{\gamma Z}^{\rm (DIS)}$ proceeds in a similar manner, and will be discussed elsewhere [20].

To evaluate the moments in Eq. (11a) we use several NLO parametrizations of PDFs determined from global fits [18, 19]. The results are summarized in Fig. 1. Variations in the values of $\alpha_s(M_Z^2)$ among the datasets considered had a negligible effect on the $n=1$ value of 0.0033. The $n=3$ moments for different datasets are virtually identical, and give negligibly small contributions.

The E dependent terms in Eq. (11a) should also be small, since these depend on $n \geq 3$ moments. However, the expansion in Eq. (11a) is not strictly valid when $E > Q_0^2/2M$. To describe the E dependence in this region we evaluate the difference $\Box_{\gamma Z}^{\rm A(DIS)}(E) - \Box_{\gamma Z}^{\rm A(DIS)}(0)$ in Eq. (9) by replacing $f(r,t)$ by $f(r,t) - f(r,0)$. The results are indeed small for E in the few GeV region, as the dotted line in Fig. 1 indicates.

For $Q^2 < Q_0^2$ a partonic description of the structure functions is not valid. In particular, since the integral over Q^2 in Eq. (9) extends down to $Q^2 = 0$, and the upper limit on the x-integral, $x_{\max}$, is also limited by Q^2, one requires the behavior of the structure functions at both low x and low Q^2. In the case of the vector $F_2^{\gamma Z}$ structure function, conservation of the two vector currents requires $F_2^{\gamma Z} \sim Q^2$ as $Q^2 \to 0$. By contrast, $F_3^{\gamma Z}$ depends on both vector and axial-vector currents, and the nonconservation of the latter means that no similar constraint exists [16].

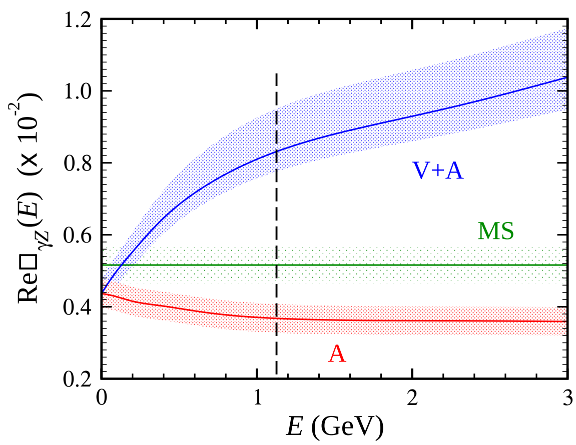

FIG. 2: Total (el+res+DIS) axial-vector hadron correction $\Box^{\rm A}_{\gamma Z}(E)$ (labeled "A") and the sum of axial and vector hadron [8] corrections (labeled "V+A"), together with the $E=0$ result of MS [3] (extended to finite E for comparison). The vertical dashed line indicates the energy at $\rm Q_{weak}$ kinematics.

In the absence of data on $F_3^{\gamma Z}(x,Q^2)$ in the low-x, low-Q^2 region, we consider models for the possible x and Q^2 dependence, obeying the following conditions: (1) $F_3^{\gamma Z}(x_{\rm max},Q^2)$ should not diverge in the limit $Q^2\to 0$; (2) $F_3^{\gamma Z}(x,Q^2)$ should match the partonic structure function at $Q^2=Q_0^2$. For the parametrization of Ref. [18] we note that $F_3^{\gamma Z}(x,Q_0^2)\sim x^{-0.7}$ as $x\to 0$. With this in mind, we consider two models for $Q^2<Q_0^2$.

Model 1 sets

$$F_3^{\gamma Z}(x,Q^2)=\left(\frac{1+\Lambda^2/Q_0^2}{1+\Lambda^2/Q^2}\right)F_3^{\gamma Z}(x,Q_0^2), \qquad (14)$$

which has the property that $F_3^{\gamma Z}(x_{\rm max},Q^2)\sim(Q^2)^{0.3}$ as $Q^2\to 0$. Here Λ^2 is a parameter that can be adjusted to examine the model sensitivity of the integral in Eq. (9). For Λ^2 in the range $(0.4-1.0)$ GeV^2, we obtain a $\pm 10\%$ variation in the values for $\Box^{\rm A}_{\gamma Z}(E)$ shown in Fig. 1.

Model 2 freezes $F_3^{\gamma Z}$ at the $Q^2=Q_0^2$ value for all W^2, which is equivalent to setting $F_3^{\gamma Z}(x,Q^2)=F_3^{\gamma Z}(x_0,Q_0^2)$, with $x_0=xQ_0^2/\left((1-x)Q^2+xQ_0^2\right)$. For this model, $F_3^{\gamma Z}$ is constant as $Q^2\to 0$, and yields a 15% larger contribution to $\Box^{\rm A}_{\gamma Z}(E)$ than Model 1, as illustrated in Fig. 1.

The total correction to $\Box^{\rm A}_{\gamma Z}$ is given by the sum (el+res+DIS), and is shown in Fig. 2 as a function of E. As demonstrated, the E dependence arises predominantly from the elastic and resonance contributions. We assign a very conservative uncertainty estimate equal to twice the low-Q^2 DIS value. This allows for uncertainties in the resonance and low-Q^2 DIS contributions, and in the effect of the running coupling constants on the dominant $n=1$ contribution. The total contribution to $\Box^{\rm A}_{\gamma Z}$ is 0.0044(4) at $E=0$, and 0.0037(4) at $E=1.165$ GeV (the $\rm Q_{weak}$ energy). This should be compared to the value 0.0052(5) used in Ref. [2], which is assumed to be energy independent. Also shown in Fig. 2 is the total $\Box_{\gamma Z}=\Box^{\rm V}_{\gamma Z}+\Box^{\rm A}_{\gamma Z}$ using the result for $\Box^{\rm V}_{\gamma Z}$ from Ref. [8], which has an uncertainty that grows with E.

Our value shifts the theoretical estimate for Q_W^p from 0.0713(8) to 0.0705(8), with a total energy dependent correction $\Box_{\gamma Z}(E)-\Box_{\gamma Z}(0)$ of $0.0040^{+0.0011}_{-0.0004}$ at $E=1.165$ GeV. A similar uncertainty would be obtained using the estimate of $\Box^{\rm V}_{\gamma Z}$ from Ref. [9], while a larger uncertainty on the vector hadron correction was quoted in Ref. [10]. These uncertainties can be reduced with future PV structure function measurements at low Q^2, such as those planned at Jefferson Lab. The high precision determination of Q_W^p would then allow more robust extraction of signals for new physics beyond the Standard Model.

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