Signature of Local Motion in the Microwave Sky

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The Signature of Local Motion in the Microwave Sky

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For observers moving with respect to the cosmic rest frame, the microwave background temperature fluctuations will no longer be statistically isotropic. Aside from the familiar temperature dipole, an observer's velocity will also induce changes in the temperature angular correlation function and create non-zero off-diagonal correlations between multipole moments. We show that both of these effects should be detectable in future full-sky maps from the Planck satellite, and can constrain modifications of the standard cosmological model proposed to explain anomalous current observations.

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The most prominent feature in the microwave background radiation is the large dipole modulation, at a part in a thousand of the mean background temperature [1, 2]. This is generally attributed to our local peculiar velocity with respect to the cosmic rest frame of 370 km/s [3–5]. However, changing frames from the cosmic rest frame to a boosted frame also induces small, distinctive changes in both the cross-power spectrum and the correlation function of the microwave radiation, both of which are potentially detectable in full sky maps with the angular resolution of the Planck satellite. Here we present a straightforward calculation of the signals and discuss their detectability, and note that subtle microwave background distortions are a promising route for constraining "tilted" cosmological models where an isocurvature perturbation on the scale of the horizon contributes to the microwave dipole and to large-scale streaming motions of galaxies and galaxy clusters. The signals are also degenerate to the dipole distortion induced by gravitational lensing, so these signals can constrain models with suppressed fluctuations on large scales, proposed to explain the low value of the microwave background correlation function at large angles.

Similar calculations were pioneered by Challinor and van Leeuwen [6], but they were primarily concerned with demonstrating that the effects were small enough to be neglected when constraining cosmological parameters with the microwave background power spectrum; they did not consider the correlation function, and did not consider detectability of the signals or the resulting constraints. Burles and Rappaport [7] considered detectability of the aberration in the microwave radiation via the shift in angular scale of acoustic peaks it introduces; this effect is related to the correlation function analysis we present here. In a related paper, Menzies and Mathews [8] showed how to de-aberrate the microwave sky, with an eye towards full-sky analysis such as cosmic topology searches. Amendola and collaborators [9] have recently published similar calculations after a draft of this work appeared, and Pereira et al. [10] have argued that motion-induced effects can be important when estimating the power spectrum from data covering less than the full sky. Doppler shifts and dipole aberration due to local motion have also been probed at other wavelengths (e.g., infrared [11], optical [12, 13], X-ray [14, 15], radio [16, 17], and gamma ray [18]), but selection effects make precise measurements difficult.

Lorentz Transformation of the Temperature Field. Consider a frame S' which is the rest frame of the microwave background, so that $T'(\mathbf{n}')$ is the temperature distribution in this frame. Now take frame S to be our observation frame which is boosted from the cosmic rest frame by a velocity $\mathbf{v}$, with resulting sky temperature $T(\mathbf{n})$ (we use velocity units with $c = 1$). Theories of cosmology predict a sky map in the cosmic rest frame, while we observe a sky map in our boosted frame.

For a photon with wavevector $\mathbf{k}' = k'\mathbf{n}'$ in the rest frame and wavevector $\mathbf{k} = k\mathbf{n}$ in the observation frame, we have the usual wavenumber transformation $k = (1 + \mathbf{v}\cdot\mathbf{n}')(1 - v^2)^{-1/2}k' \equiv Dk'$ and the aberration equation

$$\hat{\mathbf{v}}\cdot\mathbf{n} = \frac{\hat{\mathbf{v}}\cdot\mathbf{n}' + v}{1 + \mathbf{v}\cdot\mathbf{n}'} \tag{1}$$

with $\hat{\mathbf{v}}$ a unit vector in the boost velocity direction. A clear discussion of how a radiation field then transforms under Lorentz boosts has been given by Ref. [19], which clarifies some misconceptions in earlier literature. The simple result is $T(\mathbf{n}) = DT'(\mathbf{n}')$, which gives the transformation between the microwave background rest-frame temperature distribution $T'(\mathbf{n}')$ and the observation-frame temperature distribution $T(\mathbf{n})$. The factor of D gives the usual dipole distortion, while the change in direction is the result of aberration.

Transformation of the Power Spectrum. Starting from this transformation of the temperature field on the sky, we now derive the transformation of observables which are commonly extracted from cosmological models, namely correlations of multipole moments and the angular correlation function. The (rest-frame) microwave sky temperature is commonly expressed in terms of spherical harmonics,

$$T'(\mathbf{n}') = \sum_{lm} a'_{lm} Y_{lm}(\mathbf{n}'), \tag{2}$$

where the angular power spectrum in terms of these coefficients is $C'_l = \langle a'^*_{lm} a'_{lm} \rangle$. Here the angle brackets refer to an ensemble average over realizations of a random temperature field on the sky with the same underlying statistical properties. If the rest frame universe is statistically isotropic, then each moment C'_l of the angular power spectrum is independent of m, and the average value of coefficients with different indices vanishes: $\langle a'^*_{l'm'} a'_{lm} \rangle = 0$ if $l \neq l'$ or $m \neq m'$. We want the transformation law connecting the coefficients in the two frames.

The individual a_{lm} values transform as follows:

$$a_{lm} = \int d\mathbf{n}\, T(\mathbf{n}) Y^*_{lm}(\mathbf{n}) = \int d\mathbf{n} \frac{1 + \mathbf{v} \cdot \mathbf{n}'}{\sqrt{1 - v^2}} T'(\mathbf{n}') Y^*_{lm}(\mathbf{n}). \tag{3}$$

Now we choose a spherical coordinate system with the $\mathbf{z}$-axis aligned with the boost direction, and change integration variables to the rest-frame angles with $\mathbf{n}' = (\theta', \phi)$,

$$a_{lm} = \int_0^\pi \sin\theta' d\theta' \int_0^{2\pi} d\phi \frac{\sqrt{1 - v^2}}{1 + v\cos\theta'} T'(\theta', \phi) Y^*_{lm}\left(\frac{\cos\theta' + v}{1 + v\cos\theta'}, \phi\right). \tag{4}$$

Then expanding the rest-frame temperature distribution in spherical harmonics and doing the trivial integral over ϕ gives the exact expression

$$a_{lm} = \sum_{l'=0}^{\infty} a'_{l'm} I^m_{l'l}(v) \tag{5}$$

(no sum over m) where we have defined

$$I^m_{l'l}(v) \equiv 2\pi\sqrt{1 - v^2} \int_{-1}^{1} \frac{dx}{1 + vx} \tilde{P}^m_{l'}(x) \tilde{P}^m_l\left(\frac{x + v}{1 + vx}\right) \tag{6}$$

with the abbreviation

$$\tilde{P}^m_l(x) = \left(\frac{2l + 1}{4\pi} \frac{(l - m)!}{(l + m)!}\right)^{1/2} P^m_l(x) \tag{7}$$

for the spherical-harmonic-normalized associated Legendre functions.

Some care must be taken in the numerical evaluation of Eq. (6), since the integrand is rapidly oscillating for large values of $l - |m|$ and standard numerical integrators have difficulty converging (but see [20] for a recursive technique). Direct numerical integration reveals the orthonormality relation

$$\sum_{l'} I^m_{l'l_1} I^m_{l'l_2} = \delta_{l_1 l_2} \tag{8}$$

(no sum on m) which is valid for any velocity v.

From Eq. (5), we have the boosted-frame products of coefficients

$$\left\langle a^*_{l_1 m_1} a_{l_2 m_2} \right\rangle = \delta_{m_1 m_2} \sum_{l'} C'_{l'} I^{m_1}_{l'l_1} I^{m_2}_{l'l_2}. \tag{9}$$

Note that the statistical ensemble averaging procedure on the left side of this expression is independent of frame. The fact that Eq. (9) has nonzero values for $l_1 \neq l_2$ is a direct reflection of the breaking of statistical isotropy due to the preferred direction of our local motion.

Using the rest-frame power spectrum C'_l given by the WMAP+ACT best-fit cosmology [21], direct numerical evaluation of Eq. (9) with $v = 0.00123$ gives that $C_1 \approx 5000 C'_1$ for the observed dipole, and the fractional corrections $(C'_2 - C_2)/C_2 \approx 6 \times 10^{-3}$ for the quadrupole and $(C'_l - C_l)/C_l$ ranging between 10^{-6} and 5×10^{-5} for various l between 3 and 1500. These corrections to the power spectrum are too small to be observed, given the cosmic variance. Correlations with $|l_2 - l_1| \geq 2$ are at most $\mathcal{O}(v^2)$ or smaller and also undetectably small as verified by direct numerical calculation.

However, for the case $l_2 = l_1 + 1$, a linear asymptotic expansion in v for $I^m_{ll'}$ is sufficient for a consistent evaluation for $l_1 - |m_1| \lesssim 1/v$, yielding

$$\left\langle a^*_{l+1,m} a_{lm} \right\rangle \sim (C'_{l+1} - C'_l) v (l+1) A_{lm} + \mathcal{O}(v^2) \tag{10}$$

with the abbreviation $A_{lm} \equiv [(l+m+1)(l-m+1)/((2l+1)(2l+3))]^{1/2}$. Since roughly $C'_l \approx l^{-2} C'_2$ for l up to roughly 1000 (neglecting acoustic oscillations), $C'_{l+1} - C'_l \approx -2C'_l/l$ and $\langle a^*_{l+1,m} a_{lm} \rangle \approx -vC'_l$ for large l and small m. This signal can be detected statistically, as shown below.

Transformation of the Correlation Function. The change in the two-point correlation function is also interesting, since distortions in the shapes of microwave hot and cold spots due to the boost is an effect in angle θ space and not in multipole l space. For two sky directions $\mathbf{n}_1$ and $\mathbf{n}_2$, the two-point correlation function is defined as

$$C(\mathbf{n}_1, \mathbf{n}_2) \equiv \langle T(\mathbf{n}_1) T(\mathbf{n}_2) \rangle \,. \tag{11}$$

In the rest frame, which we assume to be statistically isotropic, the correlation function $C'(\mathbf{n}'_1, \mathbf{n}'_2)$ depends only on the angle between the two observation directions $\mathbf{n}'_1 \cdot \mathbf{n}'_2$. Substituting the temperature transformation directly into Eq. (11) yields the exact result

$$C(\mathbf{n}_1, \mathbf{n}_2) = \frac{1 + \mathbf{v} \cdot \mathbf{n}'_1 + \mathbf{v} \cdot \mathbf{n}'_2 + (\mathbf{v} \cdot \mathbf{n}'_1)(\mathbf{v} \cdot \mathbf{n}'_2)}{1 - v^2} C'(\mathbf{n}'_1 \cdot \mathbf{n}'_2). \tag{12}$$

As statistical isotropy is broken by the Lorentz boost, the correlation function now depends on the two directions separately. The rest-frame correlation function is modified at $\mathcal{O}(v)$ and varies with the angle between the boost direction and the observation direction. To linear order in v, we have $C(\mathbf{n}_1, \mathbf{n}_2) \simeq (1 + \mathbf{v} \cdot \mathbf{n}'_1 + \mathbf{v} \cdot \mathbf{n}'_2) C'(\mathbf{n}'_1 \cdot \mathbf{n}'_2)$.

Detectability. The off-diagonal correlation Eq. (10) can be detected over cosmic variance noise with high-resolution full-sky maps. For a full-sky microwave temperature map with $N_{\rm pix}$ pixels, each with Gaussian noise $\sigma_{\rm pix}$, and a Gaussian beam of width σ_b, the measured amplitude of each a_{lm} is approximately normally distributed with a variance $\sigma_l \equiv C_l \exp(-l^2 \sigma_b^2) + w^{-1}$ [22] and uncorrelated with other a_{lm} values, where $w^{-1} \equiv 4\pi \sigma^2_{\rm pix}/N_{\rm pix}$ is the inverse statistical weight per unit solid angle. A measurement of the multipoles a_{lm} up to a maximum multipole moment $l = l_{\rm max}$ provides $l^2_{\rm max} - 4$ estimates of the boost velocity, namely the quantities

$$v^{\rm est}_{lm} \simeq \frac{a^*_{l+1,m} a_{lm}}{(l+1) A_{lm} (C'_l - C'_{l+1})} \tag{13}$$

for $2 < l < l_{\rm max} - 1$ and $-l \leq m \leq l$. Each of these velocity estimates has a standard error of approximately $\sigma_{lm} \simeq \sqrt{2C_l\sigma_l}/((l+1)A_{lm}(C_l - C_{l+1}))$ which uses the approximations $C'_l \simeq C_l$ and $C_{l+1} C_l \simeq C^2_l$. Then estimating the velocity as a signal-to-noise weighted sum over l and m of the individual estimators $v^{\rm est}_{lm}$ each with signal-to-noise ratio of v/σ_{lm} gives a standard error on v from a full-sky map using multipoles $2 \leq l < l_{\rm max}$ of

$$\sigma_v \simeq \frac{\pi l_{\rm max}}{2^{3/2}} \left[\sum_{l=2}^{l_{\rm max}-1} \frac{(l+1)^3}{\sqrt{(2l+3)(2l+1)}} \left(1 - \frac{C_{l+1}}{C_l}\right) \left(\frac{C_l}{\sigma_l}\right)^{1/2} \right]^{-1} . \tag{14}$$

where the sum over m of A_{lm} has been approximated by an elementary integral.

The Planck satellite's 143 GHz channel has approximately $\sigma_b = 3.1$ arcminutes, and $N_{\rm pix} = 2.9 \times 10^6$ with a target noise level of $\sigma_{\rm pix} = 6.0$ μK, giving $w^{-1} = 1.6 \times 10^{-4}$ μK^2. The upper limit $l_{\rm max}$ is determined by the largest l for which systematic errors in beam characterization do not dominate the error model for a_{lm}. For $l_{\rm max} = 2000$, Eq. (14) gives $\sigma_v = 2.5 \times 10^{-4}$. If the dipole is due entirely to our peculiar velocity, $v = 0.00123$ and Planck can detect this signal through the off-diagonal cross-power signal at a signal-to-noise ratio of 5. For $l_{\rm max} = 2500$, the signal-to-noise ratio increases to 6. Other Planck channels will provide independent estimates and further increase the signal-noise ratio, which will allow probing the l and m dependence of the signal: if more frequencies reduce the noise per pixel by a factor of 2, the signal-to-noise ratio increases to 10. These estimates rely on the linear approximation

Eq. (10), which is not necessarily accurate for some terms with large l and small m; more accurate calculations require direct numerical calculation of all terms and efficient techniques for this will be presented elsewhere [23]. Foreground emission and partial sky coverage may in practice reduce somewhat the significance of a detection, although neither has greatly impacted measurements of the temperature power spectrum.

Detectability of the small corrections in Eq. (12) is harder to estimate, since values of the correlation function for similar angles are highly correlated. At a separation θ, a map has approximately $(2\pi\theta/\sigma_b)N_{\rm pix}$ pairs; for the Planck map above and, e.g., $\theta = 10^\circ$, this is about 3.5×10^9 pairs. Averaging over all pairs of pixels, each with Gaussian error $\sigma_{\rm pix}$, and propagating through the statistical errors on each pixel gives the standard error on $C(\theta)$ as $\sigma_\theta = \sigma_{\rm pix}\sqrt{2C(0)/N_{\rm pairs}}$. For a monopole and dipole-subtracted map, $C(0) = \sum_l (2l+1)C_l \exp(-l^2\sigma_b^2)/(4\pi) = 1.1 \times 10^4$ μK^2 for the Planck beam above, so $\sigma_\theta = 0.015\ \mu{\rm K}^2$, compared to a signal of $C(\theta = 10^\circ) \gtrsim 1000\ \mu{\rm K}^2$ [24]. Testing the form of Eq. (12) requires comparing different portions of the sky for a variation in the correlation function of a part in a thousand. The correlation function can be estimated at many different angles, with each providing a moderate signal-to-noise measurement of the difference in the correlation function between different sky regions. However, this estimate includes only instrumental noise, and does not account for cosmic variance between regions; more precise detectability estimates require evaluation of both the signal covariance for different angles and cosmic variance for different regions (e.g., [25]). Correlations from foreground emission are an additional challenge for this measurement.

Cosmological Implications. While calculations in this paper have been motivated by the effect of our local motion on the cosmic microwave background, an identical effect is induced by gravitational lensing. Microwave background photons are deflected by a few arcminutes on average due to large-scale mass fluctuations; this signal has recently been detected [26]. If the deflection field is decomposed into multipole moments, the dipole moment of the deflection field has an identical functional form as the aberration from a boost, and with a similar amplitude in the standard cosmological model, but with a random direction with respect to our local velocity [27]. In the present analysis we have assumed we know the direction of the boost; to detect the lensing signal requires understanding how the signal varies with assumed direction [9, 23]. To the extent that the direction can be measured, then the dipole lensing signal can be extracted by comparing with the boost signal inferred from the temperature dipole. This would put interesting constraints on modifications of the standard cosmology which suppress power on horizon scales [28, 29] in order to explain the anomalously low correlation function of the microwave background at angles larger than 60 degrees [30].

Aside from being a consistency check on a fundamental cosmological property, the distinctive microwave background signals from a local velocity with respect to the microwave background rest frame will constrain "tilted" cosmological models where the dipole arises partly due to primordial superhorizon-scale isocurvature fluctuations [31–36]. Such models might naturally explain surprising recent claims of a substantial galaxy cluster bulk flow on Hubble volume scales [37] and galaxy bulk flow on somewhat smaller scales [38].

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