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# Flavor decomposition of the elastic nucleon electromagnetic form factors

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The  $u$ - and  $d$ -quark contributions to the elastic nucleon electromagnetic form factors have been determined using experimental data on  $G_E^n$ ,  $G_M^n$ ,  $G_E^p$ , and  $G_M^p$ . Such a flavor separation of the form factors became possible up to negative four-momentum transfer squared  $Q^2 = 3.4 \text{ GeV}^2$  with recent data on  $G_E^n$  from Hall A at JLab. For  $Q^2$  above  $1 \text{ GeV}^2$ , for both the  $u$ - and  $d$ -quark, the ratio of the Pauli and Dirac form factors,  $F_2/F_1$ , was found to be almost constant in sharp contrast to the behavior of  $F_2/F_1$  for the proton as a whole. Also, again for  $Q^2 > 1 \text{ GeV}^2$ , both  $F_2^d$  and  $F_1^d$  are roughly proportional to  $1/Q^4$ , whereas the drop off of  $F_2^u$  and  $F_1^u$  is more gradual.

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Electron-nucleon scattering has been extensively studied in two cases. The first case is in elastic scattering which is characterized by the electromagnetic form factors [1]. The second case is in deep inelastic scattering characterized by the structure functions which exhibit Bjorken scaling [2]. The study of the proton form factors in elastic scattering by Hofstadter *et al.* provided some of the first information on the size of the proton and the distribution of charge and magnetization [3]. Deep inelastic scattering resulted in the discovery of quarks [4], and also taught us about the nucleon's spin structure [5]. At Jefferson Laboratory, precise measurements of the electric form factor of the proton showed unexpected behavior [6], triggering a reconsideration of nucleon structure [7, 8]. Historically, the observation of unexpected behavior in form factors and structure functions has brought about significant new understanding of the strong interaction.

Experimental data on the proton Dirac form factor  $F_1^p$  [9] have been found to be in fair agreement with a scaling prediction based on perturbative QCD (pQCD),  $F_1^p \propto Q^{-4}$  [10], where  $Q^2$  is the negative four-momentum transfer squared. It has been argued, however, that pQCD is not applicable for exclusive processes at experimentally accessible values of momentum transfer [11]. Indeed, experimental results from Thomas Jefferson Laboratory (JLab) [6] for the ratio of the proton Pauli form factor  $F_2^p$  and the Dirac form factor  $F_1^p$  have been found to be in disagreement with the suggested scaling  $F_2^p/F_1^p \propto 1/Q^2$  [10]. These same data, however, are in reasonable agreement with an updated pQCD prediction  $Q^2 F_2/F_1 \propto \ln^2[Q^2/\Lambda^2]$  [12] even at modest  $Q^2$  of several  $\text{GeV}^2$ . Here  $\Lambda$  is a soft scale parameter related to the size of the nucleon. The prediction has the important feature that it includes components of the quark wave function with nonzero orbital angular momentum (OAM). Various relativistic constituent quark models have also reproduced the data from [6], one example being Ref. [13]. These models also incorporate nonzero quark OAM. Given the importance of the physical interpretation that has been attributed to the  $Q^2$  de-

pendence of the quantity  $S_p \equiv Q^2 F_2^p/F_1^p$ , it is important to better understand the details underlying the observed behavior.

We report here on the flavor-separated elastic form factors for the up and down quarks up to  $Q^2 = 3.4 \text{ GeV}^2$ , and observe two interesting behaviors that have not previously been reported. First, for values of  $Q^2$  above roughly  $1 \text{ GeV}^2$ , we find that the  $Q^2$  dependence of the ratio  $F_2/F_1$  for the  $d$  and  $u$ -quark contributions to the nucleon form factors is surprisingly constant. This is in sharp contrast to both the expectation that  $F_2/F_1 \sim 1/Q^2$  and the observed behavior for the proton and the neutron (which also differ from each other). Furthermore, we find that for both  $F_2$  and  $F_1$ , the  $d$  quark form factors are reasonably consistent with  $1/Q^4$  scaling above roughly  $1 \text{ GeV}^2$ , whereas the  $u$  quark form factor has a very different behavior, and drops off significantly less quickly. Both of these behaviors have potentially interesting implications. At an empirical level, the  $Q^2$  dependence of  $S_p$  can be understood as resulting from these two behaviors. It is interesting to note that the authors of [12] did not expect the asymptotic predictions for the form factors to work at a few  $\text{GeV}^2$  and considered the possibility that “... the observed consistency might be a sign of precocious scaling as a consequence of delicate cancellations in the ratio”. Indeed, we see that it is the interplay between the behavior of the up and down quark contributions that gives rise to the observed  $Q^2$  dependence of  $S_p$ .

In the one-photon exchange approximation, the amplitude for electron-nucleon elastic scattering can be written  $M^{\text{EM}} = -(4\pi\alpha/Q^2)l^\mu J_\mu^{\text{EM}}$ , where  $\alpha$  is the fine structure constant,  $l^\mu = \bar{e}\gamma^\mu e$  is the leptonic vector current, and

$$J_\mu^{\text{EM}} = \langle p(n) | (\frac{2}{3}\bar{u}\gamma_\mu u + \frac{-1}{3}\bar{d}\gamma_\mu d) | p(n) \rangle \quad (1)$$

is the hadronic matrix element of the electromagnetic current operators for the proton (neutron). Here we neglect heavier quarks because experimental data on parity non-conserving polarized electron scattering from the proton are consistent with zero up to  $\sim 0.6 \text{ GeV}^2$  [14]. We submit at least the possibility of significant nonzero

strange matrix elements at higher  $Q^2$ , but note that existing data constrains at least some models to negligible values at the  $Q^2$  range of interest [15]. While we cannot evaluate the matrix elements of  $\bar{u}\gamma_\mu u$  and  $\bar{d}\gamma_\mu d$  explicitly, from symmetry considerations we know that the matrix element shown in Eq. 1 must have the form (considering the proton for definiteness)

$$J_\mu^{\text{EM}} = \bar{p}(k') \left[ \gamma^\mu F_1^p(Q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2M} F_2^p(Q^2) \right] p(k), \quad (2)$$

where  $p(k)$  and  $\bar{p}(k')$  are the proton Dirac spinors for the initial and final momenta  $k$  and  $k'$ , respectively. The definition of the neutron form factors  $F_1^n(Q^2)$  and  $F_2^n(Q^2)$  follows similarly.

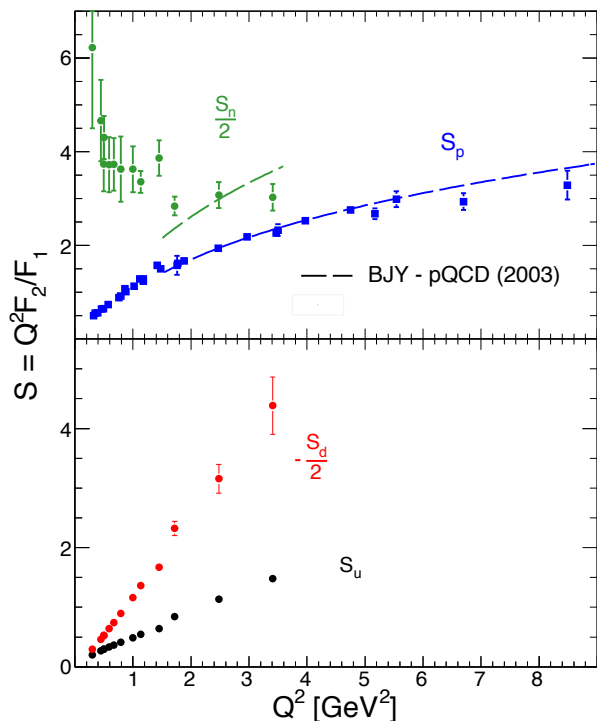


FIG. 1: The ratio of the Pauli and Dirac form factors, multiplied by  $Q^2$ ,  $S = Q^2 F_2 / F_1$ , vs. the negative four-momentum transfer squared  $Q^2$ . The upper panel shows  $S_p$  for the proton and  $S_n$  for the neutron using data from Refs.[16-21], as well as the curves of the prediction [12]:  $\ln^2[Q^2/\Lambda^2]$  for  $\Lambda=300$  MeV which is normalized to the data at 2.5 GeV<sup>2</sup>. The bottom panel shows the individual flavor quantities  $S_u$  and  $S_d$  for the  $u$  and  $d$  quarks, respectively.

The JLab data for  $G_E^p/G_M^p$  from Refs. [6] were used to plot  $S_p \equiv Q^2 F_2^p / F_1^p$  in the upper panel of Fig. 1, which also shows the prediction [12] with  $\Lambda = 300$  MeV. Data on  $G_E^n/G_M^n$  for the neutron up to  $Q^2=3.4$  GeV<sup>2</sup> were recently published by Riordan *et al.* [16]. For the first time, it is possible to examine the behavior of the neutron ratio  $F_2^n/F_1^n$  in the same  $Q^2$  range as that where the interesting behavior was first seen for the proton [6]. Using the data of Riordan *et al.* as well as those of Refs.[17-21], we also show in Fig. 1 the quantity  $S_n \equiv Q^2 F_2^n / F_1^n$ . Scaling

of  $S_n$  is clearly not evident at the lower  $Q^2$  values shown, although the data do not rule out this type of behavior at a moderately higher  $Q^2$ .

Thus far, by discussing  $F_1^{p(n)}$  and  $F_2^{p(n)}$  we are explicitly examining the behavior of the matrix element of the electromagnetic operators ( $\frac{2}{3}\bar{u}\gamma_\mu u + \frac{-1}{3}\bar{d}\gamma_\mu d$ ) in the proton (neutron). If we assume charge symmetry (thus implying  $\langle p|\bar{u}\gamma_\mu u|p\rangle = \langle n|\bar{d}\gamma_\mu d|n\rangle$ ), it is possible to perform a flavor decomposition of the form factors  $F_1^{p(n)}$  and  $F_2^{p(n)}$ , and construct form factors corresponding to the matrix elements of  $\bar{u}\gamma_\mu u$  and  $\bar{d}\gamma_\mu d$  individually [22]. Here we use the relations

$$F_{1(2)}^u = 2F_{1(2)}^p + F_{1(2)}^n \quad \text{and} \quad F_{1(2)}^d = 2F_{1(2)}^n + F_{1(2)}^p.$$

In what follows, we use the (usual) convention that  $F_{1(2)}^u$  and  $F_{1(2)}^d$  refer to the up and down quark contributions to the Dirac (Pauli) form factors of the proton. At  $Q^2=0$ , the normalizations of the Dirac form factors are given by:  $F_1^u(0) = 2$  ( $F_1^d(0) = 1$ ) so as to yield the normalization of 2 (1) for the  $u$  ( $d$ )-quark distributions in the proton. The normalizations of the Pauli form factors at  $Q^2=0$  are given by  $F_2^q(0) = \kappa_q$ , where  $\kappa_u$  and  $\kappa_d$  can be expressed in terms of the proton ( $\kappa_p$ ) and neutron ( $\kappa_n$ ) anomalous magnetic moments as

$$\kappa_u \equiv 2\kappa_p + \kappa_n = +1.67 \quad \text{and} \quad \kappa_d \equiv \kappa_p + 2\kappa_n = -2.03.$$

Having defined the flavor-separated Dirac and Pauli form factors, we can also define the quantities

$$S_u \equiv Q^2 F_2^u / F_1^u \quad \text{and} \quad S_d \equiv Q^2 F_2^d / F_1^d,$$

which we have plotted in the bottom panel of Fig. 1. Each individual data point corresponds to an experimental result on  $G_E^n/G_M^n$  from Refs.[16-21]. Only the uncertainties in the ratio  $G_E^n/G_M^n$  are included in the error bars of the flavor-separated results because the other form factors (calculated with the Kelly fit [23]) are known to much higher accuracy, albeit dependent on the particular parameterization chosen. The behavior we see is completely different from that of the proton and the neutron. There is a striking lack of saturation, and indeed the variation of  $S_u$  and  $S_d$  with  $Q^2$  appears to be quite linear. It is interesting also that the slope associated with the  $d$  quark is about six times larger than that of the  $u$  quark. When we consider the matrix elements of  $\bar{u}\gamma_\mu u$  and  $\bar{d}\gamma_\mu d$  individually, the relationship between the Pauli and the Dirac amplitudes is quite different from when we consider the sum of the amplitudes that results in the full hadronic matrix element (Eq. 2).

While it is instructive to plot  $S_u$  and  $S_d$  so that we can compare them directly with the widely discussed  $S_p$  for the proton, the inclusion of the factor of  $Q^2$  masks the detailed behavior as  $Q^2$  approaches zero. We thus plot in the top two panels of Fig. 2 the quantities  $\kappa_u^{-1} F_2^u / F_1^u$  and  $\kappa_d^{-1} F_2^d / F_1^d$ . Here, a second aspect of the behavior of the flavor decomposed form factors appears that is

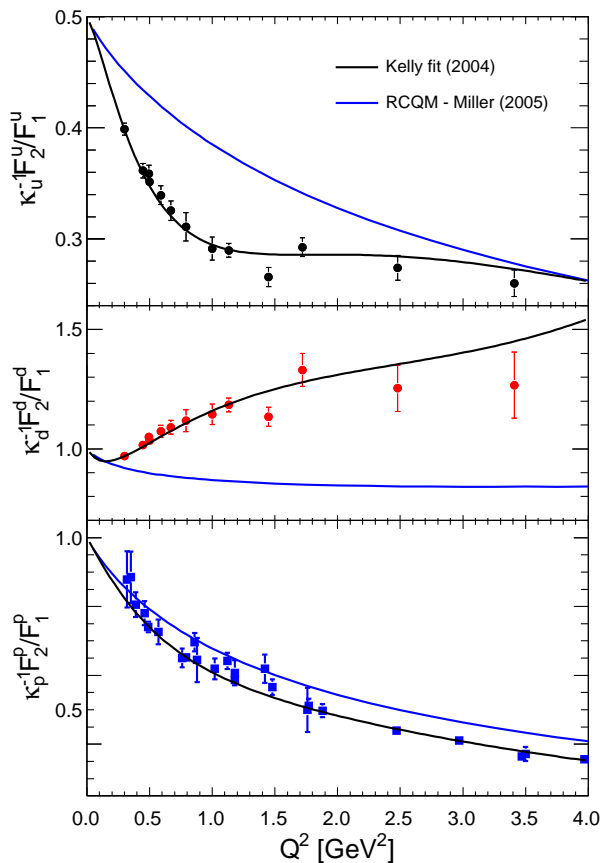


FIG. 2: The ratios  $\kappa_d^{-1} F_2^d / F_1^d$ ,  $\kappa_u^{-1} F_2^u / F_1^u$  and  $\kappa_p^{-1} F_2^p / F_1^p$  vs. momentum transfer  $Q^2$ . The data and curves are described in the text.

quite intriguing. These ratios are relatively constant for  $Q^2$  greater than  $\sim 1$  GeV<sup>2</sup>, but have a more complex behavior for lower values of  $Q^2$ . This might be interpreted as a transition between a region where the virtual photon coupling to the three-quark component in the wave function dominates (higher  $Q^2$ ) and a region where the inclusion of a coupling to a five-quark component is essential (lower  $Q^2$ ). It is tempting to interpret the simple behavior of  $F_2/F_1$  for each of the two quark flavors at  $Q^2 > 1$  GeV<sup>2</sup> as evidence of an underlying symmetry that is only evident when the three-quark component in the wave function dominates. We note also that the ratio  $F_2/F_1$  for the proton does not show a different behavior above and below 1 GeV<sup>2</sup> as one can see in the bottom panel of Fig. 2. The calculation of the form factors in a relativistic constituent quark model (RCQM) [24] (shown by the blue curves in Fig. 2) deviates considerably from the data which illustrates the discriminating power of the flavor separated form factors. The empirical Kelly fit (which predates Ref. [16]), corresponds to the black curves, and is in reasonable agreement with the data, particularly at lower  $Q^2$ .

The form factors  $F_1^u$ ,  $F_1^d$ ,  $F_2^u$  and  $F_2^d$  are shown in Fig. 3, all multiplied by  $Q^4$  for better clarity in the high- $Q^2$  range. We have also normalized  $F_2^u$  and  $F_2^d$  by

their respective anomalous magnetic moments. The (un-normalized) values are given in Table I.

TABLE I: The flavor contributions to the proton form factors, obtained using  $G_E^n/G_M^n$  form factor data from Refs.[16-21] and the Kelly fit [23] for the other form factors. The  $Q^2$  values are given in GeV<sup>2</sup>.

$Q^2$	Ref.	$F_1^u$	$F_1^d$	$F_2^u$	$F_2^d$
0.30	[20]	1.075(6)	0.505(12)	0.716(6)	-0.995(12)
0.45	[21]	0.853(6)	0.377(12)	0.515(6)	-0.777(12)
0.50	[17]	0.789(6)	0.332(12)	0.473(6)	-0.708(12)
0.50	[19]	0.789(4)	0.340(7)	0.463(4)	-0.713(7)
0.59	[20]	0.695(6)	0.283(13)	0.394(6)	-0.617(13)
0.67	[18]	0.628(6)	0.249(12)	0.342(6)	-0.552(12)
0.79	[20]	0.544(8)	0.206(15)	0.283(8)	-0.467(15)
1.00	[19]	0.434(5)	0.154(10)	0.211(5)	-0.357(10)
1.13	[21]	0.379(3)	0.124(5)	0.183(3)	-0.298(5)
1.45	[21]	0.290(3)	0.093(6)	0.128(3)	-0.213(6)
1.72	[16]	0.2257(22)	0.0529(43)	0.1103(22)	-0.1429(43)
2.48	[16]	0.1380(18)	0.0278(35)	0.0632(18)	-0.0707(35)
3.41	[16]	0.0851(12)	0.0131(24)	0.0370(12)	-0.0337(24)

Up to  $Q^2 \approx 1$  GeV<sup>2</sup> there is a constant scaling factor of  $\sim 2.5$  for  $F_1$  and  $\sim 0.75$  for  $F_2$ , between the  $u$ - and  $d$ -quark contributions. Above 1 GeV<sup>2</sup> the  $d$ -quark contributions to both nucleon form factors multiplied by  $Q^4$  become constant in contrast to the  $u$ -quark contributions which continue to rise. These experimental results are in qualitative agreement with the predictions for the moments of the generalized parton distributions reported in Ref. [25]. It is interesting to note that the  $d$ -contributions correspond to the flavor that is represented singly in the proton, whereas the  $u$ -contributions correspond to the flavor for which there are two quarks. In the framework of Dyson-Schwinger equation calculations, the reduction of the ratios  $F_1^d/F_1^u$  and  $F_2^d/F_2^u$  at high  $Q^2$  is related to diquark degrees of freedom [8, 26]. The reduction of these ratios has the immediate consequence that  $S_p$  has its observed shape despite the fact that  $S_u$  and  $S_d$  are almost linear with  $Q^2$ .

Another representation of the Dirac form factor is the infinite momentum frame density,  $\rho_D$ , given by the expression  $\rho_D(b) = \int(QdQ/2\pi)J_0(Qb)F_1(Q^2)$  [27], where  $J_0$  is the zeroth order Bessel function and  $b$  is the impact parameter. The faster drop off of the  $d$ -quark Dirac form factor in Fig. 3 implies that the  $u$  quarks have a significantly tighter distribution than the  $d$  quarks in impact-parameter space, as was noticed in Ref. [28].

In summary, we have performed a flavor separation of the elastic electromagnetic form factors of the nucleon. We find that for large  $Q^2$  the  $d$ -quark contributions to both proton form factors are reduced relative to the  $u$ -quark contributions. Possible explanations might include the importance of diquark degrees of freedom [8, 26, 29],

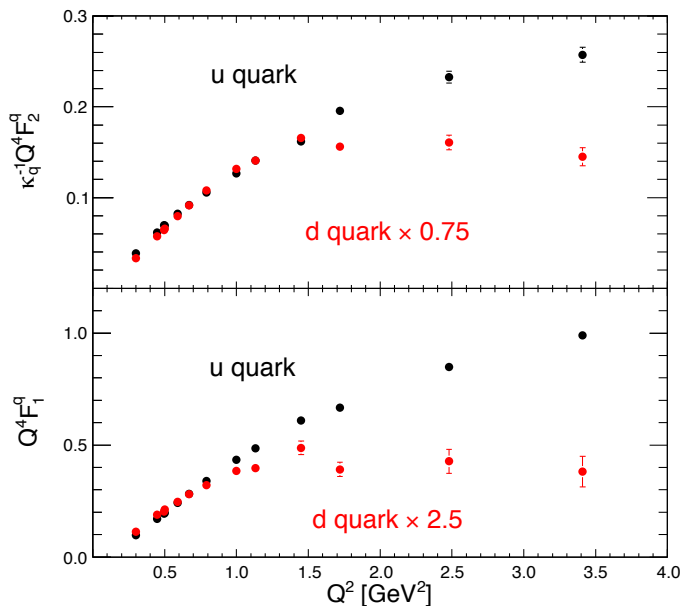


FIG. 3: The  $Q^2$ -dependence for the  $u$ - and  $d$ -contributions to the proton form factors (multiplied by  $Q^4$ ). The data points are explained in the text.

or the fact (mentioned earlier) that relatively little is known about strange quark matrix elements at high  $Q^2$ . We find also that the  $Q^2$ -dependencies of the flavor-decomposed quantities  $S_u$  and  $S_d$  are relatively linear in contrast to the more complicated behavior of  $S_p$  and  $S_n$ . This linearity is due to the fact, as yet unexplained, that the ratios  $F_2^u/F_1^u$  and  $F_2^d/F_1^d$  are constant within experimental errors for  $Q^2 > 1 \text{ GeV}^2$ . At  $Q^2 < 1 \text{ GeV}^2$ , however, these same ratios show significant variation. Given the linearity of  $S_u$  and  $S_d$ , it is quite clear that the precocious scaling of the proton form factors and the consistency of the proton data with the updated pQCD description of Ref. [12] are the result of the different behaviors of the  $u$ - and  $d$ -quark contributions to the proton form factors. Further measurements of  $G_E^n/G_M^n$  [30] will allow the flavor decomposition to be extended to  $Q^2=10 \text{ GeV}^2$  and the exploration of the  $Q^2$  range over which the apparent constant behavior of  $F_2^u/F_1^u$  and  $F_2^d/F_1^d$  persists.

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