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Majorana edge states in interacting one-dimensional systems

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We show that one-dimensional electron systems in proximity of a superconductor that support Majorana edge states are extremely susceptible to electron-electron interactions. Strong interactions generically destroy the induced superconducting gap that stabilizes the Majorana edge states. For weak interactions, the renormalization of the gap is nonuniversal and allows for a regime, in which the Majorana edge states persist. We present strategies how this regime can be reached.

Introduction. The possibility of realizing Majorana bound states at the ends of one-dimensional (1D) conductors formed by topological insulator edge states, semiconductor nanowires or carbon nanotubes in the proximity of a superconductor [1–8], as well as by quasi-one-dimensional superconductors [9] has led recently to much activity. An important factor for the interest is the potential application of the Majorana edge states as elementary components of a topological quantum computer [7, 10–13]. In a nanowire the Majorana edge modes exist because of the p-wave nature of the induced superconductivity, which is the result of the projection of the superconducting order parameter onto the band structure of the wire, consisting of helical, i.e., spin (or Kramers doublet) filtered left and right moving conducting modes. In such a setup, the Majorana edge states appear as particle-hole symmetric Andreev bound states at both ends of the wire, with a localization length ξ inversely proportional to the induced superconducting gap Δ, and their wave function overlap is proportional to $\exp(-L/\xi)$ with L the wire length. The independence and the particle-hole symmetry of the two bound states is only guaranteed if this overlap is vanishingly small, therefore large L and Δ are required.

Electron-electron interactions strongly renormalize the properties of a one-dimensional conductor [14]. In particular, it has been shown that classifications of the topological phases in interacting and non-interacting systems differ greatly [15, 16]. Further, since the elementary excitations in 1d interacting systems are generally collective excitations of bosonic character, the fate of the fermionic Majorana edge states is not obvious. In this paper we quantitatively answer this question. We focus on interaction effects in system with helical conduction states that are in contact with a superconductor. We show that the induced gap Δ is substantially reduced, and thus the Majorana edge states gets delocalized. The physics in this regime can usually only be described qualitatively. Remarkably, however, within the renormalization group analysis we show that it is possible to map the interacting system by refermionization onto an effective noninteracting fermion system *before* the strong coupling limit is reached. Due to this, we not only can prove the existence of the Majorana edge states in the interacting system, but also can quantitatively describe their wave function and extension ξ. Counterintuitively, the relevant gap size determining ξ is not the strong coupling value but the value $\Delta = \Delta(l_1)$ (see below) at which the system is mapped on the effective noninteracting system.

This result gives a precise prescription by how much ξ increases for given interaction strength and induced gap size Δ. To reach this regime and to guarantee minimal overlap of both Majorana edge states, an experiment should aim for a large induced Δ, best screened electron-electron interactions and, roughly, a system length L exceeding the minimal length estimate of the noninteracting system by at least a factor of 10.

In the following, we first illustrate the effect of electron interactions on the Majorana bound states using the fermion chain model of Ref. [10]. In particular, we show that for strong interactions the gap can entirely close and the system becomes equivalent to a gapless free electron gas. Motivated by this insight, we turn to a continuum theory for the nanowires, allowing us to include the interactions more effectively and to move beyond the restriction to a half-filled chain.

Fermionic chain. The prototype model for Majorana edge states is a one-dimensional open lattice of sites $i = 1, \ldots, N$ described by the model [10, 17]

$$H = -\sum_{i=1}^{N-1} \left[t c_i^\dagger c_{i+1} + \Delta c_i^\dagger c_{i+1}^\dagger + \text{h.c.} \right] - \mu \sum_{i=1}^{N} n_i, \quad (1)$$

where c_i are tight-binding operators of spinless fermions, for example the electron operators of the helical conduction bands, $t > 0$ is the hopping integral, $\Delta > 0$ the triplet superconducting gap, μ the chemical potential, and $n_i = c_i^\dagger c_i$. In terms of the Majorana fermion basis [18] $\gamma_i^1 = c_i + c_i^\dagger$ and $\gamma_i^2 = i(c_i - c_i^\dagger)$, the model is rewritten as $H = -i \sum_{i=1}^{N-1} \left[w_+ \gamma_i^2 \gamma_{i+1}^1 - w_- \gamma_i^1 \gamma_{i+1}^2 \right] - i\frac{\mu}{2} \sum_{i=1}^{N} \gamma_i^2 \gamma_i^1$, with $w_\pm = (t \pm \Delta)/2$. At $t = \Delta$ and $\mu = 0$, the only nonzero interaction is w_+, and the ground state corresponds to pairing of Majorana fermions between neighboring sites $\gamma_i^2 \gamma_{i+1}^1$, with an excitation gap of $2w_+$. In the open chain, γ_1^1 and γ_N^2 no longer appear in H and remain unpaired. They form the two Majorana bound states that are localized on a single lattice site at each edge of the wire and can be occupied at no energy cost. For $\mu \neq 0$ or $\Delta \neq t$, the two edge Majorana modes are coupled to the bulk system and their spatial extension becomes larger, on the order of $\xi \sim a/\ln|w_+/w|$, with $w = \max\{|\mu|, |w_-|\}$ and a the lattice constant [10]. In the finite system, the overlap of the two Majorana states at both ends of the chain is proportional to $e^{-Na/\xi}$, and the two states are independent only for $Na \gg \xi$.

In such a system, interactions between the fermions critically affect the existence and stability of the Majorana edge states. Indeed, they lead not only to a further coupling of the Majorana edge states to the bulk system, but also can substantially reduce the bulk gap size. As an illustration, we include into the model the repulsive nearest neighbor interaction $H' = U\sum_{i=1}^{N-1}(n_i - 1/2)(n_{i+1} - 1/2)$, with $U > 0$. It is now straightforward to show that interactions can entirely close the superconducting gap. For strongly interacting $t = \Delta = U/4$ we can map H by a Jordan-Wigner transformation to the spin chain $H = t\sum_{i=1}^{N-1}(\sigma_i^x\sigma_{i+1}^x + \sigma_i^z\sigma_{i+1}^z)$, where $\sigma_i^{x,y,z}$ are spin 1/2 operators (normalized to ±1) defined by $c_i = \frac{1}{2}(\sigma_i^x + i\sigma_i^y)\prod_{j<i}\sigma_j^z$. By a further Jordan-Wigner transformation to new fermion operators $\tilde{c}_i = \frac{1}{2}(\sigma_i^z + i\sigma_i^x)\prod_{j<i}\sigma_j^y$ we then see that $H = -2t\sum_{i=1}^{N-1}(\tilde{c}_i^\dagger\tilde{c}_{i+1} + \tilde{c}_{i+1}^\dagger\tilde{c}_i)$, which describes a free gapless fermion gas in which the localized states have disappeared. Although we have selected special interactions strengths, it is well known that in one dimension the renormalization due to weaker interactions can drive the system into such a gapless phase. To quantitatively include this renormalization and to allow a treatment beyond the half-filled ($\mu = 0$) case, we use in the following a continuum description, first at half filling, then away from half filling.

Continuum model. For the continuum theory, we focus on a quantum wire with Rashba spin-orbit interaction in a magnetic field with proximity induced singlet superconductivity [3–6]. Since the interacting system eventually allows a mapping onto an effective Fermi liquid, we first discuss the noninteracting case by reducing the previously considered models [3–6] to a minimal model that captures the same physics in a transparent way. The noninteracting part of the Hamiltonian for the quantum wire can be written as a sum of two parts, $H_0 = H_0^{(1)} + H_0^{(2)}$, where $H_0^{(1)}$ is given by (throughout the paper $\hbar = 1$)

$$H_0^{(1)} = \int dr \Psi_\alpha^\dagger \left[\left(\frac{p^2}{2m} - \mu\right)\delta_{\alpha\beta} + \alpha_R p\,\sigma_{\alpha\beta}^x - \Delta_Z\sigma_{\alpha\beta}^z\right]\Psi_\beta, \tag{2}$$

where Ψ_α is the electron operator for spin α, the summation over repeated spin indices, α, β, is assumed, r is the coordinate along the wire, $p = -i\partial_r$, α_R is the spin-orbit velocity, and Δ_Z is the Zeeman energy of the magnetic field applied along the spin z direction perpendicular to the spin-orbit selected spin x direction. The second part, $H_0^{(2)}$, includes the induced singlet superconducting term with order parameter Δ_S and is expressed as, $H_0^{(2)} = i\int dr \Delta_S \Psi_\alpha^\dagger \sigma_{\alpha\beta}^y \Psi_\beta^\dagger/2 +$ h.c. Without interactions, $H_0^{(1)}$ has the eigenvalues $\epsilon_\pm = p^2/2m \pm \sqrt{(\alpha_R p)^2 + (\Delta_Z/2)^2}$ and corresponding eigenmodes $\Psi_\pm(p)$. Expanding the singlet superconducting term in this eigenbasis leads to superconducting order parameters of the triplet (within Ψ_- and Ψ_+ subbands) as well as of the singlet type (mixing Ψ_- and Ψ_+ subbands). The Majorana edge states require triplet pairing [2–7, 19, 20], which is achieved by tuning the chemical potential to lie within the magnetic field gap such that only the Ψ_- subband is occupied. In Ref. [6], Majorana edge modes were derived using the full Hamiltonian $H_0^{(1)} + H_0^{(2)}$ and were shown to exist in the limit $\Delta_Z > \sqrt{\Delta_S^2 + \mu^2}$. The same physics is also obtained by restricting to the occupied Ψ_- subband, which will be assumed in the following. For $\Delta_Z \gg \Delta_S,\ \alpha_R k_F$, with $k_F \approx \sqrt{m\Delta_Z}$, the pairing then takes the compact form [2–7, 19, 20]

$$H_0^{(2)} \approx (\Delta/k_F)\int dr \Psi_-^\dagger(r) p \Psi_-^\dagger(r) + \text{h.c.}, \tag{3}$$

with the effective triplet superconducting gap $\Delta = \Delta_S(\alpha_R k_F/\Delta_Z)$.

In the following we work in the diagonal basis [21] with the fermions confined in the $r > 0$ region. The open boundary condition forces the fermion fields to vanish at both ends of the wire, $\Psi_-(r = 0) = \Psi_-(r = L) = 0$. In terms of the slowly varying right, $\mathcal{R}(r)$, and left, $\mathcal{L}(r)$, moving fields, the field $\Psi_-(r)$ acquires the form, $\Psi_-(r) = \sum_k \sin(kr)c_-(k) = e^{ik_F r}\mathcal{R}(r) + e^{-ik_F r}\mathcal{L}(r)$, where $c_-(k)$ is the annihilation operator in the Ψ_- subband. We note that $\mathcal{R}(r) = -\mathcal{L}(-r)$. The noninteracting case can therefore be written in terms of $\mathcal{R}(r)$ only as $H_0 = \int_{-L}^{L} dr\ \mathbf{R}^\dagger(r)\mathcal{H}\mathbf{R}(r)$, with

$$\mathcal{H} = \begin{pmatrix} -i\frac{v_F}{2}\partial_r & -\Delta\text{sgn}(r) \\ -\Delta\text{sgn}(r) & i\frac{v_F}{2}\partial_r \end{pmatrix} \tag{4}$$

and $\mathbf{R}(r) = [\mathcal{R}(r), \mathcal{R}^\dagger(-r)]^T$. Using $\mathbf{R}(r) = (e^{i3\pi/4}/\sqrt{2})\sum_\epsilon [u_\epsilon(r), v_\epsilon(r)]^T\gamma_\epsilon$, where the normalized functions $u_\epsilon(r)$ and $v_\epsilon(r)$ satisfy the eigenvalue equation $\mathcal{H}[u_\epsilon(r), v_\epsilon(r)]^T = \epsilon[u_\epsilon(r), v_\epsilon(r)]^T$, we obtain $H_0 = \sum_\epsilon \epsilon\gamma_\epsilon^\dagger\gamma_\epsilon$. For $\epsilon = 0$ there exists a localized mode at each edge. At $r = 0$ it is of the form $u_{\epsilon=0}(r) \propto e^{-2\Delta|r|/v_F}$, with $v_0(r) = iu_0(r)$. The operator corresponding to the edge mode, $\gamma_0 = \int_{-L}^{L} dr u_0(r)\mathcal{R}(r)$, satisfies the Majorana condition $\gamma_0 = \gamma_0^\dagger$. Thus the Majorana edge mode obtained by combining the right and left modes is given by,

$$\Psi_{\epsilon=0}^M(r) = C\gamma_0\sin(k_F r)e^{-r/\xi}, \tag{5}$$

for $L \gg \xi$, where C is the normalization constant and $\xi = v_F/2\Delta$ the localization length. Note that in 1D the decay is purely exponential.

Interaction effects. Next we include interactions between the fermions, given by $\int drdr' V(r-r')\rho(r)\rho(r')$ with $V(r)$ the repulsive potential and $\rho(r)$ the fermion density. To quantitatively include the interactions we bosonize the Hamiltonian, taking into consideration that the low-energy physics is described by a single species of fermions in the Ψ_- subband. Using the standard procedure [14], the bosonic Hamiltonian reads,

$$H = \int \frac{dr}{2}\Big[vK(\partial_r\theta)^2 + \frac{v}{K}(\partial_r\phi)^2 + \frac{4\Delta}{\pi a}\sin(2\sqrt{\pi}\theta) - \frac{U}{\pi^2 a}\cos(4\sqrt{\pi}\phi - 4k_F r)\Big], \tag{6}$$

where a is the lattice constant, the $\partial_r\phi$ field describes the density fluctuations and θ is the conjugated field. The quadratic part in Eq. (6) includes the repulsive interaction $V(r)$ between the fermions ($K<1$) and the velocity v modified by interactions. The sine term in Eq. (6) is due to the triplet superconducting term $H_0^{(2)}$ given in Eq. (3), and the cosine term describes umklapp scattering by $V(r)$.

The umklapp terms play a role only in lattice systems but are absent in quasi-one-dimensional quantum wires fabricated on a two-dimensional electron gas. For fermions on a lattice near half-filling, $4(k_F - \pi/2a)L \ll 1$ and the oscillatory part inside the cosine term can be neglected. The interactions then lead to the renormalization of the coupling constants Δ, U, and K, which by standard renormalization group (RG) theory [14] is expressed by the RG equations

$$\frac{d\ln K}{dl} = \frac{\delta^2}{2K} - 2Ky^2, \tag{7}$$

$$\frac{d\delta}{dl} = (2-\frac{1}{K})\delta, \quad \frac{dy}{dl} = (2-4K)y, \tag{8}$$

where $l = \ln[a/a_0]$ is the flow parameter with a_0 the initial value of the lattice constant. $\delta(l)$ and $y(l)$ are dimensionless quantities at length scale a, defined as $\delta(l) = 4a\Delta(l)/v_F$ and $y(l) = U(l)a/\pi v_F$. The initial values of the rescaled parameters are given by K_0, Δ_0, δ_0, U_0, and y_0. For $K<1/2$ the umklapp term is relevant and superconductivity irrelevant, leading to a Mott phase, whereas for $K>1/2$ the opposite is true and the system is superconducting. Near $K=1/2$ the low-energy physics depends critically on the relative strength of δ_0 and y_0. A large δ_0 compared to y_0 favors superconductivity over the Mott phase and vice-versa. An interesting scenario corresponds to the line of fixed points $\delta_0 = y_0$ and $K_0 = 1/2$, where the parameters remain invariant under the RG flow. Following Refs. [14, 22], we find that under a change of quantization axis the theory is described by a quadratic Hamiltonian. Therefore, similar to the discrete model with $t=\Delta=U/4$, the spectrum is gapless. The Majorana edge states are thus absent on the line of fixed points, as well as in the Mott phase. On the other hand, in the superconducting phase, $K(l)$ grows as well and eventually crosses $K(l_1)=1$ at the scale $a(l_1)$. As we show below, this allows to refermionize the system and to prove the existence of the Majorana edge states.

Away from half-filling, the umklapp term in Eq. (6) becomes strongly oscillating and can be neglected, allowing us to set $y=0$ in Eq. (8). The remaining RG equations reduce to the standard Kosterlitz-Thouless (KT) equations under the change of variables $K \to 1/2\bar{K}$ and $\delta \to \bar{\delta}/\sqrt{2}$ [14]. The flow equation of $\Delta(l)$ differs from $\delta(l)$ due to the difference in the factor of $a(l)$ and is given by, $d\Delta/dl = (1-K^{-1})\Delta$. Its solution in terms of $K(l)$ acquires the form,

$$\Delta(l) = \Delta_0 \frac{\sqrt{8[K(l)-K_0] - 4\ln[K(l)/K_0] + \delta_0^2}}{\delta_0 \exp[l]}. \tag{9}$$

The equation for the separatrix is obtained by choosing $\delta_0 = 0$ and $K_0 = 1/2$ in Eq. (9). For small deviations of K from an arbitrary initial value K_0, l is given by,

$$l \approx \frac{K_0}{\sqrt{\alpha}}\cot^{-1}\left[\frac{\alpha + k_0(k_0+x)}{x\sqrt{\alpha}}\right], \tag{10}$$

where $x = (K-K_0)/K_0$, $k_0 = 2K_0 - 1$, and $\alpha = \delta_0^2/2 - k_0^2$. Rather than linearizing the KT flow eqs. around the

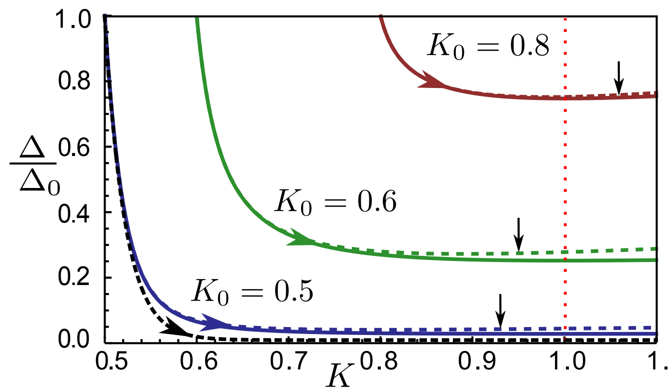

FIG. 1. (Color online) RG flow of Δ/Δ_0 as a function of K for $\Delta_0 = 0.05v_F/a_0$ and the three initial values $K_0 = 0.5$, $K_0 = 0.6$, and $K_0 = 0.8$. The solid lines are obtained from the numerical integration of the KT eqs. The dashed lines are obtained from Eqs. (9) and (10) [the dashed line with the steepest decay for $K_0 = 0.5$ marks an exponential drop, obtained from Eq. (9) with $l \approx (2K_0/\delta_0^2)x$]. The flow reaches the non-interacting limit at $K=1$ (shown by the red dotted line). The vertical arrows indicate the position where $\delta = 1$ is reached.

fixed point as is often done [14], the solutions given by Eqs. (9) and (10) are obtained by integrating the KT equations. Figure 1 shows Δ/Δ_0 as a function of K for $\Delta_0 = 0.05v_F/a_0$ and three different values of K_0, $K_0 = 0.5,\ 0.6$ and 0.8. For all the K_0's considered, Δ reduces from its initial value and acquires its minimum at $K=1$. Note that near $K=1$, Δ shows very little variation. For the strongly repulsive case, $K_0 = 0.5$, Δ is reduced by an order of magnitude as K reaches $K \lesssim 1$. In particular, for $K \approx 0.5$ and $x \ll 1$, Eq. (10) can be approximated as $l \approx (2K_0/\delta_0^2)x$ and thus Δ has an exponential drop. More generally, the exponential decay persists as long as $x \ll \delta_0^2/(2\max\{k_0, \sqrt{|\alpha|}\})$ is satisfied. At $x \sim \delta_0^2/(2\max\{k_0, \sqrt{|\alpha|}\})$, one has to consider the full form for l as given by Eq. (10).

Refermionization. We stress that the mere reduction of Δ does not tell much about the Majorana edge states yet. Indeed, their existence and the shape of their wave function has been derived in a noninteracting system only, and their fate under interactions remains still to be shown. To achieve this, we first note that although everywhere in the repulsive regime ($K < 1$) K has a monotonic increase and Δ a monotonic decrease, the flow can be divided into two regions based on the initial values of δ_0 and K_0. In the first region, characterized by initial values (K_0, δ_0) with $K_0 > 1/2$ (screened regime) or with $K_0 < 1/2$ together with $\delta_0 > 2\sqrt{2K_0 - \ln(2K_0 e)}$ (i.e., above the separatrix), the flow is toward the strong coupling regime $\delta, K \to \infty$. Under the RG, Δ decreases to a minimum at the length scale $a(l_1)$ at which $K(l_1) = 1$, and continues to increase afterwards. We note that $K = 1$ marks a special line where all interactions have scaled to zero, and our bosonic theory can be mapped via the refermionization procedure into an effective noninteracting fermionic system with a superconducting gap $\Delta(l_1)$. Thus, instead of continuing the RG flow to the strong coupling limit we stop the flow at $K(l_1) = 1$ and solve the problem exactly using the renormalized superconducting gap $\Delta(l_1)$. This is justified since the long wave-length physics remains invariant along the flow trajectory. While it would be difficult to extract information about the true electrons from the refermionization mapping, it allows us to prove the existence of the Majorana edge states. The edge wave functions calculated in this way is described very well by Eq. (5) with $\xi = v/2\Delta$, and Δ given by $\Delta(l_1)$. For initial K_0 and Δ_0 the value of $\Delta(l_1)$ is quantitatively calculated using Eqs. (9) and (10). The same refermionization mapping applies at half-filling for $K_0 > 1/2$, where $\Delta(l_1)$ is determined by Eqs. (7) and (8). Our conclusions on the shape of Majorana edge states have indeed been confirmed by a numerical approach [28].

To preserve the Majorana property of the edge states and so their usefulness for quantum computational application [2, 10], the two Majorana states at each end of the system must have minimal overlap, i.e., $2\Delta(l_1)L/v \gg 1$. This can be achieved by increasing the wire length L by at least the factor $\Delta_0/\Delta(l_1)$ as compared with the naive noninteracting picture. This result is valid if the RG flow crosses $K = 1$, which occurs if the length scale $a(l_1)$ is shorter than any cut-off length, i.e., $a(l_1) < \min\{L, L_T, a(l_\delta)\}$ [where l_δ is defined as $\delta(l_\delta) = 1$ and $L_T = v/k_B T$ is the thermal length]. If, however, $a(l^*) = \min\{L, L_T, a(l_\delta)\} < a(l_1)$, the RG is cut-off before $K = 1$ is reached. Since from Fig. 1 we see that in most cases still $\Delta(l^*) \approx \Delta(l_1)$, we expect that the Majorana edge states persist and can be approximated by Eq. (5) with $\Delta = \Delta(l^*)$. This conclusion is also supported by numerics [28].

The second region is the unscreened regime with $K_0 < 1/2$ and $\delta_0 < 2\sqrt{2K_0 - \ln(2K_0 e)}$. Here the flow is towards the line of Luttinger-liquid fixed points, $\Delta = 0$ and $K_0 < K < 1/2$. In a realistic scenario the flow is stopped before the fixed points are reached at a length scale given by, $a(l^*) = \min\{L, L_T\}$. If $a(l^*) = L_T$, then $\Delta(l^*) < k_B T$ and thermal fluctuations overcome superconductivity. On the other hand, if $a(l^*) = L$, then the superconducting term is renormalized down to $\Delta(l^*) \approx \Delta_0(L/a_0)^{1-1/K_0}$. In either case the bulk spectrum remain gapless and all correlations exhibit power-law decay. Thus, the Majorana edge states which require the presence of gapped bulk modes are absent.

One way to ensure a gapped phase in the bulk is to consider a larger value for δ_0. A large δ_0 may be difficult to achieve as the proximity induced gap Δ_S is further suppressed by the small ratio, $\alpha_R k_F/\Delta_Z$. Moreover, in contrast to K_0, controlling and scaling up the strength of the superconducting order parameter is non-trivial. A simpler alternative would be to apply gates on top of the wire to screen the interactions and to increase K_0 to a larger K_0' that pushes the initial point (K_0', δ_0) above the separatrix, $\delta_0 > 2\sqrt{2K_0' - \ln(2K_0' e)}$ or beyond $K_0' > 1/2$, so that the flow is towards the strong coupling regime. After the first preprint of this paper appeared on the arXiv server, other groups arrived at similar conclusions [28–30].

Potential candidate systems for the observation of Majorana edge states are the helical conductors formed at the boundaries of topological insulators [23, 24], InAs nanowires with strong spin-orbit interaction [2, 6, 25, 26], quasi-1D unconventional superconductors [9], carbon nanotubes [8], and quantum wires with nuclear spin ordering [27]. The latter two systems may be particularly interesting because they are readily available and support helical modes without external magnetic fields.

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