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Phys. Rev. Lett. **106**, 251301 — Published 23 June 2011

DOI: 10.1103/PhysRevLett.106.251301

Non-Gaussianity Consistency Relation for Multi-field Inflation

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While detection of the "local form" bispectrum of primordial perturbations would rule out all single-field inflation models, multi-field models would still be allowed. We show that multi-field models described by the δN formalism obey an inequality between $f_{\rm NL}$ and one of the local-form *trispectrum* amplitudes, $\tau_{\rm NL}$, such that $\tau_{\rm NL} > \frac{1}{2}(\frac{6}{5}f_{\rm NL})^2$ with a possible logarithmic scale dependence, provided that 2-loop terms are small. Detection of a violation of this inequality would rule out most of multi-field models, challenging inflation as a mechanism for generating the primoridal perturbations.

Can we rule out inflation as a mechanism for generating primordial curvature perturbations? Inflation is indispensable for explaining homogeneity and flatness of the observable universe [1]. Yet, its predictions for the statistical properties of primordial curvature perturbations may be falsifiable.

The basic predictions that inflation generates adiabatic, nearly scale-invariant, and nearly Gaussian primordial curvature perturbations [2, 3] are all consistent with the current observations (see, e.g., [4]). Notably, many inflation models predict that the amplitude of fluctuations on large scales is greater than that on small scales. In terms of the power spectrum of primordial curvature perturbations ζ, we say $k^3P_\zeta(k) \propto k^{n_s-1}$ with $n_s < 1$. The power spectrum is defined by $\langle\zeta_{\mathbf{k}}\zeta_{\mathbf{k}'}\rangle = (2\pi)^3\delta(\mathbf{k}+\mathbf{k}')P_\zeta(k)$. The latest observations give $n_s = 0.96 \pm 0.01$ [4, 5], which may be taken as evidence for inflation.

The future, more sensitive experiments will continue to test the other predictions: adiabaticity and Gaussianity of fluctuations. In this paper, we shall focus on the latter. Departure from Gaussianity, called *non-Gaussianity*, has emerged as a powerful test of inflation over the last decade (see [6] for reviews).

One of the major theoretical discoveries made from these studies is that *all* single-field inflation models yield a specific amount of non-Gaussianity in the so-called squeezed limit of the bispectrum (Fourier transform of the three-point correlation function) of ζ, given by $f_{\rm NL} = \frac{5}{12}(1-n_s) \simeq 0.02$ [7] (also see [8]). Here, $f_{\rm NL}$ characterizes the amplitude of the so-called "local form" bispectrum [9, 10]:

$$B_\zeta = \frac{6}{5}f_{\rm NL}\left[P_\zeta(k_1)P_\zeta(k_2) + (2\text{ perm.})\right], \quad (1)$$

where $\langle\prod_{i=1}^3\zeta(\mathbf{k}_i)\rangle = (2\pi)^3\delta^3(\sum_i\mathbf{k}_i)B_\zeta(k_1,k_2,k_3)$, and the "squeezed limit" is given by taking $k_3 \ll k_1 \approx k_2$, i.e., $B_\zeta(k_1,k_2,k_3) \to \frac{12}{5}f_{\rm NL}P_\zeta(k_1)P_\zeta(k_3)$. All single-field inflation models predict $(1-n_s)P_\zeta(k_1)P_\zeta(k_3)$ in this limit.

The current best limit is $f_{\rm NL} = 32 \pm 21$ (68% CL; [4]). As various second-order effects generate $f_{\rm NL} = \mathcal{O}(1)$ (see [11] for a review and references therein), a convincing detection of $f_{\rm NL} \gg 1$ would rule out all single-field inflation models. The Planck satellite is expected to reduce the error bar by a factor of four [10].

However, detection of $f_{\rm NL}$ would not rule out *multi*-field models. How can we test them also? Our work in this paper is motivated by the Suyama-Yamaguchi inequality, $\tau_{\rm NL} \geq (\frac{6}{5}f_{\rm NL})^2$ [12]. Here, $\tau_{\rm NL}$ is one of the amplitudes of the local-form *trispectrum* defined by [13]

$$T_\zeta = \tau_{\rm NL}[P_\zeta(|\mathbf{k}_1+\mathbf{k}_3|)P_\zeta(k_3)P_\zeta(k_4) + (11\text{ perm.})], \quad (2)$$

where $\langle\prod_{i=1}^4\zeta(\mathbf{k}_i)\rangle = (2\pi)^3\delta^3(\sum_i\mathbf{k}_i)T_\zeta(k_1,k_2,k_3,k_4)$.

As emphasized in [11], if the new experimental data (such as Planck) detect $f_{\rm NL}$ (hence ruling out single-field models) but do not see $\tau_{\rm NL}$ large enough to satisfy the above inequality, then a large class of multi-field models may be ruled out. The crucial question is then, "how generic is the Suyama-Yamaguchi inequality?" It was pointed out in [11] that this inequality may not be generic enough, as there are cases where this inequality is not satisfied. Recently, Suyama et al. [14] considered the same issue, where they have truncated the δN expansion (given below) at the second order and have considered a part of 1-loop corrections. The goal of this paper is to find a more general inequality than theirs. We shall retain the terms up to the fourth order of δN expansion, as these terms are required for the consistent calculations up to the 1-loop level. As a result, we find a weaker bound than the original Suyama-Yamaguchi inequality. This is relevant because, as shown in [15], large and observable primordial non-Gaussianity can be generated when the loop contributions dominate over the tree contributions in the bispectrum and/or in the trispectrum.

Throughout this paper, we shall consider a class of multi-field models which satisfy the following conditions:

1. Scalar fields are responsible for generating curvature perturbations; thus, potential contributions from vector fields (see [16] for a review and references therein) are ignored.

2. Fluctuations in scalar fields at the horizon crossing are scale invariant and Gaussian.

Therefore, we assume that non-Gaussianity is generated only on super horizon scales, according to the δN formalism [3, 17]. While the "quasi-single-field inflation" model proposed by Chen and Wang [18] is an example to which this condition may not apply, their model yields $\tau_{\rm NL} \gg f_{\rm NL}^2$, satisfying the inequality. Yet, the condition 2 is probably too strong. Whether this condition can be relaxed significantly merits further investigation.

According to the δN formalism, the curvature perturbation, ζ, is given by derivatives of the number of e-fold, $N(t,t_*) = \int_{t_*}^t H dt'$, with respect to scalar fields, φ^a, at the horizon-crossing time t_* ($a_* H_* = k$):

$$\zeta(\mathbf{x},t) = N_a(t,t_*)\delta\varphi^a_*(\mathbf{x}) + \frac{1}{2}N_{ab}(t,t_*)\delta\varphi^a_*(\mathbf{x})\delta\varphi^b_*(\mathbf{x})\cdots \tag{3}$$

where $\delta\varphi^a_*$ is a fluctuation of φ^a evaluated at t_*, i.e., $\delta\varphi^a_*(\mathbf{x}) \equiv \delta\varphi^a(t_*,\mathbf{x})$. Note that $N_a \equiv \partial N/\partial\varphi^a_*$ and $N_{ab} \equiv \partial^2 N/\partial\varphi^a_*\partial\varphi^b_*$.

The second condition above implies that the power spectrum of scalar fields is given by

$$\langle \delta\varphi^a_{\mathbf{k}}(t_*)\delta\varphi^b_{\mathbf{k}'}(t_*)\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')\delta^{ab}\frac{2\pi^2}{k^3}\mathcal{P}_*, \tag{4}$$

where $\mathcal{P}_* \equiv (H_*/2\pi)^2$. Note that we have assumed that scalar field fluctuations with different indices are uncorrelated, $\langle\delta\varphi^a\delta\varphi^b\rangle \propto \delta^{ab}$. This can be done without loss of generality: we could, for example, write the correlation matrix as $\langle\delta\varphi^a\delta\varphi^b\rangle \propto M^{ab}$, where M is a real positive symmetric matrix. One can then diagonalize M as $M = UDU^t$. Redefining scalar field fluctuations as $\delta\varphi \to \tilde{\delta\varphi} = U\delta\varphi$ will recover Eq. (4).

Now, we impose the third condition:

3. Truncate the δN expansion [Eq. (3)] at the order of $\delta\varphi^4$, i.e., $\zeta = N_a\delta\varphi^a_* + \frac{1}{2}N_{ab}\delta\varphi^a_*\delta\varphi^b_* + \frac{1}{3!}N_{abc}\delta\varphi^a_*\delta\varphi^b_*\delta\varphi^c_* + \frac{1}{4!}N_{abcd}\delta\varphi^a_*\delta\varphi^b_*\delta\varphi^c_*\delta\varphi^d_*$. Thus, we shall ignore the contributions in the power spectrum, bispectrum, or trispectrum coming from $O(\delta\varphi^5)$.

The 4th-order term is needed when we calculate all of the 1-loop contributions in $f_{\rm NL}$ and $\tau_{\rm NL}$. In the following, we shall include all of the 1-loop contributions, while some of the higher-order loop contributions are also included.

The power spectrum is given, up to the 4th order, by

$$\begin{aligned}\mathcal{P}_\zeta =& \mathcal{P}_*\Big[N_aN_a + {\rm Tr}(N^2)\mathcal{P}_*\ln(kL) + N_aN_{abb}\mathcal{P}_*\ln(k_{\rm max}L)\\ &+\frac{1}{4}N_{acc}N_{abb}\mathcal{P}_*^2\ln^2(k_{\rm max}L)\\ &+N_{abcc}N_{ab}\mathcal{P}_*^2\ln(kL)\ln(k_{\rm max}L)\dots\Big],\end{aligned} \tag{5}$$

where we have used the following notations: $N_aN_a \equiv \sum_a N_a^2$ and ${\rm Tr}(N^2) \equiv \sum_{ab}N_{ab}N_{ab}$. The L is a finite size of a box which is chosen to be much larger than the region of interest, such that the condition $Lk \gg 1$ is satisfied for arbitrary k, and $k_{\rm max}$ is the ultra-violet cutoff. The 1st term is the tree contribution; the 2nd and 3rd terms are the 1-loop contributions; and the 4th and 5th terms are the 2-loop contributions.

This result can be simplified by using the following quantities (see Eq. (25) of [19]):

$$\tilde{N}_a \equiv N_a + \frac{1}{2}N_{abb}\mathcal{P}_*\ln(k_{\rm max}L), \tag{6}$$

$$\tilde{N}_{ab} \equiv N_{ab} + \frac{1}{2}N_{abcc}\mathcal{P}_*\ln(k_{\rm max}L). \tag{7}$$

Then Eq. (5) becomes

$$\mathcal{P}_\zeta = \tilde{N}_a\tilde{N}_a\mathcal{P}_*\left(1+\mathcal{P}_{\rm loop}+\dots\right), \tag{8}$$

where we have defined a positive-definite quantity

$$\mathcal{P}_{\rm loop} \equiv \frac{{\rm Tr}(\tilde{N}^2)}{\tilde{N}_a\tilde{N}_a}\mathcal{P}_*\ln(kL). \tag{9}$$

Here, the dots in Eq. (8) include the higher-order terms such as $N_{abcc}^2{\mathcal{P}_*}^2$. This is a nice way of writing the power spectrum etc., as the results do not include the ultra-violet cutoff, $k_{\rm max}$, explicitly: the cutoff can be absorbed by redefining the derivatives of N.

As we can take L such that $kL \gg 1$, $\mathcal{P}_{\rm loop}$ is essentially a constant factor, rescaling the overall amplitude of the power spectrum without destroying the observed scale invariance of the power spectrum. Without loss of generality, we shall take k to be the usual normalization scale used by the *WMAP* collaboration, $k_0 = 0.002$ Mpc^{-1}.

Kawakami et al. [20] have derived the expressions for $f_{\rm NL}$ and $\tau_{\rm NL}$ up to the 4th order (also see [14]). These expressions are again simplified by using the redefinition of the derivatives of N and ignoring the higher-order terms:

$$\begin{aligned}\frac{6}{5}f_{\rm NL} \simeq& \left[\tilde{N}_a\tilde{N}_a + {\rm Tr}(\tilde{N}^2)\mathcal{P}_*\ln(k_0L)\right]^{-2}\\ &\times\left[\tilde{N}_a\tilde{N}_b\tilde{N}_{ab} + \left({\rm Tr}(\tilde{N}^3) + 2\tilde{N}_a\tilde{N}_{bc}\tilde{N}_{abc}\right)\mathcal{P}_*\ln(k_0L)\right],\end{aligned} \tag{10}$$

$$\begin{aligned}\tau_{\rm NL} \simeq& \left[\tilde{N}_a\tilde{N}_a + {\rm Tr}(\tilde{N}^2)\mathcal{P}_*\ln(k_0L)\right]^{-3}\\ &\times\Big[\tilde{N}_a\tilde{N}_{ab}\tilde{N}_{bc}\tilde{N}_c + \Big(2\tilde{N}_a\tilde{N}_{ab}\tilde{N}_{cd}\tilde{N}_{bcd} + {\rm Tr}(\tilde{N}^4)\\ &+2\tilde{N}_a\tilde{N}_{bc}\tilde{N}_{bd}\tilde{N}_{acd} + \tilde{N}_a\tilde{N}_b\tilde{N}_{acd}\tilde{N}_{bcd}\Big)\mathcal{P}_*\ln(k_0L)\Big],\end{aligned} \tag{11}$$

where $\tilde{N}_{abc} \equiv N_{abc} + \frac{1}{2} N_{abcdd} \mathcal{P}_* \ln(k_{\max} L)$. Although the loop terms of the bispectrum and trispectrum have terms like $\ln(k_b L)$, $\ln(k_t L)$ and $\ln(k_p L)$ where $k_b \equiv \min\{k_i\}$ with $i = \{1,2,3\}$ or $\{1,2,3,4\}$, $k_t \equiv \min\{k_i, |\vec{k}_j + \vec{k}_l|\}$ with $(i,j,l) = \{1,2,3,4\}$ and $\ln(k_p L) \sim \ln(k_i L) \sim \ln(|\vec{k}_j + \vec{k}_l| L)$ with $(i,j,l) = \{1,2,3,4\}$, we assume that these are similar to $\ln(k_0 L)$, i.e., $\ln(k_0 L) \sim \ln(k_b L) \sim \ln(k_t L) \sim \ln(k_p L)$. From now on, we shall remove the tildes from the equations, i.e., $\tilde{N} \to N$.

Now, we are ready to derive the new inequality. First of all, we use the inequality between arbitrary real numbers α and β: $\alpha^2 + \beta^2 \geq \frac{1}{2}(\alpha+\beta)^2$. Choosing α and β as

$$\alpha \equiv [N_a N_a (1+\mathcal{P}_{\rm loop})]^{-2} \left[N_a N_b N_{ab} + N_a N_{bc} N_{abc} \mathcal{P}_* \ln(k_0 L)\right],$$
$$\beta \equiv [N_a N_a (1+\mathcal{P}_{\rm loop})]^{-2} \left[{\rm Tr}(N^3) + N_a N_{bc} N_{abc}\right] \mathcal{P}_* \ln(k_0 L), \quad (12)$$

we find

$$[N_a N^a (1+\mathcal{P}_{\rm loop})]^{-4} \times \left[\Big(N_a N_b N_{ab} + N_a N_{bc} N_{abc} \mathcal{P}_* \ln(k_0 L)\Big)^2 + \left({\rm Tr}(N^3) + N_a N_{bc} N_{abc}\right)^2 \mathcal{P}_*^2 \ln^2(k_0 L)\right] \geq \frac{1}{2}\left(\frac{6}{5} f_{\rm NL}\right)^2 . \quad (13)$$

Next, pick up the first term of the LHS in (13), and use the Cauchy-Schwarz inequality. When we define the inner product of arbitrary vectors v_a and u_b as $\langle v, u\rangle \equiv \sum_a v_a u_a$, then the Cauchy-Schwarz inequality leads to $\langle v,u\rangle^2 \leq \langle v,v\rangle \langle u,u\rangle$. Choosing v_a and u_a as $v_a \equiv N_a$ and $u_a \equiv N_b N_{ba} + N_{bc} N_{abc} \mathcal{P}_* \ln(k_0 L)$, we find

$$\frac{\Big(N_a N_b N_{ab} + N_a N_{bc} N_{abc} \mathcal{P}_* \ln(k_0 L)\Big)^2}{(N_a N_a)^4 (1+\mathcal{P}_{\rm loop})^4} < \frac{N_b N_{ba} N_{ad} N_d + 2 N_d N_{da} N_{abc} N_{bc} \mathcal{P}_* \ln(k_0 L)}{(N_a N_a)^3 (1+\mathcal{P}_{\rm loop})^3} + \frac{N_{ab} N_{abc} N_{cde} N_{de} \mathcal{P}_*^2 \ln^2(k_0 L)}{(N_a N_a)^3 (1+\mathcal{P}_{\rm loop})^3}, \quad (14)$$

where we have also used $1/(1+\mathcal{P}_{\rm loop}) < 1$ with $\mathcal{P}_{\rm loop} > 0$ on the RHS. Note that the last term on the RHS is a 2-loop contribution, which becomes important later.

Finally, pick up the second term of the LHS in (13), and use the Cauchy-Schwarz inequality again: for arbitrary real symmetric matrices M, L, we have ${\rm Tr}^2(LM) \leq {\rm Tr}(M^2){\rm Tr}(L^2)$. Choosing L and M as $L_{ab} \equiv N_{ab}$ and $M_{ab} \equiv N_{ac} N_{cb} + N_c N_{cab}$, we find

$$\frac{\left({\rm Tr}(N^3) + N_a N_{bc} N_{abc}\right)^2 \mathcal{P}_*^2 \ln^2(k_0 L)}{(N_a N_a)^4 (1+\mathcal{P}_{\rm loop})^4} < \frac{\left({\rm Tr}(N^4) + 2 N_{ac} N_{cb} N_{dab} N_d + N_c N_{cab} N_{abd} N_d\right) \mathcal{P}_* \ln(k_0 L)}{(N_a N_a)^3 (1+\mathcal{P}_{\rm loop})^3}, \quad (15)$$

where we have also used $\mathcal{P}_{\rm loop}/(1+\mathcal{P}_{\rm loop}) < 1$. Here, let us reconsider the effect of our approximation that all the logarithmic factors are similar: $\ln(k_0 L) \sim \ln(k_b L) \sim \ln(k_t L) \sim \ln(k_p L)$. If we relax this assumption, then we should replace $\ln(k_0 L)$ in the right hand side of Eq. (15) with $\ln(k_0 L) \to \ln(k_t L) R$, where $R \equiv \ln^2(k_b L)/\ln(k_t L)\ln(k_p L)$. Therefore, our approximation is valid also when the geometric mean of $\ln(k_t L)$ and $\ln(k_p L)$ is similar to $\ln(k_b L)$ (but not necessarily $\ln(k_t L) \sim \ln(k_p L)$).

Collecting these results, we obtain

$$\tau_{\rm NL} + (2\ {\rm loop}) > \frac{1}{2}\left(\frac{6}{5} f_{\rm NL}\right)^2 , \quad (16)$$

where the "2 loop" term is the last term in the RHS of Eq. (14). This result shows that, when we allow ourselves for completely general models in which this particular 2-loop term can become important, the Suyama-Yamaguchi inequality, $\tau_{\rm NL} \geq \left(\frac{6}{5} f_{\rm NL}\right)^2$, may be violated badly. This illustrates the limitation of this inequality.

Still, from a model-building point of view, it is reasonable to assume that the 2-loop terms are sub-dominant compared to the tree or 1-loop terms; otherwise, we would have to require fine-tunings between the derivatives of N. Let us then study the consequence of ignoring this particular 2-loop term. We shall impose the following conditions:

$$\frac{N_{ab} N_{abc} N_{cde} N_{de} \mathcal{P}_*^2 \ln^2(k_0 L)}{N_b N_{ba} N_{ac} N_c} \ll 1,$$
$$\frac{N_{ab} N_{abc} N_{cde} N_{de} \mathcal{P}_*^2 \ln^2(k_0 L)}{N_b N_{ba} N_{ac} N_c} \ll \left|\frac{N_d N_{da} N_{abc} N_{bc} \mathcal{P}_* \ln(k_0 L)}{N_b N_{ba} N_{ac} N_c}\right|. \quad (17)$$

The first condition is (tree)≫(2-loop), and the second is (1-loop)≫(2-loop) for the terms in the RHS of Eq. (14). Interestingly, from the Cauchy-Schwarz inequality for $N_{ab} N_b$ and $N_{abc} N_{bc}$, we find

$$\left(\frac{N_d N_{da} N_{abc} N_{bc} \mathcal{P}_* \ln(k_0 L)}{N_b N_{ba} N_{ac} N_c}\right)^2 \leq \frac{N_{ab} N_{abc} N_{cde} N_{de} \mathcal{P}_*^2 \ln^2(k_0 L)}{N_b N_{ba} N_{ac} N_c} \ll \left|\frac{N_d N_{da} N_{abc} N_{bc} \mathcal{P}_* \ln(k_0 L)}{N_b N_{ba} N_{ac} N_c}\right|, \quad (18)$$

from which we obtain the following bound on a particular form of 1-loop contributions:

$$\left|\frac{N_d N_{da} N_{abc} N_{bc} \mathcal{P}_* \ln(k_0 L)}{N_b N_{ba} N_{ac} N_c}\right| \ll 1. \quad (19)$$

As a result, if we ignore the last term in the RHS of Eq. (14), we must also ignore the second term, leaving only the tree-level term in the RHS of Eq. (14). This is a peculiar feature of these terms, whose physical meaning is not clear.

In any case, provided that the following additional condition is met:

4. The 2-loop contributions are sub-dominant compared to the tree-level or 1-loop contributions (or at least the particular 2-loop term in the RHS of Eq. (14) is small compared to the others),

we finally arrive at the new inequality:

$$\tau_{\rm NL} > \frac{1}{2}\left(\frac{6}{5}f_{\rm NL}\right)^2, \qquad (20)$$

which is the main result of this paper, and is valid as long as the 2-loop contributions are small. This result generalizes the Suyama-Yamaguchi inequality (which included only the tree-level terms) as well as Ref. [14] (which included up to the second-order terms). This relation can have a logarithmic scale dependence via $R = \ln^2(k_bL)/[\ln(k_tL)\ln(k_pL)]$.

What are the implications for inflation? The most interesting case would be the observation of a complete violation of the inequality, i.e., $\tau_{NL} \ll \frac{1}{2}\left(\frac{6}{5}f_{NL}\right)^2$, which implies that inflation cannot be responsible for generating the observed fluctuations, *provided that* (1) scalar fields are the source of fluctuations; (2) fluctuations at the horizon crossing are scale invariant and Gaussian; (3) the evolution of fluctuations obeys the δN formalism; and (4) the 2-loop contributions are small.

We may not be so far away from testing this prediction. If the value of $f_{\rm NL}$ is as large as what is implied from the current data, $f_{\rm NL} \sim 30$, then the threshold value, $\tau_{\rm NL} \sim 650$, is close to the 2-σ limit expected from *Planck* [21, 22]. The large-scale structure observations should also help improving the limits on $\tau_{\rm NL}$ [23]. Therefore, in the event that *Planck* sees $f_{\rm NL}$ (thus ruling out single-field models, one of the two things can happen: (1) $\tau_{\rm NL}$ is also detected in excess of $\frac{1}{2}(\frac{6}{5}f_{\rm NL})^2$, confirming predictions from multi-field models, or (2) $\tau_{\rm NL}$ is either *not* detected, or detected below $\frac{1}{2}(\frac{6}{5}f_{\rm NL})^2$, ruling out most of the multi-field models that satisfy the above 4 conditions. This argument [11] and our result provide a strong science case for measuring the local-form trispectrum of the cosmic microwave background as well as that of the large-scale structure of the universe.

This work is supported in part by NSF grant PHY-0758153 and by the GCOE Program "Weaving Science Web beyond Particle-matter Hierarchy" at Tohoku University and by a Grant-in-Aid for Scientific Research from JAPA (Nos. 18072001, 20540245 for TF) as well as by Core-to-Core Program "International Research Network for Dark Energy."

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[1] A. H. Guth, Phys. Rev. **D23**, 347 (1981); A. D. Linde, Phys. Lett. **B108**, 389 (1982); **B129**, 177 (1983); A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982).

[2] V. F. Mukhanov and G. V. Chibisov, JETP Lett. **33**, 532 (1981); A. H. Guth and S. Y. Pi, Phys. Rev. Lett. **49**, 1110 (1982); S. W. Hawking, Phys. Lett. **B115**, 295 (1982); J. M. Bardeen, P. J. Steinhardt, and M. S. Turner, Phys. Rev. **D28**, 679 (1983).

[3] A. A. Starobinsky, Phys. Lett. **B117**, 175 (1982).

[4] E. Komatsu *et al.* (WMAP), Astrophys. J. Suppl. **180**, 330 (2009); **192**, 18 (2011).

[5] J. Dunkley *et al.* (ACT), (2010), arXiv:1009.0866.

[6] E. Komatsu, (2001), ph.D. thesis at Tohoku University, astro-ph/0206039; N. Bartolo, E. Komatsu, S. Matarrese, and A. Riotto, Phys. Rept. **402**, 103 (2004); E. Komatsu *et al.*, Astro2010: The Astronomy and Astrophysics Decadal Survey, Science White Papers, no. 158 (2009), arXiv:0902.4759; K. Koyama, Class.Quant.Grav. **27**, 124001 (2010); X. Chen, Adv.Astron. **2010**, 638979 (2010); D. Wands, Class.Quant.Grav. **27**, 124002 (2010).

[7] P. Creminelli and M. Zaldarriaga, JCAP **10**, 6 (2004).

[8] J. M. Maldacena, JHEP **05**, 013 (2003); V. Acquaviva, N. Bartolo, S. Matarrese, and A. Riotto, Nucl. Phys. **B667**, 119 (2003); D. Seery and J. E. Lidsey, JCAP **0506**, 003 (2005); X. Chen, M.-x. Huang, S. Kachru, and G. Shiu, **0701**, 002 (2007); C. Cheung, A. L. Fitzpatrick, J. Kaplan, and L. Senatore, **0802**, 021 (2008); J. Ganc and E. Komatsu, **1012**, 009 (2010); S. Renaux-Petel, **1010**, 020 (2010).

[9] A. Gangui, F. Lucchin, S. Matarrese, and S. Mollerach, Astrophys. J. **430**, 447 (1994); L. Verde, L. Wang, A. F. Heavens, and M. Kamionkowski, Mon. Not. R. Astron. Soc. **313**, 141 (2000).

[10] E. Komatsu and D. N. Spergel, Phys. Rev. **D63**, 063002 (2001).

[11] E. Komatsu, Class. Quant. Grav. **27**, 124010 (2010).

[12] T. Suyama and M. Yamaguchi, Phys. Rev. **D77**, 023505 (2008).

[13] L. Boubekeur and D. H. Lyth, Phys. Rev. **D73**, 021301 (2006).

[14] T. Suyama, T. Takahashi, M. Yamaguchi, and S. Yokoyama, (2010), arXiv:1009.1979.

[15] Y. R. Heiner R. S. Cogollo and C. A. Valenzuela-Toledo, JCAP **0808**, 029 (2008); C. A. V.-T. Yeinzon Rodriguez, Phys.Rev. **D81**, 023531 (2010).

[16] E. Dimastrogiovanni, N. Bartolo, S. Matarrese, and A. Riotto, Adv.Astron. **2010**, 752670 (2010).

[17] A. A. Starobinsky, JETP Lett. **42**, 152 (1985); D. S. Salopek and J. R. Bond, Phys. Rev. **D42**, 3936 (1990); M. Sasaki and E. D. Stewart, Prog. Theor. Phys. **95**, 71 (1996); D. H. Lyth and Y. Rodriguez, Phys. Rev. Lett. **95**, 121302 (2005).

[18] X. Chen and Y. Wang, Phys. Rev. **D81**, 063511 (2010); JCAP **1004**, 027 (2010).

[19] C. T. Byrnes, K. Koyama, M. Sasaki, and D. Wands, JCAP **0711**, 027 (2007).

[20] E. Kawakami, M. Kawasaki, K. Nakayama, and F. Takahashi, JCAP **0909**, 002 (2009).

[21] N. Kogo and E. Komatsu, Phys.Rev. **D73**, 083007

(2006).
[22] J. Smidt, A. Amblard, C. T. Byrnes, A. Cooray, A. Heavens, *et al.*, Phys.Rev. **D81**, 123007 (2010).
[23] D. Jeong and E. Komatsu, Astrophys.J. **703**, 1230 (2009); V. Desjacques and U. Seljak, Classical and Quantum Gravity **27**, 124011 (2010).