Comment on “Do Gluons Carry Half of the Nucleon Momentum?”
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Comment on “Do Gluons Carry Half of the Nucleon Momentum?” by X. S. Chen et al. (PRL103, 062001 (2009))

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In a recent paper by Chen et al. [1], the textbook definition of a charged-particle’s momentum and angular momentum in gauge theories has been questioned [2, 3]. The authors claim they have found a “proper” definition, and challenge the well-known result in perturbative quantum chromodynamics (QCD) that the gluons carry one-half of the nucleon momentum in asymptotic limit [3]. Here I argue that the textbook result stands, and one-half of the nucleon momentum in asymptotic limit shall be measurable in deep-inelastic scattering (DIS) and shall contribute 1/5 of the nucleon momentum. In Ref. [1], a “sound” definition of a charged particle’s momentum in a U(1) gauge field $A_\mu$ is purported to be

$$P_\mu^q = P_\mu - qA_{\text{pure}}^\mu/c, \quad (1)$$

where $P_\mu$ is the canonical momentum and $A_{\text{pure}}^\mu$ is a pure gauge term transforming in the same manner as does the full $A_\mu$ and always gives “null field strength.” This magical $A_{\text{pure}}^\mu$ allows a “gauge-invariant” definition of $P_\mu^q$ and “physical” $A_{\text{phys}}^\mu = A_\mu - A_{\text{pure}}^\mu$. The authors claim that the quark’s $P_\mu^q$ shall be measurable in deep-inelastic scattering (DIS) and shall contribute 1/5 of the nucleon momentum.

First of all, separating $\vec{A}$ into $\vec{A}_{\text{phys}} + \vec{A}_{\text{pure}}$ cannot be uniquely done by the conditions $\vec{\nabla} \cdot \vec{A}_{\text{phys}} = 0$ and $\vec{\nabla} \times \vec{A}_{\text{pure}} = 0$, contrary to authors’ claim. In fact, one can always add/subtract a term $\vec{\nabla} \phi$ with $\nabla^2 \phi = 0$ to change the separation. A simple counter example is that of a constant magnetic field in the z-direction. $\vec{A}_1 = (By, 0, 0)$ and $\vec{A}_2 = (B_y - Bz, 0, 0)/2$ both must be “physical” according to the authors. Of course, one can add more constraints to make the separation unique. However, this amounts to defining $\vec{A}_{\text{phys}}$ by gauge fixing and performing calculations under a fixed gauge.

Next, what is theoretically sound to define and experimentally measurable in electromagnetism are already well-known. The kinematic momentum of a charge particle is

$$\vec{p} = \vec{P} - q\vec{A}/c, \quad (2)$$

with the full gauge field $\vec{A}$ required. It is $\vec{p}$ which gives rise to the kinetic energy of the particle $E = \vec{p}^2/2m$, and it is $\vec{p}$ which generates the electric current, $\vec{j} = (q/m)\vec{p}$. Feynman in his famous lectures provided a beautiful example (Sec. 21-3) to demonstrate that $\vec{p}$ is the momen-