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I. L. Aleiner, B. L. Altshuler, and K. B. Efetov

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Localization and critical diffusion of quantum dipoles in two dimensions

I.L. Aleiner,¹ B.L. Altshuler,¹ and K. B. Efetov^{2,3}

¹*Physics Department, Columbia University, New York, N.Y. 10027, USA*

²*Theoretische Physik III, Ruhr-Universität Bochum, 44780 Bochum, Germany*

³*International Institute of Physics, UFRN, 59.078-400 - Natal/RN- Brazil*

We discuss quantum propagation of dipole excitations in two dimensions. This problem differs from the conventional Anderson localization due to existence of long range hops. We found that the critical wavefunctions of the dipoles always exist which manifest themselves by a scale independent diffusion constant. If the system is T-invariant the states are critical for all values of the parameters. Otherwise, there can be a “metal-insulator” transition between this “ordinary” diffusion and the Levy-flights (the diffusion constant logarithmically increasing with the scale). These results follow from the two-loop analysis of the modified non-linear supermatrix σ -model.

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Anderson [1] showed that a quenched disorder can localize a quantum particle, *i.e.* completely suppress its diffusion. Later [2, 3], it was realized that in two dimensions (2D) localization occurs for an arbitrary weak disorder. This conclusion was reached by studying the scaling behavior of the dimensionless Thouless conductance $g(L)$ as a function of the linear size of the system L (the observable electrical conductance of the system of the e charged particles is given by $g \times e^2/\hbar$, and we will set Planck constant $\hbar = 1$ hereinafter). Localization implies that $g_L \rightarrow 0$ as $L \rightarrow \infty$. This is always true when the time reflection symmetry (T-invariance) is broken (so called unitary ensemble, GUE). For T-invariant systems it is still correct if the orbital and spin degrees of freedom are decoupled or when the particles have an integer spin (orthogonal ensemble, GOE). For the particles with half-integer spin, the theory [4] predicts that the spin-orbital coupling causes antilocalization $g(L \rightarrow \infty) \rightarrow \infty$ if disorder is weak, while for a stronger disorder $g(L \rightarrow \infty) \rightarrow 0$ (metal-insulator transition for symplectic ensemble) [5].

Besides the current carrying charged particles important objects in the many-body theory are neutral excitations (NEX) – the bound states of two particles with opposite charges. One can name excitons in semiconductors, optical phonons in polar crystals, dipole excitations in granular superconductors, vacancy-interstitial excitations in Wigner crystals or vortex lattices, etc. Being the lowest energy excitations, the NEX not only determine the low temperature energy transport but also provide a thermal bath for more energetic charge excitations, thus influencing the charge transport. As for any neutral particles, the number of NEX is not conserved (*e.g.* electron and hole can annihilate each other). Each NEX has a finite energy and cannot simply disappear. However, their number non-conservation facilitates long range hops mediated by virtual photons (this leads also to dipole-dipole interactions). In this Letter we investigate the effect of the long-range hops on the localization of NEX.

We will be interested in 2D quantum dipoles – NEX whose annihilation [creation] operators $\hat{b}_\alpha(\mathbf{r})$ [$\hat{b}_\alpha^\dagger(\mathbf{r})$] are

characterized by the additional index $\alpha = x, z$. The pair (\hat{b}_x, \hat{b}_z) transforms under rotations similar to a vector in 2D plane (x, z) . For a small density of NEX, we can neglect the interaction between them and use a bilinear form of $\hat{b}_\alpha, \hat{b}_\alpha^\dagger$ as the Hamiltonian $\hat{\mathcal{H}} = \int d^2\mathbf{r}_1 d^2\mathbf{r}_2 \hat{b}_\alpha^\dagger(\mathbf{r}_1) H_{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2) \hat{b}_\beta(\mathbf{r}_2)$, (we imply summation over the repeated indices $\alpha, \beta = x, z$ hereinafter). The absorption-emission of the virtual photons (with infinite speed) results in a long-range hopping term [6]

$$H_{\alpha\beta}^{lr} = \lambda (\delta_{\alpha\beta} |\mathbf{r}|^2 - 2r_\alpha r_\beta) / (2\pi |\mathbf{r}|^4), \quad \mathbf{r} \equiv \mathbf{r}_1 - \mathbf{r}_2, \quad (1)$$

additional to the local Hamiltonian

$$H_{\alpha\beta}^{sr} = \delta(\mathbf{r}_1 - \mathbf{r}_2) [H_{\alpha\beta}^0(-i\nabla_2) + V_{\alpha\beta}(\mathbf{r}_2)]. \quad (2)$$

Constant λ in Eq. (1) encodes dipole-photon transitions matrix elements and the energy denominators.

The hops (1) qualitatively modify the Anderson localization. We derived and analyzed the renormalization group (RG) equations describing the scaling of the Thouless conductance $g(L, \lambda)$. For the GOE we found the *stable* fixed line $g_c(\lambda)$ on the $g - \lambda$ plane, which corresponds to the critical state, see Fig. 1a. Remarkably, $g_c(\lambda)$ may be large, which makes this line accessible in the perturbative RG. For the GUE, we discover the *unstable* line $g_{\text{mit}}(\lambda)$ separating the antilocalization and the localization behaviors, see Fig. 1b. We argue that the latter is the precursor of the critical state similar to the one in the GOE, which, unfortunately, occurs at $g \lesssim 1$ and thus, is out of reach of the perturbative RG analysis.

The quantum motion of the dipoles in the rotationally symmetric clean system is described by

$$H_{\alpha\beta}^0 = \frac{\mathbf{k}^2 \delta_{\alpha\beta}}{2m_1(\mathbf{k})} + \frac{k_\alpha k_\beta}{2m_2(\mathbf{k})} + \hbar [\hat{\sigma}_y]_{\alpha\beta}, \quad (3a)$$

where $m_{1,2}(\mathbf{k})$ are analytic functions of the momentum \mathbf{k} , invariant with respect to the lattice symmetry group. The Pauli matrices $\hat{\sigma}_i$, $i = x, y, z$ act in 2×2 space of dipole components xz . The last term in Eq. (3a) breaks

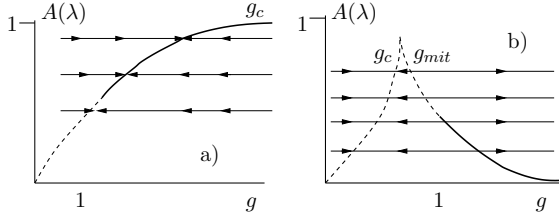


FIG. 1. RG flows for (a) GOE and (b) GUE, and $A(\lambda) < 1$ is defined in Eq. (10). Solid lines are accessible by the perturbative RG, and dashed ones are conjectures.

the T-invariance by the magnetic field $\propto h$ (The Hamiltonian of NEX cannot contain the vector potential). At $h = 0$, we can diagonalize $H^0 + H^{lr}$ (see Fig. 2a):

$$E_-(\mathbf{k}) = \frac{\mathbf{k}^2}{2m_1(\mathbf{k})}; \quad E_+(\mathbf{k}) = E_-(\mathbf{k}) + \frac{\mathbf{k}^2}{2m_2(\mathbf{k})} + \lambda. \quad (3b)$$

Note that $E_{\pm}(\mathbf{k})$ are analytic functions of \mathbf{k} , even though H^{lr} lifts the degeneracy at $k = 0$ protected by symmetry.

The 2×2 matrix $\hat{V}(\mathbf{r})$ in xz -space is a Gaussian disorder breaking all the system spatial symmetries,

$$\langle \hat{V}(\mathbf{r}) \otimes \hat{V}(\mathbf{r}') \rangle = \delta(\mathbf{r} - \mathbf{r}') \sum_{i=0,x,y,z} u_i^2 \hat{\sigma}_i \otimes \hat{\sigma}_i, \quad (3c)$$

where $\hat{\sigma}_0 \equiv \hat{\mathbb{1}}$. For T-invariant systems, $u_y = 0$. The rotational symmetry after the disorder averaging requires $u_x = u_z$. The elastic mean free time τ is given by $1/\tau = \pi\nu \sum_{i=0,\dots,z} u_i^2$, with the total density of states being

$$\nu(\epsilon) = \sum_{\pm} \int \delta[\epsilon - E_{\pm}(\mathbf{k})] d^2\mathbf{k} / (2\pi)^2. \quad (4)$$

We assume that the disorder is weak, $\epsilon\tau \gg 1$.

For the specific form of Eq. (1), the time evolution $\hat{U}(t) = \exp\left(it \int d^2\mathbf{r}_1 d^2\mathbf{r}_2 \hat{b}_{\alpha}^{\dagger}(\mathbf{r}_1) H_{\alpha\beta}^{lr}(\mathbf{r}_{12}) \hat{b}_{\beta}(\mathbf{r}_2)\right)$ can be described locally by introducing additional fields

$$\hat{U}^{lr}(t) = \int \mathcal{D}\mathbf{a}(\mathbf{r}) \mathcal{D}\mathbf{a}^*(\mathbf{r}) \exp\left(\frac{i\lambda t}{2} \int d^2\mathbf{r} \hat{c}^{\dagger} \hat{M} \hat{c}\right); \quad (5)$$

$$\hat{c}^{\dagger} = [(a_x^*, a_z^*); (b_x^{\dagger}, b_z^{\dagger})]^{ab}; \quad \hat{M} = \begin{pmatrix} \nabla^2 \hat{\sigma}_z & -\nabla_{\alpha} \hat{\sigma}_{\alpha} \\ \nabla_{\alpha} \hat{\sigma}_{\alpha} & \mathbb{1} \end{pmatrix}^{ab}$$

Due to the locality of the Poisson equation describing virtual photons, operator \hat{M} is differential rather than integral one and this enables us to develop a renormalizable theory of localization.

To understand the effect of the long hops (1) on the localization, consider the dipole with high energy, $\epsilon \gg \lambda$, see Fig. 2a. For the clean system, the wave functions are comprised by plane waves with wavevectors $k_{\pm}(\epsilon)$ and oscillate rapidly. The long-range hops are thus irrelevant.

In the presence of disorder a dipole is characterized by the probability density $\mathbf{n}(\mathbf{r})$ of finding it at a point \mathbf{r} .

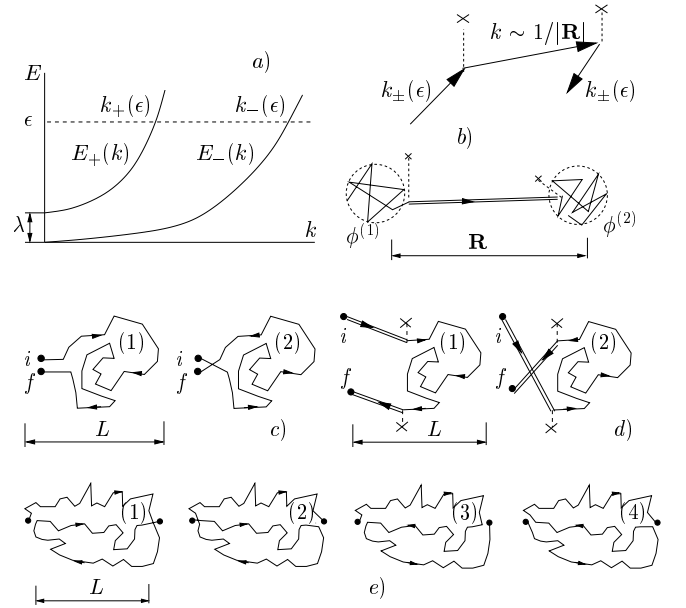


FIG. 2. a) Band structure for the clean system; b) Long hop assisted by the impurities (crosses); c) Semiclassical paths for one-loop weak localization; d) Long range hops and semiclassical paths for two-loop renormalization; e) Two loop semiclassical paths including only local evolution.

Under the assumption of Markovian time evolution,

$$\partial_t n(\mathbf{r}) = \int d^2\mathbf{R} W(\mathbf{R}) [n(\mathbf{r} + \mathbf{R}) - n(\mathbf{r})]. \quad (6a)$$

At large enough distances and times, Eq. (6a) reduces to the diffusion equation with diffusion constant D determined by the rates kernel $W(\mathbf{R})$:

$$D = \int d^2\mathbf{R} W(\mathbf{R}) (\mathbf{R}^2/4). \quad (6b)$$

To estimate $W(\mathbf{R})$, consider the transition between wave-packets $\phi_{\alpha}^{(1,2)}$, formed by plane-waves close to the energy shell and normalized $\int d^2\mathbf{r} \phi_{\alpha}^{(i)} [\phi_{\alpha}^{(i)}]^* = 1$, see Fig. 2b. The disorder can *e.g.* scatter $\phi_{\alpha}^{(1)}$ into a virtual small momentum state, this state can be transferred far by the Hamiltonian (1), and then scattered by disorder back onto the energy shell (upper part of Fig. 2b). The amplitude for this process, $J(\mathbf{R})$, is given by

$$J \simeq \frac{\lambda \left(\mathbf{R}^2 v_{\alpha}^{(1)} (v_{\alpha}^{(2)})^* - 2v_{\alpha}^{(1)} R_{\alpha} R_{\beta} (v_{\beta}^{(2)})^* \right)}{2\pi\epsilon^2 |\mathbf{R}|^4}, \quad (7)$$

where $v_{\alpha}^{(j)} = \int d^2\mathbf{r} \phi_{\alpha'}^{(j)} V^{\alpha'\alpha}$. We substitute Eq. (7) into the golden rule formula, average it over the disorder (3c), and use the normalization of $\phi_{\alpha}^{(j)}$. We find

$$W_{lr}(\mathbf{R}) = 2\pi\nu \langle |J(\mathbf{R})|^2 \rangle = \lambda^2 / (4\pi^3 \nu \epsilon^4 \tau^2 |\mathbf{R}|^4), \quad (8)$$

Substituting Eq. (8) into Eq. (6b) and taking into account only hops with $L < |R| < L + \delta L$, we obtain a positive correction to the Thouless conductance $g = \nu D$:

$$\delta g_{lr} = \delta \ell A / (2\pi^2); \quad \ell = \ln(L/L_0), \quad (9)$$

where L_0 is a microscopic scale, and [see Eq. (8)] $A = \lambda^2 / (4\tau^2\epsilon^4)$; $\tau\epsilon \gg 1$, $\epsilon \gg \lambda$. For ϵ in resonance with E_{\pm} , the summation of all orders of perturbation theory in λ gives the Breit-Wigner type formula

$$A = \frac{4\lambda^2\tau^2}{[4 + \tau^2\xi_+^2][4 + \tau^2\xi_-^2]} < 1, \quad \xi_{\pm} \equiv \epsilon - E_{\pm}(0), \quad (10)$$

i.e. even at $\lambda > 1$ ($\epsilon \rightarrow \lambda$), the contribution of the long-range hops cannot exceed the unitary limit.

Levy flight term (9) is not the only logarithmical contribution to the conductance. The interference of close time-reversed paths ($h, u_y = 0$) controlled by local term H^{sr} (Fig. 2c) yields the weak localization correction [3]

$$\delta g_{wl} = -\delta \ell / (2\pi^2); \quad (11)$$

Since $A < 1$, long hops (9) never overcome weak localization but can almost compensate it if $|A - 1| \ll 1$.

In this case, it is worthwhile to evaluate the two-loop contribution. When included as intermediate steps into interference contribution of Fig. 2c, the long range hops simply change the bare diffusion constant D . However, δg_{wl} does not contain D at all, *i.e.* irreducible interference processes are important. One of them, Fig. 2e, vanishes as the path (1) interferes destructively with path (2) and constructively with paths (3), (4). It is the long hop part of the Hamiltonian, Fig. 2d, that leads to the logarithmic correction to g . Modified non-linear σ -model described below yields a new RG equation for GOE

$$\frac{\partial g}{\partial \ell} = \frac{A - 1}{2\pi^2} + \frac{A}{8\pi^4 g} + \mathcal{O}(1/g^2), \quad g \gg 1. \quad (12)$$

yielding the RG flow of Fig. 1a and the *stable* fixed line

$$g_c(A) = A / [4\pi^2(1 - A)]. \quad (13)$$

Note, that $g_c \gg 1$ in the limit $1 - A \ll 1$ *i.e.* the two loop approximation is sufficient. At $g < 1$, Eq. (13) is not applicable. However, there is a strong reason to believe that the critical line terminates at $g = 0$, $A = 0$ point. Indeed, if $g < 1$, the local part of the evolution can produce only small contributions $\delta g \simeq \delta \ell g \ln g$. Arguments leading to the estimate (9) remain valid. Indeed, one can choose the exact localized eigenfunctions of the H_{sr} instead of the wave packets and obtain

$$\partial g / \partial \ell = A / (2\pi^2) + \mathcal{O}(g \ln g) > 0, \quad g \ll 1, \quad (14)$$

i.e., $g = 0$ is unstable for any $A > 0$. As $\partial g / \partial \ell < 0$ for $g \gg 1$, see Eq. (12), the fixed line has the form of Fig. 1a.

Application of the magnetic field removes the time reversal symmetry, $h, u_y \neq 0$, and thus suppresses the contribution of time reversal paths of Fig. 2 c,d at distances larger than $L_h = \sqrt{D/\omega_h}$, where $\omega_h \simeq h^2\tau + \pi\nu u_y^2$. The interference between path (1) and paths (3,4) is also destroyed and there is no more cancelation of the local interferences in the second loop. We find for GUE

$$\frac{\partial g}{\partial \ell} = \frac{A}{2\pi^2} - \frac{1}{8\pi^4 g}, \quad g \gg 1, \quad (15)$$

similarly to results of Ref. [7] for an electron in random magnetic field with long-range correlations.

Equation (15) has an *unstable* fixed line

$$g_{mit}(A) = (4\pi^2 A)^{-1}, \quad (16)$$

corresponding to the metal-insulator transition. This result is controllable for $A \ll 1$. The RG flow of Fig. 2b follows from Eq. (16) and Eq. (14) for $g < 1$.

Let us discuss the relation of our results to earlier works. At first glance, the long jumps are equivalent to artificial models with long-range random links known as random band matrices, RBM [8, 9], (the matrix elements are the Gaussian variable with $\langle h_{ij}^2 \rangle \simeq 1/|i - j|^\alpha$, $\langle h_{ij} \rangle = 0$) The dipole hop model (1) is different as it contains an infinite number of the long hop loops (*e.g.* the product of the matrix elements $\langle h_{12}h_{23}h_{34}h_{41} \rangle$ for non-coinciding points 1, ..., 4 vanishes for RBM and it is finite for the dipolar interaction) and determines all of the interference contributions of Fig. 2d. The quantum dipole problem with long range hops was considered by Levitov [10]. He started from the strongly localized states, $g \rightarrow 0$, and replaced the strongly localized pairs of Fig. 2b by one effective dipole on each linear scale. This approach misses the logarithmic contributions of the multiple short range hops. We believe that in such a way one can show the instability of $g = 0$ state but can not obtain the critical line ($g \propto \lambda$ claimed in Ref. [11] is neither Thouless conductance nor ac-conductance. Both those quantities are $\propto \lambda^2$, see Eq. (14)).

Supersymmetric non-linear σ ($nl\sigma$) model is usually formulated [12] in terms of the 8-component supervector. The 8-dimensional space where this vector resides can be presented as a direct product of 3 two-dimensional subspaces, RA , N and g , for the retarded-advanced, Gorkov-Nambu, and the fermion-boson sectors. The matrix structure of the Hamiltonians (1) and (3), brings up an additional two dimensional subspace (without subscript), so the resulting supervector b is 16-component. One more supervector a with the same structure as b is introduced to decouple the long-range hops as it is shown in Eq. (5). Those two vectors can be united in one 32-dimensional supervector $\psi^T = (a^T, b^T)^{ab}$. The disorder

averaging is then performed as

$$\begin{aligned} \langle \dots \rangle &= \int \dots \exp(-L[\psi]) \mathcal{D}\psi, \quad \psi^\dagger \hat{\Lambda} = \bar{\psi} = [\hat{\mathbb{C}}\psi]^T, \\ \hat{\Lambda} &= \hat{\Lambda} \otimes \mathbb{1}^{ab} \otimes \mathbb{1}; \quad \hat{\Lambda} = \hat{\sigma}_z^{RA} \otimes \mathbb{1}^N \otimes \mathbb{1}^g; \\ \hat{\mathbb{C}} &= \hat{C} \otimes \mathbb{1}^{ab} \otimes \mathbb{1}; \quad \hat{C} = \mathbb{1}^{RA} \otimes (\hat{\sigma}_-^N \otimes \mathbb{1}^g - \hat{\sigma}_+^N \otimes \hat{\sigma}_z^g), \end{aligned} \quad (17a)$$

where \dots in the LHS stands for any combination of advanced/retarded Green functions $\hat{G}^{A,R} = (\varepsilon \mp \omega/2 - \hat{H}_0 - \hat{V} - \hat{H}_{lr} \mp i0)^{-1}$ and \dots in the RHS for the corresponding sources [12], whose form is not important for RG. Term L_0 in the Lagrangian $L = L_0 + L_{int}$ describes the property of the clean system

$$\begin{aligned} L_0[\psi] &= \int d^2\mathbf{r} \left\{ \bar{\psi} \left[i\lambda \hat{\mathbb{M}}/2 + \hat{\Lambda} 0^+ \right] \psi + i\bar{\mathbf{b}} \hat{\mathbb{H}}_0 (-i\nabla) \mathbf{b} \right\}; \\ \mathbb{H}_0 &= \left[\left(\frac{k^2}{2m_1} - \epsilon \right) \hat{I} - \frac{\omega}{2} \hat{\Lambda} \right] \otimes \mathbb{1} + \left[\frac{\mathbf{k}^T \otimes \mathbf{k}}{2m_2} \right] \otimes \hat{I} \\ &+ h\hat{\sigma}_y \otimes \hat{\Sigma}_3; \quad \hat{\mathbb{M}} = \begin{pmatrix} \nabla^2 \hat{\sigma}_z & \nabla_\alpha \hat{\sigma}_\alpha \\ -\nabla_\alpha \hat{\sigma}_\alpha & \mathbb{1} \end{pmatrix}^{ab} \otimes \hat{I}, \end{aligned} \quad (17b)$$

where $\hat{I} \equiv \mathbb{1}^{RA} \otimes \mathbb{1}^N \otimes \mathbb{1}^g$, and $\hat{\Sigma}_3 = \mathbb{1}^{RA} \otimes \hat{\sigma}_z^N \otimes \mathbb{1}^g$. The term L_{int} originates from the disorder averaging (3c):

$$L_{int}[\psi] = \frac{1}{2} \sum_{i=0,x,y,z} u_i^2 \int (\bar{b}\hat{\sigma}_i b)^2 d^2\mathbf{r}. \quad (17c)$$

As for the conventional $n\sigma$ -model the interaction is decoupled by introduction the additional field $\hat{Q} \propto \mathbf{b} \otimes \bar{\mathbf{b}}$ and fixing $Q^2 = \hat{I}$ in the saddle point approximation is valid as long as $\epsilon\tau \gg 1$. For $\mathbf{a} = 0$, the field \mathbf{b} for a fixed Q is massive and can be integrated out producing convergent gradient expansion in $n\sigma$ model. However, field \mathbf{a} is massless and its fluctuations lead to the logarithmic corrections discussed above. Therefore, only “fast” field ψ with the momenta close to the energy shell $k_\pm(\epsilon)$ can be included into the effective theory for Q , whereas fields with the smaller momenta must be kept in the theory:

$$\langle \dots \rangle = \int \dots e^{-F[\hat{Q}] - \mathcal{L}[\psi, \hat{Q}]} \mathcal{D}\hat{Q} \mathcal{D}\psi, \quad \hat{Q}^2 = \hat{I}; \quad (18a)$$

$$\hat{Q} = \hat{C} \hat{Q}^T \hat{C}^T; \quad \hat{Q}^\dagger = \hat{\mathbb{K}} \hat{Q} \hat{\mathbb{K}}; \quad \hat{\mathbb{K}} = \begin{pmatrix} \mathbb{1}^g & 0 \\ 0 & \hat{\sigma}_z^g \end{pmatrix}^{RA} \otimes \mathbb{1}^N.$$

Here, ψ is the 32-component *smooth* on the scale of the mean free path field satisfying the constraints of Eq. (17b); \hat{Q} is the smooth 8×8 supermatrix [12]. The entries in Eq. (18a) are

$$\begin{aligned} F &= \frac{\pi}{8} \text{str} \int d^2\mathbf{r} \left\{ g \left(\nabla \hat{Q} \right)^2 + \nu \left[2i\omega \hat{\Lambda} \hat{Q} - \omega_h \left(\hat{\Sigma}_3 \hat{Q} \right)^2 \right] \right\}; \\ \mathcal{L} &= \frac{i\lambda}{2} \int d^2\mathbf{r} \bar{\psi}(\mathbf{r}) \mathbb{B}(\hat{Q}) \psi(\mathbf{r}); \\ \mathbb{B} &= \hat{\mathbb{M}} + \frac{2}{\lambda} \begin{pmatrix} 0; & 0 \\ 0; & -\left(\epsilon \hat{I} + \frac{i\hat{Q}(\mathbf{r})}{2\tau} \right) \otimes \mathbb{1} + h\hat{\sigma}_y \otimes \hat{\Sigma}_3 \end{pmatrix}^{ab}, \end{aligned} \quad (18b)$$

where the bare conductance is given by

$$g = \sum_{\pm} \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{\tau}{2} \frac{\partial E_{\pm}}{\partial k_{\alpha}} \frac{\partial E_{\pm}}{\partial k_{\alpha}} \delta[\epsilon - E_{\pm}(\mathbf{k})].$$

Note that, in contrast to non-local *RBM* models [8], the theory (18) is renormalizable. Indeed, $F[\hat{Q}]$ of Eq. (18a) includes only relevant terms allowed by $\hat{Q}^2 = \hat{I}$. The same applies to the Lagrangian \mathcal{L} as the natural dimensions of fields $[\nabla \mathbf{a}] = [\mathbf{b}] = 1$ while \mathbf{a} itself can not enter: \mathcal{L} is invariant with respect to constant shift of \mathbf{a} .

We checked that the fluctuations of Q -matrix do not change the Lagrangian \mathcal{L} and, thus, the coefficient A in Eqs. (12)-(15). This relates to the fact that the averaged density of states cannot have corrections from mixing of retarded and advanced sectors described by Q matrix.

In order to sum up leading logarithmic divergences, we performed two-loop RG analysis of theory (18b), see Ref. [13] for details. We represent $\hat{Q} = \hat{V} \left[(1 + i\hat{P})/(1 - i\hat{P}) \right] V$ and $\psi = \psi_{<} + \hat{V}\psi_{>}$, where $\psi_{<}, \hat{V}$ are slow variables and the other are fast. We integrated out the fast variables in the second loop approximation to obtain the effective free energy for slow ones. Varying the result of the integration with respect to a gauge invariant cutoff we obtain Eqs. (12) and (15).

In conclusion, we performed the scaling analysis of the localization problem of 2D quantum dipoles, see Fig. 1. For T-invariant systems, the Thouless conductance tends to a finite value. Breaking the T-invariance leads to the transition between the antilocalization and a critical behavior. To describe those phenomena we constructed a novel version of the non-linear σ -model, Eq. (18), and derived the second loop RG equations. This model also allows studies [13] of the multifractal properties of the wave-function using methods of Ref. [12].

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