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# Dihadron fragmentation functions for large invariant mass

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Using perturbative Quantum Chromodynamics, we compute dihadron fragmentation functions for a large invariant mass of the dihadron pair. The main focus is on the interference fragmentation function  $H_1^\triangleleft$ , which plays an important role in spin physics of the nucleon. Our calculation also reveals that  $H_1^\triangleleft$  and the Collins fragmentation function have a closely related underlying dynamics. By considering semi-inclusive deep-inelastic scattering, we further show that collinear factorization in terms of dihadron fragmentation functions, and collinear factorization in terms of single hadron fragmentation functions provide the same result in the region of intermediate invariant mass.

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*I. Introduction.*—Fragmentation functions (FFs) for quarks and gluons parameterize the hadronization taking place in high-energy scattering processes with identified strongly interacting particles in the final state. The main focus is typically on FFs describing the transition of a parton into a single hadron — see Ref. [1] for a field-theoretic definition of such objects. However, already in the late 1970's dihadron fragmentation functions (DiFFs) were introduced in order to quantify the hadron structure of jets [2]. Moreover, it was shown that DiFFs are needed to obtain a consistent result for the production of two hadrons in electron-positron annihilation when working beyond leading order in perturbative Quantum Chromodynamics (QCD) [3]. In the meantime, DiFFs also play a considerable role in heavy ion physics [4].

In 1993, it was proposed [5] that quark fragmentation into two hadrons can also be used to address the transversity distribution  $h_1$  of the nucleon [6]. To this end, one can study the production of two hadrons in semi-inclusive deep-inelastic scattering (DIS) in the current fragmentation region [5]. (A very similar idea was put forward in Ref. [7].) If the target is transversely polarized, there exists a correlation between the spin vector of the target and the orientation of the plane given by the momenta of the two hadrons. This observable contains the product of  $h_1$  and a new fragmentation function ( $H_1^\triangleleft$  in the notation of Ref. [8]), which describes the strength of a correlation between the transverse polarization of the fragmenting quark and the orientation of the hadron plane. Like the transversity,  $H_1^\triangleleft$  is chiral-odd, and it results from the interference between two different production amplitudes why it is normally referred to as interference fragmentation function in the literature.

Data on the mentioned observable in semi-inclusive DIS have already been obtained by the HERMES and COMPASS Collaborations [9, 10]. The major difficulty is that, *a priori*, both  $h_1$  and  $H_1^\triangleleft$  are unknown. Existing models for  $H_1^\triangleleft$  [11] are still in a too early stage for getting a quantitative constraint on the transversity. (For a related discussion we refer to [12].) However, as was shown in [13], one can measure two back-to-back hadron pairs

in electron-positron annihilation in order to get a handle on  $H_1^\triangleleft$  — see also [14], and Ref. [15] for preliminary data from the Belle Collaboration. A combined analysis of the two processes allows one, in principle, to extract both unknown functions [16]. Such a strategy would be very similar to the combined analysis of the Collins effect [17] in semi-inclusive DIS and in electron-positron annihilation [18].

The interference FF  $H_1^\triangleleft$  and the Collins function ( $H_1^\perp$  in the notation of Refs. [19]) can be considered as complementary tools for getting a handle on the transversity distribution, with both having advantages and drawbacks. An important advantage in the case of  $H_1^\triangleleft$  is the fact that one can integrate over the total transverse momentum of the two hadrons in the final state, leading to a collinear factorization formula. In contrast, the Collins effect relies on factorization in terms of transverse momentum dependent parton correlators (TMD-factorization) [20], which has additional technical complications. On the other hand, when using  $H_1^\triangleleft$ , the dependence on the relative transverse momentum of the two hadrons must be kept in order to have a well-defined hadron plane. This implies that  $H_1^\triangleleft$  must also depend on the invariant mass  $M_{hh}$  of the dihadron system. If  $M_{hh}$  is of the order of  $\Lambda_{QCD}$ , DiFFs are entirely non-perturbative objects. In this kinematical region, one can only fit the DiFFs to experimental data or try to estimate them by using some model for the strong interaction in the non-perturbative regime [11].

In the present paper, we apply perturbative QCD in order to evaluate DiFFs for  $M_{hh} \gg \Lambda_{QCD}$ . The main focus is on the interference FF  $H_1^\triangleleft$ . For large  $M_{hh}$ , the DiFFs can be expressed as a convolution of hard coefficients and (collinear) single hadron fragmentation correlators. In particular, the calculation determines the behavior of DiFFs as a function of  $M_{hh}$ . We also argue that  $H_1^\triangleleft$  and the Collins function  $H_1^\perp$  (at large transverse momentum [21]) depend on the same two collinear twist-3 fragmentation correlators. In addition, we compute the (transverse) spin dependent cross section for dihadron production in semi-inclusive DIS for  $\Lambda_{QCD} \ll M_{hh} \ll Q$

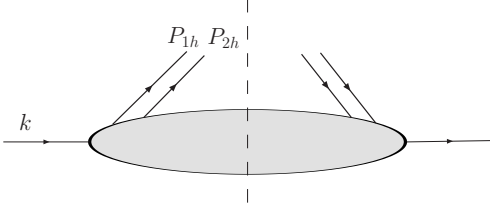


FIG. 1: Dihadron fragmentation function for a quark.

( $Q$  denoting the virtuality of the exchanged photon) with single hadron FFs. By comparing this result with the cross section obtained in the framework of DiFFs, we show that collinear factorization in terms of DiFFs holds as long as  $M_{hh} \ll Q$ . In its spirit, our study is similar to recent work in which certain transverse momentum dependent parton correlators were evaluated for large transverse momenta, and the matching between collinear factorization and TMD-factorization was explicitly shown for intermediate transverse momenta [21, 22].

*II. Kinematics and definition of dihadron fragmentation functions.*— We start by discussing the kinematics for the fragmentation of a quark into two hadrons (displayed for the squared amplitude also in Fig. 1),

$$q(k) \rightarrow h_1(P_{1h}) + h_2(P_{2h}) + X. \quad (1)$$

We assume that the quark has a large light-cone minus momentum  $k^-$ . For later convenience, we choose a reference frame in which one hadron has no transverse momentum. In the hadron-1 frame, for instance, the light-cone components of the hadron momenta can be represented as

$$\begin{aligned} P_{1h} &= (0, z_{1h}k^-, 0), \\ P_{2h} &= \left( \frac{M_{hh}^2}{2z_{1h}k^-}, z_{2h}k^-, \sqrt{\frac{z_{2h}}{z_{1h}}} M_{hh} \right). \end{aligned} \quad (2)$$

Neglecting the hadron masses, one readily verifies that  $(P_{1h} + P_{2h})^2 = M_{hh}^2$ . We also introduce the total hadron momentum and the momentum difference according to

$$P_{hh} = P_{1h} + P_{2h}, \quad R = \frac{P_{1h} - P_{2h}}{2}. \quad (3)$$

Their minus momenta are given by  $P_{hh}^- = z_{hh}k^-$ ,  $R^- = \hat{z}_{hh}k^-$ , with  $z_{hh} = z_{1h} + z_{2h}$  and  $\hat{z}_{hh} = (z_{1h} - z_{2h})/2$ .

The operator definition of the unpolarized DiFF and the interference DiFF, for a quark flavor  $q$ , reads

$$\begin{aligned} & \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{d\hat{z}_{hh}}{32(2\pi)^3} \frac{z_{hh}^2}{z_{1h}^2} \sum_X \int \frac{dy^+}{2\pi} e^{ik^-y^+} \\ & \times \langle 0 | \psi^q(y^+) | P_{1h}, P_{2h}, X \rangle \langle P_{1h}, P_{2h}, X | \bar{\psi}^q(0) | 0 \rangle \\ & = \frac{\gamma^+}{2} D_1^q(z_{hh}, M_{hh}^2) + \frac{\sigma^{\alpha+} R_{T\alpha}}{2|\vec{R}_T|} H_1^{\alpha q}(z_{hh}, M_{hh}^2), \end{aligned} \quad (4)$$

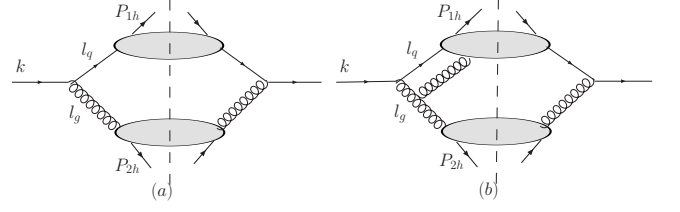


FIG. 2: Generic diagrams contributing to DiFFs for large invariant mass. For  $D_1$  the diagram (a) needs to be considered, whereas in the case of  $H_1^{\alpha}$  both diagrams are relevant.

where a gauge link has been suppressed. Note that our definition of  $H_1^{\alpha}$  differs from the one in Ref. [8] by some prefactors. We have integrated over the relative longitudinal momentum fraction  $\hat{z}_{hh}$ , which is needed if one wants to apply the collinear factorization discussed in the next section.

With these conventions, the parton model cross section for the production of two hadrons in semi-inclusive DIS (with a transversely polarized proton),  $ep^\dagger \rightarrow eh_1h_2X$ , takes the form

$$\begin{aligned} \frac{d\sigma}{dx_B dy d\phi_S dz_{hh} dM_{hh}^2 d\phi_R} &= \frac{2\alpha_{em}^2 s x_B}{Q^4} \sum_q e_q^2 \\ & \times \left[ \left( 1 - y + \frac{y^2}{2} \right) f_1^q(x_B) D_1^q(z_{hh}, M_{hh}^2) \right. \\ & \left. + (1 - y) \sin(\phi_R + \phi_S) h_1^q(x_B) H_1^{\alpha q}(z_{hh}, M_{hh}^2) \right]. \end{aligned} \quad (5)$$

Equation (5) contains both the unpolarized cross section and one component depending on the transverse target polarization. The azimuthal angle between  $\vec{R}_T$  and the lepton plane is denoted by  $\phi_R$ , the azimuthal angle of the transverse spin vector of the proton is denoted by  $\phi_S$ , while  $x_B$  and  $y$  are the commonly used DIS variables.

*III. Dihadron fragmentation functions at large invariant mass.*— When  $M_{hh} \gg \Lambda_{QCD}$ , dihadron fragmentation of a quark can be viewed as a two-step process: first, the quark splits into a quark (with momentum  $l_q$ ) and a gluon (with momentum  $l_g$ ), which is calculable in perturbative QCD. Second, each of these two partons fragments into a single hadron. This scenario is illustrated in Fig. 2.

We introduce momentum fractions  $(z_1, z_2)$  through  $l_q = P_{1h}/z_1$ ,  $l_g = P_{2h}/z_2$ , and define  $\xi = z_{1h}/z_1$ . Evaluating the diagram in Fig. 2(a) one finds for the unpolarized DiFF

$$\begin{aligned} D_1^q(z_{hh}, M_{hh}^2) &= \frac{\alpha_s}{2\pi M_{hh}^2} \int_{-\frac{1}{2}}^{\frac{1}{2}} d\hat{z}_{hh} \int_{z_{1h}}^{1-z_{2h}} \frac{d\xi}{\xi(1-\xi)} \\ & \times C_F \frac{1+\xi^2}{1-\xi} D_1^{h_1/q} \left( \frac{z_{1h}}{\xi} \right) D_1^{h_2/g} \left( \frac{z_{2h}}{1-\xi} \right), \end{aligned} \quad (6)$$

with  $D_1^{h_1/q}$  and  $D_1^{h_2/g}$  representing unpolarized single-hadron FFs. A second term, where hadron 1 originates

from the fragmentation of the gluon, is not written for brevity. The result in Eq. (6) shows that the DiFF  $D_1$  drops like  $1/M_{hh}^2$ . Note also that (6) allows one to recover the inhomogeneous part of the evolution equation for  $D_1$  by integrating over  $M_{hh}^2$  [2, 3].

Computing  $H_1^{\leftarrow}$  for large  $M_{hh}$  is much more involved as one has to use collinear twist-3 factorization [23, 24]. One contribution arises from the diagram in Fig. 2(a), if the relative transverse momentum between the quark with momentum  $l_q$  and the hadron 1 is kept. Also, 3-parton correlators including a transverse gluon field  $A_\perp$  enter the calculation — see the sample diagram in Fig. 2(b). The correlators associated with these two contributions (the so-called  $\partial_\perp$ -contribution and the  $A_\perp$ -contribution) are of the form  $\langle \bar{\psi} \partial_\perp \psi \rangle$  and  $\langle \bar{\psi} A_\perp \psi \rangle$ , respectively. In this paper, contributions from 3-gluon correlators are neglected since they do not affect any of our general results. We leave this part, details of the calculation, and the discussion about a potential singularity in (6) at  $\xi = 0$  and  $\xi = 1$  for future work [25].

The calculation of the hard coefficient for the  $A_\perp$ -contribution is essentially identical to the corresponding part in the treatment of the Collins function at large transverse momentum [21]. On the other hand, to obtain the  $\partial_\perp$ -contribution is more complicated than for the Collins function evaluation. To this end, we assign a (relative) transverse quark momentum according to  $l_q = P'_{1h}/z'_1 + l_{q\perp}$ , where  $z'_1 = z_1 + \delta z_1$ , and  $P'_{1h} = P_{1h} + \delta P_{1h}$ . To keep  $M_{hh}$  fixed,  $P_{2h}$  must also change, i.e.,  $P'_{2h} = P_{2h} + \delta P_{2h}$ . In particular,  $\delta P_{2h\perp} = -z_2 l_{q\perp}$ . The kinematics is entirely determined by the constraints

$$\begin{aligned} P_{1h} \cdot \delta P_{1h} &= 0, & P_{2h} \cdot \delta P_{2h} &= 0, \\ P_{1h} \cdot \delta P_{2h} + P_{2h} \cdot \delta P_{1h} &= 0, \\ \delta P_{1h\perp} + \delta P_{2h\perp} &= 0, & \delta P_{1h\perp} &= 0, \end{aligned} \quad (7)$$

where we use the on-shell condition for the two hadrons, constraints from keeping  $M_{hh}$  and  $z_{hh}$  fixed, and the fact that we are working in the hadron-1 frame. (Recently, we used a related approach for computing a particular single spin asymmetry in the Drell-Yan process [26].) The solution to the set of equations in (7) reads

$$\begin{aligned} \delta P_{1h} &= \left( 0, \frac{2k^- l_{q\perp}}{P_{2h\perp}} \frac{z_2 z_{1h} z_{2h}}{z_{hh}}, 0 \right), \\ \delta P_{2h} &= \left( -\frac{P_{2h\perp} l_{q\perp}}{k^-} \frac{z_2}{z_{hh}}, -\frac{2k^- l_{q\perp}}{P_{2h\perp}} \frac{z_2 z_{1h} z_{2h}}{z_{hh}}, -z_2 l_{q\perp} \right), \\ \delta z_1 &= \frac{2}{z_{hh}} z_1 z_2 (z_2 - z_1) (1 - \xi) \frac{l_{q\perp}}{P_{2h\perp}}. \end{aligned} \quad (8)$$

The partonic scattering amplitude  $M$  depends now on  $l_{q\perp}$ . We expand  $M$ ,

$$\begin{aligned} M(P'_{1h}, P'_{2h}, z'_1) &= M(P_{1h}, P_{2h}, z_1) \\ &+ \frac{\partial M(P'_{1h}, P'_{2h}, z'_1)}{\partial l_{q\perp}} \Big|_{l_{q\perp}=0} l_{q\perp} + \dots \end{aligned} \quad (9)$$

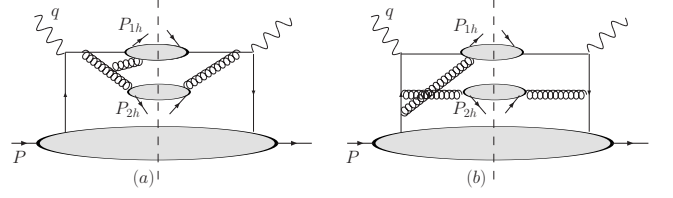


FIG. 3: Sample diagrams contributing to dihadron production in semi-inclusive DIS. Diagram (a) generates a leading power contribution and diagram (b), in the light-cone gauge, is power-suppressed for  $M_{hh} \ll Q$ .

and keep the term linear in  $l_{q\perp}$  for obtaining the relevant (twist-3)  $\partial_\perp$ -contribution [24, 25].

Both the  $\partial_\perp$ -contribution and the  $A_\perp$ -contribution can be brought into a gauge invariant form. After collecting all the pieces we find [25]

$$\begin{aligned} H_1^{\leftarrow q}(z_{hh}, M_{hh}^2) &= \frac{\alpha_s}{2\pi M_{hh}^3} \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{d\hat{z}_{hh}}{\sqrt{z_{1h} z_{2h}}} \int_{z_{1h}}^{1-z_{2h}} \frac{d\xi}{\xi} \\ &\times A^{h_1/q}(\hat{z}_{hh}, \xi) D_1^{h_2/g} \left( \frac{z_{2h}}{1-\xi} \right), \end{aligned} \quad (10)$$

where the function  $A$  is defined as

$$\begin{aligned} A &= C_F \left[ \left( z_1^3 \frac{\partial}{\partial z_1} \frac{\hat{H}(z_1)}{z_1^2} \right) 2\xi^2 \frac{z_1 - z_2}{z_{hh}} + \hat{H}(z_1) \frac{2\xi^2}{1-\xi} \right] \\ &+ \int \frac{d\bar{z}_1}{z_1^2} PV \left( \frac{1}{\frac{1}{z_1} - \frac{1}{\bar{z}_1}} \right) \hat{H}_F(z_1, \bar{z}_1) \\ &\times \left[ -C_F \frac{2z_{1h}}{z_1} \left( 1 + \frac{z_{1h}}{\bar{z}_1} - \frac{z_{1h}}{z_1} \right) \right. \\ &\left. - \frac{C_A}{2} \frac{2z_{1h}}{z_1} \frac{z_1 \bar{z}_1 (z_1 + \bar{z}_1) - z_{1h} (z_1^2 + \bar{z}_1^2)}{z_1 (z_1 - \bar{z}_1) (\bar{z}_1 - z_{1h})} \right]. \end{aligned} \quad (11)$$

This result shows that  $H_1^{\leftarrow}$  behaves like  $1/M_{hh}^3$  for large  $M_{hh}$ . It also reveals an intimate connection between  $H_1^{\leftarrow}$  and the Collins function (at large transverse momentum) [21], as both functions depend on the same collinear twist-3 correlation functions  $\hat{H}$  and  $\hat{H}_F$ , which we take in the definition of Ref. [21]. The hard coefficients (of the  $\partial_\perp$ -contribution) differ in both cases.

*IV. Single spin asymmetry for dihadron production in semi-inclusive DIS*—In this section, we investigate the validity of the factorization formula (5) for the cross section of dihadron production in semi-inclusive DIS for the case  $\Lambda_{QCD} \ll M_{hh} \ll Q$ . We focus on the discussion of the spin-dependent component of the cross section, which is nontrivial already at lowest order in perturbation theory.

For  $M_{hh}$  of the order of  $Q$ , one can use collinear factorization in terms of single hadron fragmentation correlators. Sample diagrams are shown in Fig. 3. We have evaluated the cross section for this kinematics, and then expanded the result for  $\Lambda_{QCD} \ll M_{hh} \ll Q$ . For factorization in terms of DiFFs to hold, the result has to match

with (5), if  $H_1^\triangleleft$  for  $M_{hh} \gg \Lambda_{QCD}$  from (10) is inserted. By inspecting the Feynman diagrams it becomes obvious that this matching is indeed nontrivial. For instance, diagram (b) in Fig. 3 has no counterpart in the calculation of  $H_1^\triangleleft$  for large  $M_{hh}$ . However, it turns out that for  $M_{hh} \ll Q$  this diagram is power-suppressed. This is true only in the light-cone gauge  $A^- = 0$  which we use for this analysis [25]. In the end we indeed find a matching, showing the consistency of the factorization in terms of DiFFs for  $M_{hh} \gg \Lambda_{QCD}$  as long as  $M_{hh} \ll Q$ .

*V. Conclusions.* — In this paper, we have studied DiFFs for a large invariant mass  $M_{hh}$  of the dihadron pair, where perturbative QCD can be applied. The main focus has been on the interference DiFF  $H_1^\triangleleft$ , which drops like  $1/M_{hh}^3$  for large  $M_{hh}$  and is related to the same universal twist-3 collinear fragmentation correlators that describe the Collins FF  $H_1^\perp$  (at large transverse momentum). The analysis also predicts that the transverse single spin asymmetry for dihadron production in semi-inclusive DIS behaves like  $1/M_{hh}$ . The preliminary COMPASS data [10], ranging up to  $M_{hh} \approx 2 GeV$ , are in agreement with this general result. We expect a corresponding behavior for the so-called Artru-Collins asymmetry in electron-positron annihilation [13], for which preliminary data from Belle exist [15]. For the case of semi-inclusive DIS we have also shown explicitly, to lowest nontrivial order in perturbation theory, that collinear factorization in terms of DiFFs is valid for large  $M_{hh}$  provided that  $M_{hh} \ll Q$ .

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