Comment on “Do Gluons Carry Half of the Nucleon Momentum?”

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Comment on "Do Gluons Carry Half of the Nucleon Momentum?" by X. S. Chen et. al. (PRL103, 062001 (2009))

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In a recent paper by Chen et al. [1], the textbook definition of a charged-particle's momentum and angular momentum in gauge theories has been questioned [2, 3]. The authors claim they have found a "proper" definition, and challenge the well-known result in perturbative quantum chromodynamics (QCD) that the gluons carry one-half of the nucleon momentum in asymptotic limit [3]. Here I argue that the textbook result stands, and the incorrect conclusion of the paper arises from a misunderstanding of gauge symmetry.

In Ref. [1], a "sound" definition of a charged particle's momentum in a U(1) gauge field A^μ is purported to be (see Eq. (6) in the paper)

$$P_q^\mu = P^\mu - qA^\mu_{\rm pure}/c \ , \tag{1}$$

where P^μ is the canonical momentum and $A^\mu_{\rm pure}$ is "a pure gauge term transforming in the same manner as does the full A^μ" and always gives "null field strength." This magical $A^\mu_{\rm pure}$ allows a "gauge-invariant" definition of P^μ_q and "physical" $A^\mu_{\rm phys} = A^\mu - A^\mu_{\rm pure}$. The authors claim that the quark's P^μ_q shall be measurable in deep-inelastic scattering (DIS) and shall contribute 1/5 of the nucleon momentum.

First of all, separating $\vec{A}$ into $\vec{A}^\mu_{\rm phys} + \vec{A}_{\rm pure}$ cannot be uniquely done by the conditions $\vec{\nabla}\cdot\vec{A}_{\rm phys} = 0$ and $\vec{\nabla}\times\vec{A}_{\rm pure} = 0$, contrary to authors' claim. In fact, one can always add/subtract a term $\vec{\nabla}\phi$ with $\nabla^2\phi = 0$ to change the separation. A simple counter example is that of a constant magnetic field in the z-direction. $\vec{A}_1 = (By, 0, 0)$ and $\vec{A}_2 = (By, -Bx, 0)/2$ both must be "physical" according to the authors. Of course, one can add more constraints to make the separation unique. However, this amounts to defining $\vec{A}_{\rm phys}$ by gauge fixing and performing calculations under a fixed gauge.

Next, what is theoretically sound to define and experimentally measurable in electromagnetism are already well-known. The kinematic momentum of a charge particle is

$$\vec{\pi} = \vec{P} - q\vec{A}/c \ , \tag{2}$$

with the full gauge field $\vec{A}$ required. It is $\vec{\pi}$ which gives rise to the kinetic energy of the particle $E = \vec{\pi}^2/2m$, and it is $\vec{\pi}$ which generates the electric current, $\vec{j} = (q/m)\vec{\pi}$. Feynman in his famous lectures provided a beautiful example (Sec. 21-3) to demonstrate that $\vec{\pi}$ is the momentum related to the velocity of a charge particle measurable experimentally [4]. $A^\mu_{\rm phys}$ has never been considered as a meaningful observable in electromagnetism.

In the context of QCD, it is π^μ which appears in the twist-2 operators of the operator product expansion for deep-inelastic scattering (DIS) [2]. The light-cone plus(+) component of the operators generates the light-momentum of a parton in $A^+ = 0$ gauge. There is no place for P^μ_q (Eq.(1)) in any QCD experimental observables. In particular, the parton distributions advocated by the authors do not appear in any factorization of hard processes [3].

Finally, the textbook procedure to construct gauge-invariant quantities is dictated by Lorentz symmetry, which requires a four-vector field to describe the two degrees of freedom of a massless spin-1 particle. To ensure the gauge part do not contribute to observables, one first formulates a Lorentz-invariant and gauge symmetric theory and then imposes gauge conditions in quantization. The reverse of the procedure, namely constructing observables directly in term of "physical" degrees of freedom after imposing the gauge conditions, does not lead to useful physics because 1) the observables generally do not have proper Lorentz transformation, 2) they generally are non-local, and 3) they generally have no physical measurements [5]. This, unfortunately, is exactly what Ref. [1] is advocating. Giving up locality and Lorentz symmetry, one can invent myriad gauge-invariant "observables" which can never actually be observed.

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