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Phys. Rev. Lett. **107**, 077004 — Published 10 August 2011

DOI: 10.1103/PhysRevLett.107.077004

Persistent spin oscillations in a spin-orbit-coupled superconductor

Amit Agarwal,[1] Marco Polini,[2] Rosario Fazio,[1] and G. Vignale[3]

[1] *NEST, Scuola Normale Superiore and Istituto Nanoscienze-CNR, I-56126 Pisa, Italy*
[2] *NEST, Istituto Nanoscienze-CNR and Scuola Normale Superiore, I-56126 Pisa, Italy*
[3] *Department of Physics and Astronomy, University of Missouri, Columbia, Missouri 65211, USA*

Quasi-two-dimensional superconductors with tunable spin-orbit coupling are very interesting systems with properties that are also potentially useful for applications. In this Letter we demonstrate that these systems exhibit undamped collective spin oscillations that can be excited by the application of a supercurrent. We propose to use these collective excitations to realize persistent spin oscillators operating in the frequency range of 10 GHz − 1 THz.

PACS numbers: 74.20.-z,73.20.Mf,71.45.-d,71.70.Ej

Introduction. — Spin-orbit-coupled two-dimensional (2D) electron gases (EGs) are the focus of great interest in the field of semiconductor spintronics [1]. This interest has been largely fueled by the hope to realize the visionary Datta-Das "spin transistor" [2] in which the on/off state is achieved by purely-electrical control of the electron's spin in a spin-orbit-coupled semiconductor channel placed between ferromagnetic leads. Research in spin-orbit-coupled 2DEGs has been recently revitalized by theoretical [3] and experimental [4] studies of the spin Hall effect, in which a current traversing the sample generates a spin-current in the orthogonal direction.

The study of the interplay between spin-orbit coupling (SOC) and superconductivity in 2D systems, stemming from the seminal works of Edelstein [5] and Gor'kov and Rashba [6], has also gained impetus [7]. There is a large variety of systems in which SOC and superconductivity coexist: two examples of great current interest are i) 2DEGs in InAs or GaAs semiconductor heterostructures that are proximized by ordinary s-wave superconducting leads [8, 9] – a class of systems which plays a key role in the quest for Majorana fermions [10] – and ii) 2DEGs that form at interfaces between complex oxides [11], such as $LaAlO_3$ and $SrTiO_3$, which display tunable SOC [12] and superconductivity [13].

Motivated by this body of experimental and theoretical literature, we investigate the collective spin dynamics of an archetypical 2DEG model Hamiltonian with Rashba SOC and s-wave pairing [6], in the presence of *repulsive* electron-electron (e-e) interactions. In the absence of superconductivity a Rashba 2DEG exhibits spin oscillations, which, at long wavelength and for weak repulsive interactions, have a frequency $\approx 2\alpha k_{\rm F}$, α being the strength of SOC and $k_{\rm F}$ the 2D Fermi wavenumber in the absence of SOC. These oscillations, however, are damped and quickly decay due to the emission of (double) electron-hole pairs, which, in the normal phase, are present at arbitrary low energies. In this Letter we demonstrate that in a Gor'kov-Rashba superconductor (GRSC), collective spin oscillations continue to exist in a wide range of parameters, and are undamped because they lie inside the superconducting gap where no other excitation exists. Fig. 1 shows schematically the nature of the spin oscillations in a GRSC. At variance with the Cooper pairs of a standard s-wave semiconductor, the pairs of a GRSC are in a mixture of singlet and triplet states. It is this feature that enables the pairs to respond to an oscillating magnetic field applied, say, in the $\hat{\boldsymbol{y}}$ direction. In the course of the oscillation the spins of a pair tilt in opposite directions, in a pair-breaking motion that creates a net spin polarization along the $\hat{\boldsymbol{y}}$ axis. The spin polarization produces an exchange field, which, if the electron-electron interaction is sufficiently strong, sustains oscillations of the appropriate frequency in the absence of an external field. The essential point is that these oscillations are undamped as long as their frequency falls below the quasiparticle gap: they will therefore display an extraordinarily long lifetime [14].

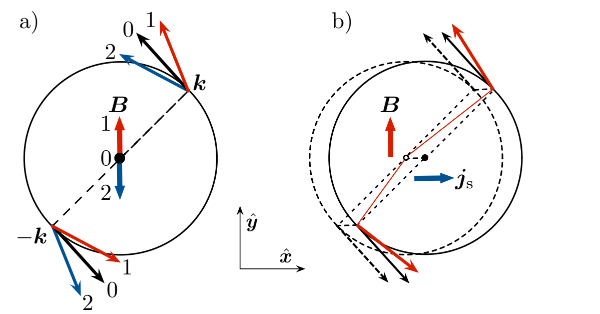

FIG. 1: (color online) a) Response of a Cooper pair in the $\lambda = +$ chirality subband of a Gor'kov-Rashba superconductor subjected to an oscillating magnetic field in the $\hat{\boldsymbol{y}}$ direction. The solid circle is the Fermi surface and the black dot is the origin of momentum space. The arrows labeled by "1", "2", and "0" describe the orientation of the spins under the action of a magnetic field that points up, down, or vanishes. Spontaneous oscillations are sustained, in the absence of a magnetic field, by the internal exchange field. b) A supercurrent boosts the Fermi surface in the $\hat{\boldsymbol{x}}$ direction (solid line) and creates a magnetic field in the $\hat{\boldsymbol{y}}$ direction. As a result, spins begin to oscillate around the new equilibrium orientation, indicated by the thick red arrows.

In order to excite these long-lived spin modes one could in principle apply a short magnetic pulse, but there is also a purely-electrical method. Namely, a supercurrent pulse applied, say, in the $\hat{\boldsymbol{x}}$ direction, will generate, *via* the Edelstein effect [5] an effective magnetic field pulse in the $\hat{\boldsymbol{y}}$ direction, and this should be sufficient to start the spin oscillations. This excitation mechanism is illustrated in Fig. 1b). The Fourier spectrum of the supercurrent pulse must not contain frequencies of the order of (or larger than) twice the superconducting gap to avoid the creation of quasiparticle excitations. We suggest that the new collective spin mode can be used to realize "persistent spin oscillators" operating in the frequency range of 10 GHz − 1 THz (for superconductors with a critical temperature in the range $10^{-1} - 10$ K).

Model Hamiltonian and effective low-energy theory. — We consider the following model Hamiltonian: $\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \hat{\mathcal{H}}_{\rm p} + \hat{\mathcal{H}}_{\rm e-e}$. Here $\hat{\mathcal{H}}_0$ is the kinetic energy term given by $\hat{\mathcal{H}}_0 = \sum_{i,j}\int d^2\boldsymbol{r}\ \hat{\psi}^\dagger_i(\boldsymbol{r})\ h_{ij}(\boldsymbol{r})\ \hat{\psi}_j(\boldsymbol{r})$, where ($\hbar = 1$ throughout this manuscript)

$$h_{ij}(\boldsymbol{r}) = \frac{(-i\boldsymbol{\nabla}_{\boldsymbol{r}})^2}{2m}\delta_{ij} + \alpha\ [\boldsymbol{\sigma}_{ij} \times (-i\boldsymbol{\nabla}_{\boldsymbol{r}})]\cdot\hat{\boldsymbol{z}} - \mu\ \delta_{ij}\ . \quad (1)$$

Here $\hat{\psi}^\dagger_i(\boldsymbol{r})$ [$\hat{\psi}_j(\boldsymbol{r})$] creates (destroys) an electron with real-spin label $i =\uparrow,\downarrow$ and band mass m, α measures the strength of Rashba SOC, $\boldsymbol{\sigma} = (\sigma^1, \sigma^2)$ is a 2D vector of 2×2 Pauli matrices σ^a, μ is the chemical potential, and $\hat{\boldsymbol{z}}$ is a unit vector normal to the 2D plane where electrons are confined to move (the $\hat{\boldsymbol{x}}-\hat{\boldsymbol{y}}$ plane). Diagonalization of $\hat{\mathcal{H}}_0$ yields two bands, $\xi_\lambda(k) = \boldsymbol{k}^2/(2m) + \lambda\alpha k - \mu$, $\lambda = \pm 1$ being the so-called "chirality" index. Rashba SOC forces spins to lie on the $\hat{\boldsymbol{x}} - \hat{\boldsymbol{y}}$ plane and to be perpendicular to $\boldsymbol{k}$ at each point in momentum space [see Fig. 1a)].

The second term in the Hamiltonian $\hat{\mathcal{H}}$, $\hat{\mathcal{H}}_{\rm p}$, is an s-wave pairing Hamiltonian which is responsible for superconductivity: it physically corresponds to an attractive interaction of strength $-g$ with $g > 0$, which is active only in a thin shell of momentum space around the Fermi surface. The microscopic mechanism responsible for the appearance of the pairing term is not important here. The problem defined by $\hat{\mathcal{H}}_0 + \hat{\mathcal{H}}_{\rm p}$ has been studied by Gor'kov and Rashba [6] who calculated the in-plane and out-of-plane spin susceptibilities $\chi_{\parallel(\perp)}(q = 0, \omega \to 0)$. Due to a mixture of spin-singlet and spin-triplet channels stemming from SOC, the GRSC develops a finite and anisotropic spin response.

In this Letter we study the spin response of a GRSC at *finite* frequency ω, taking into account also repulsive e-e interactions described by the last term in the Hamiltonian $\hat{\mathcal{H}}$,

$$\hat{\mathcal{H}}_{\rm e-e} = V\int d^2\boldsymbol{r}\ \hat{\rho}_\uparrow(\boldsymbol{r})\hat{\rho}_\downarrow(\boldsymbol{r})\ , \quad (2)$$

where $V > 0$ and the spin-resolved density operator is defined by $\hat{\rho}_i(\boldsymbol{r}) = \hat{\psi}^\dagger_i(\boldsymbol{r})\hat{\psi}_i(\boldsymbol{r})$. We are interested in studying the collective dynamics of the system described by $\hat{\mathcal{H}}$ assuming that it remains in a phase characterized by a *hard* (finite in any direction of space) gap, despite the presence of repulsive e-e interactions. These lead to an effective reduction of the parameter g, in the spirit of the Anderson-Morel pseudopotential [15].

We now derive an effective low-energy action corresponding to the full Hamiltonian $\hat{\mathcal{H}}$ in terms of spin degrees-of-freedom only. The first step is to decouple the two quartic terms, $\hat{\mathcal{H}}_{\rm p}$ and $\hat{\mathcal{H}}_{\rm e-e}$, by means of a suitable Hubbard-Stratonovich (HS) transformation (see *e.g.* Refs. 16, 17). For the pairing term $\hat{\mathcal{H}}_{\rm p}$ we introduce the complex HS field $\Delta_0(\boldsymbol{r},\tau)$, which describes the superconducting order parameter [17]. We do the decoupling in the chiral basis: this allows us to work with Cooper pairs that are protected by time-reversal symmetry [6]. Transforming back to the real-spin basis we get spin-triplet pairing in addition to the regular spin-singlet pairing [6].

It is useful to rewrite $\hat{\mathcal{H}}_{\rm e-e}$ as [16],

$$\hat{\mathcal{H}}_{\rm e-e} = \frac{V}{4}\int d^2\boldsymbol{r}\ \left\{\hat{\rho}^2(\boldsymbol{r}) - \Big[\sum_{a=1}^{3}\hat{s}_a(\boldsymbol{r})\zeta_a\Big]^2\right\}\ , \quad (3)$$

where $\hat{\rho}(\boldsymbol{r}) = \sum_i\hat{\rho}_i(\boldsymbol{r})$ is the total-density operator, $\hat{s}_a(\boldsymbol{r}) = \sum_{i,j}\hat{\psi}^\dagger_i(\boldsymbol{r})\sigma^a_{ij}\hat{\psi}_j(\boldsymbol{r})$ is the usual spin-density operator, and $\boldsymbol{\zeta} = (\zeta_1, \zeta_2, \zeta_3)$ is an arbitrary unit vector in 3D space. To decouple $\hat{\mathcal{H}}_{\rm e-e}$ by means of HS transformation we introduce four real HS fields [16]: $\phi(\boldsymbol{r},\tau)$ and $\boldsymbol{M}(\boldsymbol{r},\tau)$, which are conjugate to density fluctuations and spin fluctuations, respectively.

The notation is considerably simplified by defining a four-component spinor $\hat{\Psi}^\dagger(\boldsymbol{r},\tau) = [\hat{\psi}^\dagger_\uparrow\ \hat{\psi}^\dagger_\downarrow\ \hat{\psi}_\uparrow\ \hat{\psi}_\downarrow]$ in real-spin space. The exact microscopic action corresponding to $\hat{\mathcal{H}}$ after the HS transformation can now be expressed in a compact form as (the variables $\boldsymbol{r},\tau$ will be suppressed from now on when needed for brevity)

$$\mathcal{S} = \int_0^\beta d\tau\int d^2\boldsymbol{r}\Big[\frac{|\Delta_0|^2}{g} + \frac{\phi^2 + \boldsymbol{M}\cdot\boldsymbol{M}}{V} + \bar{\Psi}\frac{(-G_0^{-1} + \Sigma_0)}{2}\Psi\Big]\ , \quad (4)$$

where $\beta = (k_{\rm B}T)^{-1}$, $\Sigma_0(\boldsymbol{r},\tau) = i\phi(\tau^3\otimes\mathbb{1}_\sigma)$, and $\bar{\Psi}$ is the Grassmann variable corresponding to the fermionic field $\hat{\Psi}^\dagger$. Here G_0^{-1} is the Green's function of the problem defined by $\hat{\mathcal{H}}_0 + \hat{\mathcal{H}}_{\rm p}$ [6] and is a 4×4 matrix given by

$$\begin{aligned} -G_0^{-1} \ &= \ \partial_t\mathbb{1}_\tau\otimes\mathbb{1}_\sigma + \tau^3\otimes h + \alpha\ \{\boldsymbol{\Gamma}\times(-i\boldsymbol{\nabla})\cdot\hat{\boldsymbol{z}}\} \\ &+ \frac{\tau^1 + i\tau^2}{2}\otimes\boldsymbol{\Delta} + \frac{\tau^1 - i\tau^2}{2}\otimes\bar{\boldsymbol{\Delta}}\ . \end{aligned} \quad (5)$$

The Pauli matrices τ^a act in the 2×2 Nambu-Gor'kov space and $\mathbb{1}_\sigma$ ($\mathbb{1}_\tau$) is the identity matrix in real-spin (Nambu-Gor'kov) space, $\boldsymbol{\Gamma} = (\Gamma^1, \Gamma^2, \Gamma^3) \equiv (\tau^3\otimes\sigma^1, \mathbb{1}_\tau\otimes\sigma^2, \tau^3\otimes\sigma^3)$ and $\boldsymbol{\Delta}$ is a 2×2 matrix whose diagonal (off-diagonal) elements are related to the triplet (singlet) order parameter [see Eq. (S5) in Ref. 18].

At low energies, fluctuations of the amplitude of the order parameter $\Delta_0(\boldsymbol{r},\tau)$ do not play any role while phase fluctuations give rise to the Bogoliubov-Anderson mode [17]. To this end, we write $\Delta_0(\boldsymbol{r},\tau) = \Delta e^{i\theta(\boldsymbol{r},\tau)}$, with Δ real. The amplitude Δ is fixed by the saddle-point equation $\delta\mathcal{S}/\delta\Delta = 0$, which yields the BCS equation [6] [see Eq. (S4) in Ref. 18].

The role of the phase field $\theta(\boldsymbol{r},\tau)$ can be made explicit in the action $\mathcal{S}$ by performing the following gauge transformation $\hat{\varphi}_i(\boldsymbol{r},\tau) = \hat{\psi}_i(\boldsymbol{r},\tau)e^{i\theta(\boldsymbol{r},\tau)/2}$ to new fermionic fields $\hat{\varphi}_i(\boldsymbol{r},\tau)$. Writing the action $\mathcal{S}$ in terms of the new fermionic fields generates new self-energies in the round brackets in the second line of Eq. (4): $-G_0^{-1} + \Sigma_0 \to -G_0^{-1} + \Sigma$, where $\Sigma = \Sigma_1 + \Sigma_2 + \Sigma_3$ with

$$\begin{aligned}\Sigma_1(\boldsymbol{r},\tau) &= \left[i\Big(\frac{1}{2}\partial_\tau\theta + \phi\Big) + \frac{(\nabla_{\boldsymbol{r}}\theta)^2}{8m}\right]\tau^3\otimes\mathbb{1}_\sigma \\ &- \frac{i}{2m}\left[\frac{\boldsymbol{\nabla}_{\boldsymbol{r}}^2\theta}{2} + (\boldsymbol{\nabla}_{\boldsymbol{r}}\theta)\cdot\boldsymbol{\nabla}_{\boldsymbol{r}}\right]\mathbb{1}_\tau\otimes\mathbb{1}_\sigma\ , \quad (6)\end{aligned}$$

$$\Sigma_2(\boldsymbol{r},\tau) = \boldsymbol{M}\cdot\boldsymbol{\Gamma} \ \text{ and } \ \Sigma_3(\boldsymbol{r},\tau) = \frac{\alpha}{2}\left[\boldsymbol{\Gamma}\times(\boldsymbol{\nabla}_{\boldsymbol{r}}\theta)\right]\cdot\hat{\boldsymbol{z}}\ . \quad (7)$$

The fermionic part of the action can be integrated out (since it corresponds to a Gaussian functional integral for the partition function) leaving us with the following effective action

$$\begin{aligned}\mathcal{S}_{\rm eff} = \int_0^\beta d\tau\int d^2\boldsymbol{r}\ &\left[\ \frac{\Delta^2}{g} + \frac{\phi^2 + \boldsymbol{M}\cdot\boldsymbol{M}}{V}\right] \\ &- \frac{1}{2}{\rm Tr}\left[\ln\left(-G_0^{-1}+\Sigma\right)\right]\ , \quad (8)\end{aligned}$$

where the symbol "Tr" means a trace over all degrees of freedom (including space and imaginary time).

To make further progress we need to expand the last term in $\mathcal{S}_{\rm eff}$ in powers of Σ. We keep terms up to second order in the Fourier components of the fields $\phi_{\boldsymbol{q}}, \theta_{\boldsymbol{q}}$ and $\boldsymbol{M}_{\boldsymbol{q}}$. A remarkable simplification occurs in the $q\to 0$ limit where the action reduces to the sum of independent quadratic terms (see Sect. II in Ref. 18). Density and supercurrent oscillations on one hand and spin oscillations on the other hand decouple. As usual, the frequencies of collective modes are determined by the isolated poles of appropriate susceptibilities. For short range interactions, the density/current modes disperse linearly in q and their frequency vanishes at $q=0$ as expected for a regular Goldstone mode. The spin modes, on the other hand, have a finite frequency, which increases with increasing Δ [consistent with the fact that the resistance of Cooper pairs to the twisting motion described in Fig. 1a) increases with increasing Δ], but remains less than 2Δ, ensuring long lifetime.

Collective spin oscillations. — In the $q\to 0$ limit all the mixed response functions vanish (see Sect. II in Ref. 18) and the frequency of the collective spin mode $\omega_\parallel$ ($\omega_\perp$) at $q=0$ is given by the solution of the equation

$$2V^{-1} - \chi_{\parallel(\perp)}(0,\omega) = 0 \quad (9)$$

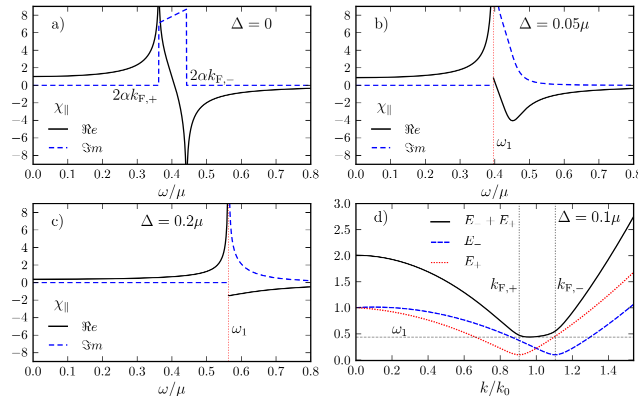

FIG. 2: (color online) Panel a) - c) The in-plane dynamical spin susceptibility $\chi_\parallel(0,\omega)$ [in units of the 2D density-of-states $m/(2\pi)$] as a function of ω (in units of μ) for increasing values of Δ (in units of μ) and $V=0$. The solid line represents $\Re e\ \chi_\parallel(0,\omega)$, while the dashed line represents $\Im m\ \chi_\parallel(0,\omega)$. Note that for finite Δ, $\Re e\ \chi_\parallel(0,\omega)$ diverges at $\omega=\omega_1$ and that $\Im m\ \chi_\parallel(0,\omega) = 0$ for $0<\omega<\omega_1$. Panel d) The quantity $E_+(k)+E_-(k)$ as a function of k (in units of k_0). In this figure we have fixed $\alpha = 0.2\mu/k_0$ with $k_0 = \sqrt{2m\mu}$.

with respect to ω. In passing, we note that Eq. (9) can also be obtained diagrammatically from a vertex equation obtained by summing up ladder diagrams (see Sect. III in Ref. 18). In Eq. (9), $\chi_\parallel = \chi_{\sigma^1\sigma^1} = \chi_{\sigma^2\sigma^2}$ and $\chi_\perp = \chi_{\sigma^3\sigma^3}$ are the in-plane and out-of-plane dynamical spin susceptibilities of the GRSC described by $\hat{\mathcal{H}}_0 + \hat{\mathcal{H}}_{\rm p}$, respectively. These are obtained from the analytical continuation, $i\nu_m \to \omega + i0^+$, of the corresponding expressions in imaginary frequency:

$$\begin{aligned}\chi_{\sigma^a\sigma^b}(0,i\nu_m) &= -\frac{1}{2\beta A}\sum_{\boldsymbol{k},n}{\rm Tr}\Big[\Gamma^a G_0(\boldsymbol{k},i\epsilon_n + i\nu_m/2) \\ &\times\ \Gamma^b G_0(\boldsymbol{k},i\epsilon_n - i\nu_m/2)\Big]\ , \quad (10)\end{aligned}$$

where "Tr" implies a trace over spin and Nambu-Gor'kov indices and ν_m (ϵ_n) is a bosonic (fermionic) Matsubara frequency. After analytic continuation we find, at $T=0$,

$$\begin{aligned}\chi_\parallel(0,\omega) &= -\frac{1}{8\pi}\int_0^\infty kdk\left(1 - \frac{\xi_+\xi_- + \Delta^2}{E_+E_-}\right) \\ &\times\left(\frac{1}{\omega + i0^+ - \mathcal{E}} - \frac{1}{\omega+i0^+ + \mathcal{E}}\right) \quad (11)\end{aligned}$$

and $\chi_\perp(0,\omega) = 2\chi_\parallel(0,\omega)$ [$\mathcal{E} \equiv E_+(k) + E_-(k)$ and $E^2_\lambda(k) \equiv \xi^2_\lambda(k) + \Delta^2$]. Due to the relation between out-of-plane and in-plane spin response functions, we will discuss only collective in-plane excitations.

We calculate $\chi_\parallel(0,\omega)$ numerically from Eq. (11) and plot its real and imaginary parts in Fig. 2. In the limit $\Delta=0$ (*i.e.* absence of superconductivity) – see panel a) – the imaginary part is non-zero only in the interval of frequencies between $2\alpha k_{\rm F,+}$ and $2\alpha k_{\rm F,-}$ [$k_{\rm F,\pm}$ being the minority (majority) Fermi wave vectors for the two Rashba

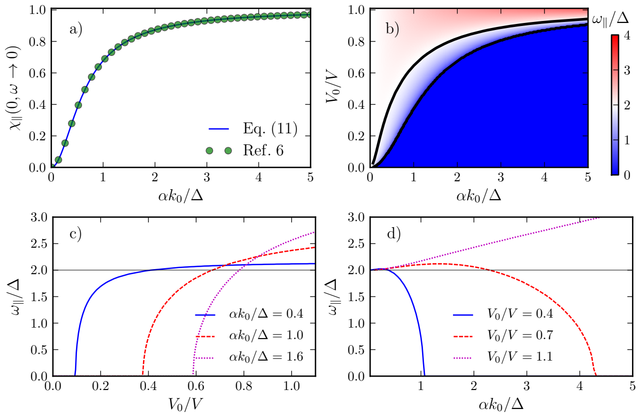

FIG. 3: (Color online) Panel a) The $\omega \to 0$ limit of $\chi_\parallel(0,\omega)$ as a function of $\alpha k_0/\Delta$. The solid line is the result obtained from Eq. (11) while the filled circles are the result of Ref. 6. Panel b) A 2D color plot of the frequency $\omega_\parallel$ of the in-plane collective spin mode (in units of Δ) as a function of the inverse of the strength of electron-electron repulsions (V_0/V, with $V_0 = 4\pi/m$) and SOC ($\alpha k_0/\Delta$). In this plot $\Delta = 0.1\ \mu$. The top contour line is for $\omega_\parallel = 2\Delta$ while the bottom contour line defines the boundary of the region in which $\omega_\parallel = 0$. The collective spin mode is undamped when it lies within the superconducting gap ($0 < \omega_\parallel < 2\Delta$), *i.e.* when $\omega_\parallel$ falls in the region enclosed by the two contour lines. Panels c) and d) represents 1D cuts of the plot in panel b).

bands $\xi_\lambda(k)$] and the real-part exhibits (logarithmic) singularities at these boundaries (see Sect. IV in Ref. 18). When this result is inserted in Eq. (9), one finds a collective spin mode, which is undamped within this approximation. In a more refined theory (beyond Gaussian fluctuations), however, low-energy double electron-hole excitations damp this mode. We now show that, at odds with the normal phase, in the superconducting state the mode lies (for a wide range of parameters) within the superconducting gap and thus cannot be damped by these excitations.

In panels b) - c) we plot $\chi_\parallel(0,\omega)$ for finite Δ. In the superconducting state $\Re e\ \chi_\parallel(0,\omega)$ exhibits a divergence at $\omega_1 \equiv \min_k[E_+(k) + E_-(k)]$. In panel d) we plot $E_+(k) + E_-(k)$ as a function of k. In the region $0 < \omega < \omega_1$, $\Im m\ \chi_\parallel(0,\omega)$ is identically zero and, since $\Re e\ \chi_\parallel(0,\omega)$ diverges for $\omega \to \omega_1$, there is always an in-plane collective spin mode with frequency $\omega_\parallel \approx \omega_1$ for weak repulsive interactions V. Our results for the frequency of the in-plane collective mode $\omega_\parallel$ as a function of V and α (for a fixed value of Δ) are summarized in Fig. 3. Note that there is a wide range of parameters such that $\omega_\parallel$ lies within the superconducting gap, $0 < \omega_\parallel < 2\Delta$. We also have checked that, as expected, $\omega_\parallel$ increases with Δ.

In summary, we have shown that quasi-two-dimensional superconductors with tunable spin-orbit coupling exhibit undamped collective spin oscillations that can be excited by the application of a magnetic field or a supercurrent. The concerted action of spin-orbit coupling and electron-electron interaction is essential to the establishment of these collective oscillations. Since the frequency $\omega_\parallel$ of these oscillations is of the order of the superconducting gap Δ we expect that our findings might enable the realization of long-lived spin oscillators operating in the frequency range of 10 GHz - 1 THz.

We acknowledge financial support by the EU FP7 Programme under Grant Agreement No. 215368-SEMISPINNET (A.A. and M.P.), No. 234970-NANOCTM and No. 248629-SOLID (R.F), and by the NSF under Grant No. DMR-0705460 (G.V.).

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