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Thermalization of nonequilibrium one-dimensional electrons in quantum wires

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We study the problem of energy relaxation in a one-dimensional electron system. The leading thermalization mechanism is due to three-particle collisions. We show that for the case of spinless electrons in a single channel quantum wire the corresponding collision integral can be transformed into an exactly solvable problem. The latter is known as the Schrödinger equation for a quantum particle moving in a Pöschl-Teller potential. The spectrum for the resulting eigenvalue problem allows for bound state solutions, which can be identified with the zero-modes of the collision integral, and a continuum of propagating modes, which are separated by a gap from the bound states. The inverse gap gives the time scale at which counter-propagating electrons thermalize.

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Introduction.—Relaxation processes and nonequilibrium dynamics in one-dimensional electron systems have moved into focus of recent theoretical [1–9] and experimental research [10–13]. One essential characteristic of one-dimensional electrons is the absence of relaxation in case of a linear energy dispersion relation and a slow relaxation if dispersion is nonlinear, inhibited by momentum and energy conservation laws [4, 7]. Based on momentum-resolved tunneling spectroscopy the peculiarities of relaxation in one-dimesional electron systems have been observed in recent experiments [10, 12]. Moreover, the violation of Wiedeman-Franz law observed [15] at the plateau of the electrical conductance in single channel quantum wires can be qualitatively understood from the different relaxation processes required for equilibration of applied temperature gradients and chemical potentials.

In presence of many transport channels relaxation of electrons at low temperature is primarily provided by pair collisions. A solution of the eigenvalue equation of the two-particle collision integral in two and three dimensions has already been given four decades ago at the early era of Fermi liquid theory [16]. A remarkable result of that theory is that the eigenvalue equation turns out to be exactly solvable. In single channel wires, on the other hand, conservation laws severely restrict the phase-space available for scattering processes, and pair collisions do not provide a relaxation mechanism. If electron density is not too low then three-particle collisions [5, 17] constitute the leading order relaxation process. Unlike the two-particle collision integral in higher dimensional systems, a spectral analysis of the corresponding three-particle collision integral in single channel wires so far is missing. This paper aims to fill this gap. We show that for specific momentum configurations of the scattering states, relevant for energy relaxation, the collision integral of spinless electrons can be diagonalized analytically. We find that zero modes of the collision integral are separated by a gap from a continuum of decaying modes. The gap value provides us the time-scale at which counter-propagating electrons, exposed to a small temperature differences, thermalize.

Formulation of the Problem.—Our motivation is to identify the time scale $\tau_{\rm th}$ at which thermalization between out-of-equilibrium counter-propagating electrons occurs in a clean single channel quantum wires. We pursue this goal by studying the spectrum of the three-particle collision integral under the assumption that left- and right-moving electrons inside the wire are initially at distinct equilibria, characterized by different temperatures, with ΔT being the temperature-difference.

Within the Boltzmann kinetic equation approach for the electron distribution function, $\dot{\mathcal{F}} = -\mathcal{I}\{\mathcal{F}\}$, microscopic details of relaxation process are stored inside the collision integral $\mathcal{I}\{\mathcal{F}\}$. In the following we specify $\mathcal{I}\{\mathcal{F}\}$ for spinless electrons with quadratic energy dispersion, interacting via Coulomb potential $V(x)$ [18]. In the high density limit interactions are weak, $e^2/\hbar v_F\kappa \ll 1$, here v_F is the Fermi velocity and κ is the dielectric constant of the host material, and the leading order relaxation process is due to three-particle collisions:

$$\mathcal{I}\{\mathcal{F}\}_{p_1} = -\sum_{p_2,p_3}\sum_{p_{1'},p_{2'},p_{3'}} W_{123}^{1'2'3'} \left[\mathcal{F}_1\mathcal{F}_2\mathcal{F}_3\mathcal{F}^h_{1'}\mathcal{F}^h_{2'}\mathcal{F}^h_{3'} - \mathcal{F}_{1'}\mathcal{F}_{2'}\mathcal{F}_{3'}\mathcal{F}^h_1\mathcal{F}^h_2\mathcal{F}^h_3\right] . \quad (1)$$

Higher order scattering processes are suppressed by the small interaction strength and phase space. In Eq. (1) we introduced notations, $\mathcal{F}_i = \mathcal{F}(t,x,p_i)$, and $\mathcal{F}^h_i = 1-\mathcal{F}_i$. The $W_{123}^{1'2'3'} = \frac{2\pi}{\hbar}|\langle 1'2'3'|\hat{V}\hat{G}_0\hat{V}|123\rangle_c|^2\delta(E_i - E_f)$ is the rate of three-particle scattering from the incoming states $p_{1,2,3}$ into the outgoing states $p_{1',2',3'}$ with energies $E_{i(f)} = \sum_i^3 \varepsilon_{i(i')}$, respectively. An explicit form of W for a generic two-body interaction potential $\hat{V} = \frac{1}{2L}\sum_{k_1k_2q} V_q \hat{c}^\dagger_{k_1+q}\hat{c}^\dagger_{k_2-q}\hat{c}_{k_2}\hat{c}_{k_1}$, where $\hat{c}^\dagger_k$ ($\hat{c}_k$) is the electron creation (annihilation) operator, has been recently derived in Ref. [5]. We here merely mention that $\hat{G}_0$

denotes the free particle Green's function, subscript "c" refers to irreducible scattering processes, L is wire length and V_q is the Fourier transform of the interaction potential $V(x)$.

Due to nonlinearity of the collision integral in Eq. (1) an exact analytical solution of the Boltzmann equation is very difficult to find. A simplification is possible within the linear response analysis in which distributions entering Eq. (1) can be linearized around an equilibrium state, $\mathcal{F}_p = f_p + f_p f_p^h \psi_p$. Here $f_p = [e^{(\varepsilon_p-\varepsilon_F)/T}+1]^{-1}$ is the equilibrium Fermi distribution with ε_F the Fermi energy and temperature T, $f_p^h = 1 - f_p$, and ψ_p in response to the externally applied perturbation, in our case $\psi_p \propto (\varepsilon_p - \varepsilon_F)\mathrm{sign}(p)\Delta T$. Restricting to the linear response regime we insert above distribution into the collision integral Eq. (1) and arrive at the Boltzmann equation, $\dot{\psi}_{p_1} = -\mathcal{L}\{\psi\}_{p_1}$, with the linear collision operator

$$\mathcal{L}\{\psi\}_{p_1} = \frac{1}{f_1 f_1^h} \sum_{\substack{p_2,p_3 \\ p_{1'},p_{2'},p_{3'}}} \mathcal{K}_{\{p_i\};\{p_{i'}\}} \sum_{i=1}^{3} (\psi_{p_i} - \psi_{p_{i'}}) \,. \quad (2)$$

Here the kernel $\mathcal{K}_{\{p_i\};\{p_{i'}\}} = W_{123}^{1'2'3'} f_1 f_2 f_3 f_{1'}^h f_{2'}^h f_{3'}^h$. A spectral analysis of the linearized collision operator in Eq. (2) under the above formulated assumptions is the problem we address in the following.

Zero-Modes and Symmetries.—Eigenfunctions of the collision integral with eigenvalue zero ('zero modes') correspond to constant in time solutions of the Boltzmann equation and are associated with the conserved quantities in the system. Indeed, it is readily checked that $\mathcal{L}$ in Eq. (2) is nullified by $\psi_E = \varepsilon_p$ (energy-conservation), $\psi_P = p$ (momentum-conservation), and $\psi_N = \mathrm{const}$ (conservation of total particle-number). Since we are interested in thermalization we can further restrict to processes in which all of the participating states are close to the Fermi points. Then the difference in number of left- and right-moving electrons, ΔN, is also conserved and $\psi_{\Delta N} = \mathrm{sign}(p)$ is an additional zero mode. We only briefly mention that three particle collisions changing ΔN require backscattering and are important for the relaxation of differences in the chemical potentials of counter-propagating electrons [7].

Defining the Hilbert space of functions endowed with scalar product $\langle \psi_p | \psi'_p \rangle = \frac{1}{2mT} \int_{-\infty}^{+\infty} dp\, f_p f_p^h \psi_p \psi'_p$ it is readily checked that $\mathcal{L}$ is positive Hermitian, implying a spectrum of eigenvalues larger or equal to zero. The zero modes form a basis of the four-dimensional subspace of conserved quantities. Any ψ_p that falls off this category evolves according to the Boltzmann equation and eventually relaxes into one of the zero modes or their linear combination. In general, the collision operator $\mathcal{L}$ may have discrete and continuous parts of the spectrum. However, only if the zero-modes are separated by a well defined gap to a smallest non-vanishing eigenvalue the concept of a relaxation rate is justified.

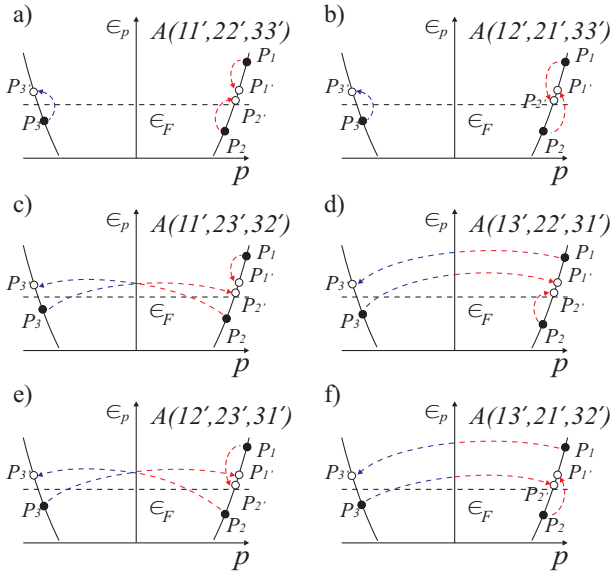

FIG. 1: [Color online] Three-particle scattering processes that allow for the energy exchange between counter-propagating electrons and thus lead to their thermalization. The matrix elements $\langle 1'2'3'|\hat{V}\hat{G}\hat{V}|123\rangle_c$ decompose into six contributions, $A(11',22',33')$ plus five permutations of primed arguments, corresponding to the direct a) and five exchange terms b)–f).

It is helpful to account for an additional symmetry of the linearized collision integral. As a consequence of invariance of the scattering kernel under reversal of momenta, $\mathcal{K}_{\{p_i\};\{p_{i'}\}} = \mathcal{K}_{\{-p_i\};\{-p_{i'}\}}$, $\mathcal{L}$ commutes with the inversion operator $\Pi\psi_p = \psi_{-p}$. Therefore, eigenfunctions of $\mathcal{L}$ have well defined parity and the operator itself can be decomposed into a direct sum $\mathcal{L} = \mathcal{L}^+ \oplus \mathcal{L}^-$ of operators $\mathcal{L}^\pm$ acting in the mutually orthogonal subspaces of even and odd parity functions. (Notice that the Hilbert space defined above decomposes into a direct sum of even and odd parity functions, mutually orthogonal to each other.) As we are interested in the relaxation of an odd parity perturbation, $\psi_{-p} = -\psi_p$, our main focus is on the spectrum of $\mathcal{L}^-$.

Spectrum of the Linearized Collision Integral.—We start the analysis from identifying small parameters in the problem. Fermi blocking in combination with conservation laws restricts participating scattering states to momentum strips of order $\delta p \sim T/v_F \ll p_F$ around the Fermi points. Denoting $q_i = p_{i'} - p_i$ the momentum-transfer in a collision we thus have the small parameters $|q_i/p_j| \sim T/\varepsilon_F \ll 1$, where $i,j = 1,2,3$. The energy is transferred via the three-particle collision in which one of the incoming electrons, say with momentum p_3, scatters off two other counter-propagating electrons (see Fig. 1 for the six possible scattering processes where an electron with $p_3 < 0$ scatters off two right-movers). Using momentum conservation $q_2 + q_1 + q_3 = 0$ we can express energy conservation as $p_2(q_1+q_3) - (p_1 q_1 + q_1^2 + p_3 q_3 + q_1 q_3 + q_3^2) = 0$ which, solved for q_3, shows that $q_3 = q_1(p_1 - p_2)/2p_F +$

$\mathcal{O}\left((p_1-p_2)^2/p_F^2, q_1^2/p_F^2\right)$. It becomes clear now that energy transfer between the counter-propagating electrons occurs via small portions of momentum exchange $v_F|q_3| \sim T^2/\varepsilon_F$, and $\{|q_3/q_1|, |q_3/q_2|\} \ll 1$ present additional small parameters, while $q_1 \simeq -q_2$ up to corrections of $\mathcal{O}\left(q_2(p_1-p_2)/p_F\right)$.

A further important ingredient for our calculation is the three-particle scattering rate for the Coulomb potential. Expanding $W_{123}^{1'2'3'} = \Lambda_{123}^{1'2'3'}\delta(E_i-E_f)\delta_{q_1+q_2+q_3=0}$ to leading order in the above small parameters, we find that the scattering rate depends only logarithmically on the momenta transfer q_i. Indeed, $\Lambda_{123}^{1'2'3'} = \frac{2\pi}{\hbar}(\frac{2e^2}{\kappa})^4[\frac{(k_F w)^2}{2L^2\varepsilon_F}]^2\Gamma^2(q_1,q_3)$, with

$$\Gamma(q_1,q_3) = [1+3(\gamma_E+\ln[k_F w])]\ln[2p_F|q_3|/q_1^2], \quad (3)$$

where γ_E is the Euler constant. This result [Eq. (3)] applies at not too low temperatures $\{Td/\hbar v_F, k_F d\} \gg 1$ at which screening of the nearby gate is not important. Technically speaking, the typical Fourier component V_k of the interaction potential then has wave number in the range $d^{-1} \ll |k| \ll w^{-1}$, so that $V_k \simeq \frac{2e^2}{\kappa}\left[\ln\frac{2e^{-\gamma_E}}{|k|w} + \frac{(kw)^2}{4}\ln\frac{2e^{1-\gamma_E}}{|k|w}\right]$ [18]. The logarithmic q_i-dependence of the scattering rate is a result of subtle cancelations between direct and various exchange terms contributing to the scattering amplitude, see also Fig 1. It is owing to this property of W that makes an analytical diagonalization of $\mathcal{L}$ possible.

Our strategy is now to split the linearized collision operator into two contributions

$$\mathcal{L}\{\psi\} = \mathcal{L}_0\{\psi\} + \delta\mathcal{L}\{\psi\}\,, \quad (4)$$

where $\mathcal{L}_0$ allows for an analytical diagonalization, while corrections to the spectrum from $\delta\mathcal{L}$ turn out to be irrelevant for our problem (and may, in principle, be calculated from perturbation theory). More precisely, we choose $\mathcal{L}_0$ as in Eq. (2) with kernel $\mathcal{K}_0$ resulting from the original $\mathcal{K}$ upon linearization of the quadratic energy dispersion, i.e. upon substituting $f_{p\pm p_F}$ and $\delta(E_i-E_f)$, respectively, by $g_{\pm p} \equiv (e^{\pm v_F p/T}+1)^{-1}$, and $\delta_0(E_i-E_f) \equiv \frac{L}{2hv_F}\left[\delta_{q_1+q_2}\delta_{q_3}\Theta(p_1,p_2,-p_3) + \delta_{q_2+q_3}\delta_{q_1}\Theta(p_1,-p_2,-p_3)\right]$, $\Theta(p_1,p_2,-p_3) = [\theta(p_1)\theta(p_2)\theta(-p_3) + \theta(-p_1)\theta(-p_2)\theta(p_3)]$. That is the operator $\mathcal{L}_0$ corresponds to a problem with linear dispersion relation. The expectation that corrections to the spectrum from $\delta\mathcal{L}$ are small is, of course, a consequence of the fact that three-particle scattering processes which provide energy exchange involve all colliding electrons near the Fermi points, while states at the band bottom are not crucial for thermalization. Since the above approximation preserves inversion symmetry, $[\mathcal{L}_0, \Pi] = 0$, eigenfunctions of $\mathcal{L}_0$ have well defined parity. Given this property we may restrict the eigenvalue equation $\mathcal{L}_0\{\psi^n\}_p = \omega_n\psi_p^n$ to momenta $p > 0$ and then extend solutions to negative values $p < 0$ by taking even and odd parity combinations $\psi_p = \theta(p)\psi_p^n \pm \theta(-p)\psi_{-p}^n$.

We next show that $\mathcal{L}_0$ can be transformed into a linear second-order differential equation. As a first step, we adopt logarithmic accuracy approximation to substitute the argument of the logarithm in the momentum dependent scattering rate Eq. (3) by its typical value, $q_1^2/(q_3 p_F) \to 1$, as dictated by the conservation laws. The linear collision operator with constant scattering rate can be reduced then to

$$\mathcal{L}_0\{\psi\}_{p_F+p_1} = \gamma_0\Big[\frac{L}{2h}\left(\varkappa^2+p_1^2\right)\psi_{p_F+p_1} - \frac{1}{g_{p_1}^h}\sum_{p_2}(p_2-p_1)g_{p_2}^h b_{p_2-p_1}\left(2\psi_{p_F+p_2}-\psi_{p_F-p_2}\right)\Big], \quad (5)$$

where $\varkappa = \pi T/v_F$ is characteristic momentum due to thermal smearing of Fermi functions, $b_p = (e^{v_F p/T}-1)^{-1}$ is the bosonic distribution function, $g_p^h = 1-g_p$, and $\gamma_0 = \frac{(Lk_F)^2}{h^2}\frac{TL\ln^2 2}{\varepsilon_F^2}(\frac{2e^2}{\kappa})^4[\frac{(k_F w)^2}{2L^2\varepsilon_F}]^2\ln^2[k_F w]$. The sequence of exact transformations leading to Eq. (5) can be summarized as follows [18]. (i) It is convenient to organize $\mathcal{L}_0$ into six contributions $\mathcal{L}_0 = \sum_{s=\pm}\sum_{j=1}^3 l_j^{s-}$ where $l_j^{s-}\{\psi\} = \frac{1}{g_1 g_1^h}\sum_{p_2p_3}\sum_{p_{1'}p_{2'}p_{3'}}\mathcal{K}_0^{s-}[\psi_j-\psi_{j'}]$ and $\mathcal{K}_0^{+-}$ ($\mathcal{K}_0^{--}$) describes processes in which the right-mover p_1 scatters off one right- and one left-mover (two left-movers). (ii) In the individual contributions l_j^{s-} we remove two out of five momentum-integrations employing conservation laws and complete two further integrations with help of the identities: $\sum_p g_p g_{p-q}^h = \frac{L}{h}qb_q$, $\sum_q g_{p+q}g_{p-q}^h = -\frac{L}{h}pb_{-p}$, and $\sum_q q g_{p+q}b_q = -\frac{L}{h}(\varkappa^2+p^2)g_p$. (iii) We find that all terms $l_j^{--}\{\psi\}$ and $l_3^{+-}\{\psi\}$ are identical zero, while $l_1^{+-}\{\psi\}+l_2^{+-}\{\psi\}$ can be summed to give Eq. (5).

We then introduce odd and even momentum combinations with respect to the Fermi point

$$\psi_p^{\pm} = \sqrt{g_p g_p^h}\left[\psi_{p_F+p} \pm \psi_{p_F-p}\right], \quad (6)$$

where the normalization factor is chosen for convenience, and recast the eigenvalue problem for odd and even combinations, $s = \pm$, as follows

$$\omega_n\psi_{p_1}^{ns} = \gamma_0\left[\frac{L}{2h}(p_1^2+\varkappa^2)\psi_{p_1}^{ns} - \sum_{p_2}\frac{3^{\delta_s}(p_2-p_1)\psi_{p_2}^{ns}}{2\sinh\frac{v_F(p_2-p_1)}{2T}}\right], \quad (7)$$

with $\delta_\pm = 0, 1$, respectively. At this stage we introduce dimensionless momentum $k = \frac{v_F p}{\pi T}$, energy $\Omega_n = \frac{2h}{\gamma_0 L}\left(\frac{\pi v_F}{T}\right)^2\omega_n$, and notice that the kernel in Eq. (7) depends on the difference of its arguments, which makes it convenient to perform a Fourier transformation $\psi_k^{n\pm} = \int\frac{dx}{2\pi}\psi_x^{n\pm}e^{ikx}$ and $\int\frac{ke^{ikx}dk}{\sinh(\pi k/2)} = \frac{2}{\cosh^2 x}$. As a result, the eigenvalue equations for the Fourier images ψ_k^{ns} reduce to Schrödinger equations of a particle moving in a Pöschl-Teller potential

$$\left[\frac{d^2}{dx^2} + \Omega_n - 1 + \frac{2\cdot 3^{\delta_s}}{\cosh^2 x}\right]\psi_x^{ns} = 0, \quad (8)$$

which can be solved with help of operator-algebra technique known from the harmonic oscillator problem [19].

The eigenvalue problem in Eq. (8) allows for one even- and one odd-parity bound-state $\Omega_n = 0$ of the form $\psi_x^{0+} = 1/\cosh x$ and $\psi_x^{0-} = -3\sinh x/\cosh^2 x$, respectively [20]. Upon inverse Fourier transformation and extension to negative momenta in the above prescribed manner this gives the four zero-modes $\psi_N^{0+} = \text{const}$, $\psi_{\Delta N}^{0-} = \text{sgn}(p)$, $\psi_P^{0-} = p$, and $\psi_E^{0+} = |p|$. As already discussed, the first three functions are consequences of conservation of total particle number, momentum and the difference in number of left- and right-moving electrons. The fourth zero mode expresses conservation of energy for the linearized spectrum. Of course, these four zero modes could have been directly inferred from $\mathcal{L}_0$.

More relevant for our problem is the fact that these bound state solutions are separated by a gap $\delta\Omega = 1$ from a continuum of propagating modes and $\mathcal{L}_0$, therefore, posses a well defined smallest non-vanishing eigenvalue. In order to associate this latter with the thermalization rate, we have to make sure that this gap is also present in the original collision operator $\mathcal{L}^-$. Employing that eigenfunctions of $\mathcal{L}_0$ form a complete set we may express $\mathcal{L}^-$ in terms of its odd parity subset. Finally, reminding that $\psi_{\Delta N}$ and ψ_P are also nullified by $\mathcal{L}^-$, it is evident that the smallest non-vanishing eigenvalue of $\mathcal{L}^-$ is of order $\delta\omega = \min_{\psi_k^-}\{\langle\psi_k^-|\mathcal{L}|\psi_{k'}^-\rangle\}$, with $\{\psi_k^-\}$ the set of eigenstates corresponding to odd parity propagating solutions of Eq. (8). Since matrix elements $|\langle\psi_k^-|\delta\mathcal{L}|\psi_{k'}^-\rangle| \ll |\langle\psi_k^-|\mathcal{L}_0|\psi_{k'}^-\rangle|$ it readily follows that $\mathcal{L}^-$ and $\mathcal{L}_0^-$ share a gap of same order. Employing then $\delta\Omega = 1$, restoring original units, and inserting the explicit form of γ_0 we arrive at the thermalization rate

$$1/\tau_{\text{th}} = c(\varepsilon_F/\hbar)(e^2/\hbar v_F\kappa)^4\lambda^2(k_F w)(T/\varepsilon_F)^3\,, \qquad (9)$$

where coefficient $c = \ln^2 2/2\pi^6$ and function $\lambda(x) = x^2\ln x$. Eq. (9) presents the main result of this paper. We note here that the temperature dependence of τ_{th}^{-1} can be understood from a simple phase-space argument like in Fermi liquid theory. To first approximation the thermalization rate follows from the out-scattering part of the collision integral, $1/\tau_{\text{out}} \propto \sum_{\{p_i\}}\mathcal{K}_{\{p_i\}}$, where two out of the five momenta $\{p_i\}$ are fixed by the conservation laws, while the remaining three extend over momentum range set by the temperature broadening $\delta p \sim T/v_F$ of the Fermi distributions. Since the scattering amplitude is only logarithmically dependent on momentum transfer then the scattering rate $W \simeq \text{const}$ and, therefore, $1/\tau_{\text{out}} \propto (T/\varepsilon_F)^3$ [21].

Conclusions.— We have analyzed the spectrum of the three-particle collision integral in a one-dimensional electron system. We found that zero modes, associated with the conservation laws, are separated by an energy gap from a continuum of propagating modes, and identified the gap with the relaxation rate τ_{th} relevant for thermalization of counter-propagating electrons. Our analysis applies to clean single channel quantum wires of spinless electrons at not too low densities. It is highly desirable, yet very challenging, to extend the analysis to lower densities where interactions become strong [14]. Also, inclusion of spin [4] presents an interesting problem, since at very low temperatures effect of spin-charge coupling becomes relevant [22].

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[1] D. B. Gutman, Y. Gefen, and A. D. Mirlin, Phys. Rev. Lett. **101**, 126802 (2008); Phys. Rev. B **81**, 085436 (2010); EPL **90**, 37003 (2010); Phys. Rev. B **80**, 045106 (2009).

[2] D. A. Bagrets, I. V. Gornyi, and D. G. Polyakov, Phys. Rev. B **80**, 113403 (2009); S. N. Dinh, D. A. Bagrets, A. D. Mirlin, Phys. Rev. B **81**, 081306 (R) (2010).

[3] S. Takei, M. Milletar, and B. Rosenow, Phys. Rev. B **82**, 041306 (2010).

[4] T. Karzig, L. I. Glazman, F. von Oppen, Phys. Rev. Lett. **105**, 226407 (2010).

[5] A. M. Lunde, K. Flensberg, and L. I. Glazman, Phys. Rev. B **75**, 245418 (2007).

[6] J. Rech and K. A. Matveev, Phys. Rev. Lett. **100**, 066407 (2008); J. Phys.: Condens. Matter **20**, 164211 (2008).

[7] J. Rech, T. Micklitz, and K. A. Matveev, Phys. Rev. Lett. **102** 116402 (2009); T. Micklitz, J. Rech and K. A. Matveev, Phys. Rev. B **81**, 115313 (2010); A. Levchenko *at. al.*, Phys. Rev. B **82**, 115413 (2010); A. Levchenko, Z. Ristivojevic, and T. Micklitz, Phys. Rev. B **83**, 041303(R) (2011).

[8] P. Degiovanni *et al.*, Phys. Rev. B **81**, 121302(R) (2010); A. M. Lunde, S. E. Nigg, and M. Büttiker, Phys. Rev. B **81**, 041311(R) (2010).

[9] M. Heyl *et al.*, Phys. Rev. B **82**, 033409 (2010).

[10] Y. F. Chen *et al.*, Phys. Rev. Lett. **102**, 036804 (2009).

[11] G. Granger, J. P. Eisenstein, and J. L. Reno, Phys. Rev. Lett. **102**, 086803 (2009).

[12] G. Barak *at. al.*, Nature Phys. **6**, 489 (2010).

[13] C. Altimiras *et al.*, Nature Phys. **6**, 34 (2009); Phys. Rev. Lett. **105**, 226804 (2010).

[14] K. A. Matveev, A. V. Andreev, and M. Pustilnik, Phys. Rev. Lett. **105**, 046401 (2010).

[15] O. Chiatti *at. al.*, Phys. Rev. Lett. **97**, 056601 (2006); Phys. Rev. B **75**, 245418 (2007).

[16] G. A. Brooker, J. Sykes, Phys. Rev. Lett. **21**, 279 (1968); H. H. Jensen, H. Smith and J. W. Wilkins, Phys. Lett. **27B**, 532 (1968).

[17] Y. M. Sirenko, V. Mitin, and P. Vasilopoulos, Phys. Rev. B **50**, 4631 (1994).

[18] See supplementary material at "http://PhysRevLett" for

additional information.
[19] F. Schwabl, *Quantum mechanics*, 2nd. ed. (Springer-Verlag, NY, 1995).
[20] Notice that the eigenvalue equation for $s = -$ allows for the second bound state solution $\psi^{-,2}(x)- = 3/\cosh^2 x$ with energy $\lambda_n = -3$. This is, however, an even parity function.
[21] For the crossover from finite temperature relaxation to zero temperature relaxation see Ref. [18].
[22] R. G. Pereira and E. Sela, Phys. Rev. B **82**, 115324 (2010); T. L. Schmidt, A. Imambekov, and L. I. Glazman, Phys. Rev. B **82**, 245104 (2010).