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Microscopic Approach to Shear Viscosities Of Unitary Fermi Gases Above and Below the Superfluid Transition

Hao Guo [1], Dan Wulin [1], Chih-Chun Chien[2] and K. Levin[1]

[1] *James Franck Institute and Department of Physics, University of Chicago, Chicago, Illinois 60637, USA and*

[2] *Theoretical Division, Los Alamos National Laboratory, MS B213, Los Alamos, NM 87545, USA*

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Recent experiments on the shear viscosity η in a unitary Fermi gas fail to see the theoretically predicted upturn in η at the lower T. In this paper we compute η in a fashion which is demonstrably consistent with conservation laws and, in the process, provide an understanding of recent experiments. We show that this disagreement with prior theories cannot be readily attributed to the trap, since (via edge effects) trap-averaged viscosities will be larger than their homogeneous counterparts. The small values of η we find can be simply understood; they reflect the fact that the Goldstone bosons (phonons) do not couple to transverse probes such as η, and fermionic excitations, which determine the viscosity are necessarily absent in the ground state.

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One of the most exciting aspects of ultracold Fermi gas experiments near the unitary limit is their strong connection to other physics sub-disciplines. The notion of a "near-perfect fluid", observed in the cold gases is of great interest to physicists working on black holes, on RHIC physics and on quark-gluon plasmas, all of which exhibit anomalously low shear viscosities η [1]. As has been argued in the literature [1], the shear viscosity is an important diagnostic of the nature of microscopic theories. For the relativistic case, much attention has focused on calculations of the shear viscosity which build on the AdS/CFT conjecture [2]. Moreover, because η is associated with anomalously short mean free paths, it has been claimed [1] that many alternative approaches such as "kinetic theory estimates and perturbative computational techniques are doomed to fail".

For the non-relativistic cold Fermi gases, considerable insight concerning thermodynamics and the theory behind various spectroscopic probes has been obtained by extending the formalism of BCS to accomodate arbitrarily strong interactions. In this paper we use this BCS to Bose Einstein condensation (BEC) crossover scheme to address the observed anomalously low viscosity in trapped Fermi gases. Our aim is to build on a significant body of literature which has yielded an understanding of the superfluid shear viscosity (η) [3, 4] in BCS-based systems such as helium-3. Historically, BCS transport theory was validated only after [5] a clarification was presented of the role of Goldstone bosons in leading to gauge invariance and conservation laws. Among the most persuasive checks one has of a given many body theory is consistency with sum rules which reflect conservation laws. In strict BCS theory these sum rules are easily established. Goldstone boson effects are shown to be absent in transverse response functions, so that sum rules are satisfied without their inclusion. The challenge, however, in extending BCS theory is that, because of the stronger-than-BCS attraction, one must include both fermionic and "bosonic" excitations of the condensate as well as of the normal, above T_c phase [6]. These bosons are associated principally with non-condensed pair degrees of freedom.

Our goals in this paper are (1) to outline this promising new formalism for addressing η which is fully consistent with sum rules [7]. (2) To address experimental data (some of which [8] appeared after these predictions) where there are important differences with other homogeneous theories in the literature. (3) To extract important information about the nature of the excitation spectrum (phononic versus fermionic) which strongly constrains microscopic theories of the unitary gas. We also address trap effects and show, as expected, that the trap-integrated viscosity will be artificially *higher* than for the homogeneous case, since η will be dominated by unpaired fermions at the trap edge. Our theory and experiments [8] show that η is monotonically decreasing with $T \to 0$, but we find that η/s appears to be relatively T independent at the lower temperatures. Thus we cannot rule out its upturn due to trap effects. After this paper was submitted we learned of nearly simultaneous work [9] which predicts that *both* the viscosity and the ratio η/s of the unitary gas necessarily exhibit a minimum, somewhat below T_c.

The shear viscosity may be one of the most important clues to our ultimate microscopic understanding of unitary gases and BCS-BEC crossover in general. This is because it reflects the normal fluid component (or excitation spectrum) in a very direct way. While thermodynamic probes do reflect the excitations, this is only in the form of power laws which are less susceptible to easy analysis for a trapped configuration. In a low T normal Fermi liquid phase with scattering lifetime γ^{-1} and effective mass m^*, $\eta = \frac{1}{5} n v_F^2 \gamma^{-1} m^*$. More generally, one can think of η as characterized by the effective number of the normal excitations ($n \to n_{eff}(T)$) as well as their lifetime which we emphasize here is a many body effect. Crucial is an understanding of how n_{eff} depends on T.

Previous BCS-based work [3, 4] allows us to anticipate our simple physical picture. There is no coupling to Goldstone bosons (phonons) in a transverse probe (such as η or the conductivity [10]) and there are no fermionic (or bosonic) excitations at low T; hence η should vanish near the ground state. Indeed, in the superfluid phase of helium-3 [11] η drops off rapidly to zero. In the helium-4 counterpart, the single particle bosonic excitations couple to the collective (Nambu-Goldstone) modes, leading to an upturn [12] in η at low T, which has also been predicted (but not seen) for the atomic Fermi superfluids [9, 13]. In past literature there has been a focus on either the fermionic [14] or bosonic [13] constituents of the unitary gas, Our Kubo-based formalism readily accomodates the simultaneous bosonic and fermionic contributions and thereby addresses n_{eff} as well as the lifetime (through interconversion of bosons and fermions); this Kubo approach includes scattering processes via the lifetimes [15] which appear in the various Green's functions and vertex diagrams.

We stress that, although there is disagreement in the literature [9, 13], our qualitative results for η are straightforward and should be rather generic. The only subtle feature is the role of the non-condensed boson contributions; here we appeal to consistency with the transverse sum rule to support our theoretical approach. A central conclusion is that both the effects of a fermionic gap (with onset temperature $T^* > T_c$) and the non-condensed pairs act in concert to reduce n_{eff} and thus *lower* the shear viscosity at all $T < T^*$. When compared with very recent shear viscosity experiments [8] (we independently infer [16] an estimated lifetime from radio frequency data) this lends further support to our microscopic starting point.

We compute viscosities using the more well controlled current-current correlation functions [7], $\overleftrightarrow{\chi}_{JJ}$ rather than stress-tensor correlation functions, via

$$\eta = -m^2 \lim_{\omega \to 0} \lim_{\mathbf{q} \to 0} \frac{\omega}{q^2} \mathrm{Im} \chi_T(\omega, \mathbf{q})$$

which is importantly constrained by the sum rule [7]

$$\lim_{\mathbf{q} \to 0} \int_{-\infty}^{\infty} \frac{d\omega}{\pi} \Big(- \frac{\mathrm{Im}\chi_T(\omega, \mathbf{q})}{\omega} \Big) = \frac{n_n(T)}{m}, \qquad (1)$$

The transverse susceptibility $\chi_T = (\sum_{\alpha=x}^{z} \chi_{JJ}^{\alpha\alpha} - \chi_L)/2$ with the longitudinal $\chi_L = \hat{\mathbf{q}} \cdot \overleftrightarrow{\chi}_{JJ} \cdot \hat{\mathbf{q}}$. Here $n_n(T)$ corresponds to the number of particles in the normal fluid and $n_n(T) \to n$ above T_c, where n is the total particle number. The current-current correlation function is $\overleftrightarrow{\chi}_{JJ} = \overleftrightarrow{P} + \frac{\overleftrightarrow{n}}{m} + C_J$, where C_J is known [17, 18] and associated with the collective modes. These latter are unimportant for the transverse response. The quantity $\overleftrightarrow{P}$ which represents the paramagnetic current, is of central interest in this paper.

Because dissipative transport in BCS-BEC theory is complex, among the most persuasive checks on a proper characterization of the "normal fluid" is consistency with sum rules. By the same token, because it is more difficult to precisely enforce the counterpart longitudinal sum rule, which also involves Goldstone boson effects, in this paper we do not discuss the bulk viscosity. We note that Eq. (1) is more general and fundamental than sum rules derived in Ref. [19].

Our theoretical scheme is based on the BCS-Leggett ground state, extended [20] to non-zero temperature T. A detailed discussion of the basis for the transport studies here can be found in Ref.10, 17, 18, 20, where the closely-related conductivity and other transport is discussed. For this reason we do not repeat the details. There are two contributions to the square of the pairing gap $\Delta^2(T) = \Delta^2_{\rm sc}(T) + \Delta^2_{\rm pg}(T)$, corresponding to condensed (sc) and to non-condensed (pg) pairs, which are associated with a pseudogap. The fermions have dispersion $E_{\mathbf{p}} \equiv \sqrt{\xi^2_{\mathbf{p}} + \Delta^2(T)}$, where $\xi_{\mathbf{p}} = \epsilon_{\mathbf{p}} - \mu$ (In what follows, we omit the subscript $\mathbf{p}$ for convenience).

Consistency in linear response theory is based on the use of Ward identities which connect transport to the fermionic self energy. Here we adopt the *literature standard* for the self energy in $T > T_c$ theories of the cuprate pseudogap [21]

$$\Sigma(\mathbf{p}, \omega) \equiv -i\gamma + \frac{\Delta^2_{pg}}{\omega + \xi_{\mathbf{p}} + i\gamma} + \frac{\Delta^2_{sc}}{\omega + \xi_{\mathbf{p}} + i0^+}. \qquad (2)$$

The condensed pairs have the usual BCS self energy contribution, Σ_{sc}, while the self energy of the non-condensed pairs Σ_{pg} contains an additional damping term parameterized by γ. In the cold gases, this temperature dependent many body lifetime associated with pair-fermion inter-conversion can be partially quantified by Radio Frequency (RF) "photoemission" experiments [22, 23] for ^{40}K. In ^{6}Li, where the viscosity experiments are performed [8, 24, 25], we have previously estimated [16] this parameter by fitting non-momentum-resolved RF experiments.

To keep the equations simple and transparent we proceed in two stages. The diagram set which satisfies Eq.(1), includes [10, 17, 18, 20], both Maki Thompson (MT) and two Aslamazov-Larkin (AL) diagrams. We begin in the weak dissipation limit, by which we mean the lifetime of the fermionic excitations is very long, so that to leading order we may set $\gamma \approx 0^+$ in Eq.(2). In this weak dissipation limit [17, 18, 20] one can write more

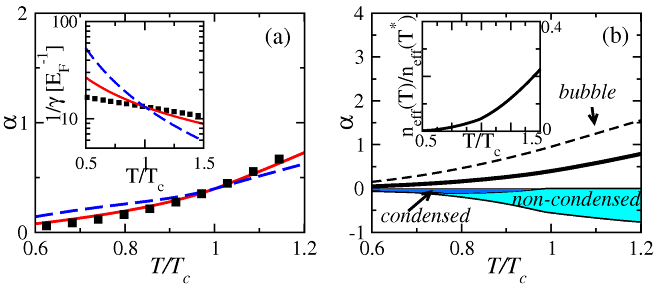

Figure 1: (Color online) (a) Calculated (homogeneous) viscosity $\eta = n\hbar\alpha$ for a unitary gas, illustrating robustness under changes in the value of the lifetime $1/\gamma$. The color coded lifetimes (in units E_F^{-1}) indicated in the inset correspond to their counterparts in the main figure. Here $1/\gamma$ is deduced from fits to RF experiments (black squares) and both linear (red) and quadratic (blue) dependences on T/T_c. (b) Contributions to α (net total = thick black curve) from condensed and non-condensed pairs (shaded) and from the simple bubble diagram (dashed). Inset plots the effective carrier number as $\propto \eta\gamma$, showing the decrease relative to high T.

simply

$$\overleftrightarrow{P}(\omega,\mathbf{q}) = \sum_{\mathbf{p}} \frac{\mathbf{pp}}{m^2}\Big[\frac{E_+ + E_-}{E_+E_-}(1-f_+-f_-) \times\frac{E_+E_- - \xi_+\xi_- - \delta\Delta^2}{\omega^2-(E_++E_-)^2} - \frac{E_+-E_-}{E_+E_-} \times(f_+-f_-)\frac{E_+E_- + \xi_+\xi_- + \delta\Delta^2}{\omega^2-(E_+-E_-)^2}\Big], \quad (3)$$

where $\hbar = 1$, $E_\pm = E_{\mathbf{p}\pm\mathbf{q}/2}$, the Fermi functions, $f_\pm = f(E_\pm)$ and $\delta\Delta^2 = \Delta_{\rm sc}^2 - \Delta_{\rm pg}^2$. If we now take the low ω, q limits, η assumes a form similar to a stress tensor correlation function. We incorporate lifetime effects [15] (which preserve the analytic sum rule consistency) by writing $\delta(\omega \pm \mathbf{q}\cdot\nabla_{\mathbf{p}}E) = \lim_{\gamma\to 0}\frac{\frac{1}{\pi}\gamma}{(\omega\pm\mathbf{q}\cdot\nabla_{\mathbf{p}}E)^2+\gamma^2}$.

Importantly, a rather simple expression for the shear viscosity emerges:

$$\eta = \int_0^\infty dp \frac{p^6}{15\pi^2m^2}\frac{E^2-\Delta_{\rm pg}^2}{E^2}\frac{\xi^2}{E^2}\Big(-\frac{\partial f}{\partial E}\Big)\frac{1}{\gamma} \quad (4)$$

From Eq.(4) one can identify the effective carrier number ($n_{eff}(T) \propto \eta\gamma$) discussed in the introduction, and verify that it is dramatically suppressed at low T. Equation (4) which is our central result, is a generally familiar BCS expression [3, 4] except for the effects associated with the non-condensed pairs. These appear via non-zero $\Delta_{\rm pg}$ which enters as a prefactor $1 - \frac{\Delta_{\rm pg}^2}{E^2}$. This deviation from unity can be traced to the AL diagrams.

The fact that the non-condensed pairs suppress η is a very important observation which comes physically from the fact that when pairs are present there are fewer fermions to contribute to the viscosity. In the weak dissipation limit, we can analytically prove this sum rule for the transverse susceptibility (Eq.(1)). We note that the total number of particles can be written as $n = \sum_{\mathbf{p}}\big(1 - \frac{\xi}{E}(1-2f)\big)$. The superfluid density at general temperatures is given by $n_s = m{\rm Re}P^{xx}(0,0) + n = \frac{2}{3}\frac{\Delta_{\rm sc}^2}{m}\sum_{\mathbf{p}}\frac{p^2}{E^2}\Big(\frac{1-2f}{2E} + \frac{\partial f}{\partial E}\Big)$. Thus from Eq.(3), the left hand side of Eq.(1) is $\sum_{\mathbf{p}}\frac{p^2}{6m^2}\Big[\frac{2\Delta_{\rm pg}^2}{E^2}\frac{1-2f}{E} - 4\frac{E^2-\Delta_{\rm pg}^2}{E^2}\frac{\partial f}{\partial E}\Big] = \frac{n-n_s}{m} = \frac{n_n}{m}$, thereby proving the sum rule. What is essential here is a validation that we have included the effects of non-condensed pairs in a consistent fashion.

Importantly, the expression in Eq.(4) can be generalized to the stronger dissipation limit [10], by introducing generalized Green's functions which represent the various pg and sc contributions. [See Supplementary material]. We stress that calculations based on the strong dissipation approach [10] show very little difference from those obtained using the weak dissipation scheme. This is principally due to nodeless s-wave pairing gap and the inferred size of γ. While one could argue that this one parameter (chosen independently of viscosity measurements) makes the theory less compelling, we stress that the *qualitative disagreement* between alternate theories [9, 13] of η is so vast that at this stage it makes little sense to assess any theory by the flexibility of a single parameter.

In Figure 1(a) we plot the shear viscosity for a unitary gas as computed via the strong dissipation approach. We have verified that the results are similar if we use the simpler form of Eq.(4) directly. The plot is for α defined as $\eta \equiv \alpha n\hbar$ versus temperature for a homogeneous system at unitarity and for a range of different lifetime parameterizations. The inset to Figure 1(a) presents a plot of the RF-deduced lifetime as black squares along with a few alternative functional forms. Each of these corresponds (via color coding) to the plots for α in the main body of the figure. In all cases α drops to zero at low temperatures, although one can see that this is slightly countered by the fact that the fermions are longer lived at low T. This figure should make it clear that the behavior shown here is quite independent of any detailed models for the inverse lifetime γ^{-1}. This largely follows from the intuition already embedded in Eq.(4) showing the dominant effect is due to an exponential decrease in the number of condensate excitations associated with s-wave pairing.

In the inset of Figure 1(b) we plot the effective carrier number defined as $\propto \eta\gamma$ as a function of T. The curve, normalized to the high temperature value where $\Delta = 0$, shows a clear suppression of the carrier number which is associated with the non-condensed pairs. The main body of the figure presents a breakdown of the various contributions to the viscosity coming from this bubble term and from the contributions of condensed (proportional to Δ_{sc}^2) and non-condensed (proportional to Δ_{pg}^2) pairs.

We turn now to calculations in a trap based on the

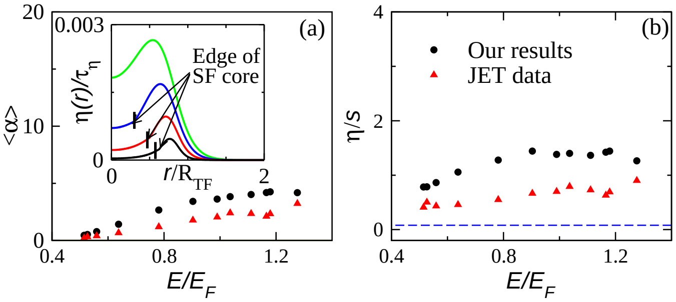

Figure 2: (Color online) (a) Comparison of shear viscosity $\eta \equiv \alpha n \hbar$ and experiments [25] (red triangles) at unitarity for a trapped gas. In theory plots (black dots) we use the calculated thermodynamics for the trap energy E and entropy density s. From bottom to top, the inset in (a) plots the shear viscosity profile (divided by lifetime) at $T/T_F = 0.15$, 0.20, 0.25 and 0.30. The edges of the superfluid core are labeled by black solid lines. (b) Comparison of η/s. The blue dashed line labels the quantum lower limit of η/s given by Ref. [2].

local density approximation. Our calculations incorporate the same trap averaging procedure as used Ref. [25]. Here we have used the calculated trap thermodynamics [26] to rescale the various axes. The inset to Figure 2(a) presents the shear viscosity profile in the trap (divided by lifetime τ_η where $\tau_\eta = 1/\gamma$) at different global trap temperatures for a unitary Fermi gas. This figure shows that the superfluid core contribution to η is considerably smaller than that of the normal cloud at the trap edge. The main body of Figure 2(a) presents a comparison of the viscosity coefficient α between theory (based on the RF-deduced lifetime), as black dots, and experiment [25] (red triangles) as a function of E. When comparing with the homogeneous calculations of Fig. 1, these plots indicate a fairly substantial increase in η from trap effects, simply due to the non-superfluid fermions at the trap edge. Since other (homogeneous) theories find an overestimate of η at low T, relative to experiments, we conclude that including the trap, in their approaches would lead to less, not more, consistency with experiment. Figure 2 (b) shows the comparison of η/s where s is the entropy density. It can be seen that our calculations show a reasonable understanding of these data [25].

There appear to be no other BCS-BEC based calculations of η in the literature which address the entire range of T below T_c and also the consistency check of Eq.(1). Bruun and Smith [14] have studied the above T_c shear viscosity and importantly recognized [14] that the pseudogap reduces η. However, the diagram set which was used was "not conserving"[14]. Rupak and Schafer [13] argued that η is dominated by the Goldstone bosons or phonons and predicted an upturn at the lowest T in both η and η/s. It has to be stressed, however, that in BCS-based fermionic superfluids the Goldstone bosons do not couple to a transverse response such as η. They similarly do not couple to the conductivity [10] or to the spin response [18]. If other modes of low energy excitation (such as phonons) are to be introduced then one must establish that these are consistent with the appropriate sum rules. This paper has established the rather subtle role of the non-condensed bosons (which disappear at $T = 0$) and which are seen to lower η. It has also shown that the viscosity of the superfluid unitary gas may be less similar to helium-4, than to helium-3.

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