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Evidence for a Bound H-dibaryon from Lattice QCD

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We present evidence for the existence of a bound H-dibaryon, an $I = 0$, $J = 0$, $s = -2$ state with valence quark structure *uuddss*, at a pion mass of $m_\pi \sim 389$ MeV. Using the results of Lattice QCD calculations performed on four ensembles of anisotropic clover gauge-field configurations, with spatial extents of $L \sim 2.0$, 2.5, 3.0 and 3.9 fm at a spatial lattice spacing of $b_s \sim 0.123$ fm, we find an H-dibaryon bound by $B^{\rm H}_\infty = 16.6 \pm 2.1 \pm 4.6$ MeV at a pion mass of $m_\pi \sim 389$ MeV.

It is now well established that quantum chromodynamics (QCD), the theory describing the dynamics of quarks and gluons, and the electroweak interactions, underlie all of nuclear physics, from the hadronic mass spectrum to the synthesis of heavy elements in stars. To date, there have been few quantitative connections between nuclear physics and QCD, but fortunately, Lattice QCD is entering an era in which precise predictions for hadronic quantities with quantifiable errors are being made. This development is particularly important for processes which are difficult to explore in the laboratory, such as hyperon-hyperon and hyperon-nucleon interactions for which knowledge is scarce, primarily due to the short lifetimes of the hyperons, but which may impact the late-stages of supernovae evolution. In this letter we report strong evidence for a bound H-dibaryon, a six-quark hadron with valence structure *uuddss*, from $n_f = 2+1$ Lattice QCD calculations at light-quark masses that give the pion a mass of $m_\pi \sim 389$ MeV.

The prediction of a relatively deeply bound system with the quantum numbers of $\Lambda\Lambda$ (called the H-dibaryon) by Jaffe [1] in the late 1970s, based upon a bag-model calculation, started a vigorous search for such a system, both experimentally and also with alternate theoretical tools. Experimental constraints on, and phenomenological models of, the H-dibaryon can be found in Refs. [2–4]. While experimental studies of doubly-strange hypernuclei restrict the H-dibaryon to be unbound or to have a small binding energy, the most recent constraints on the existence of the H-dibaryon come from heavy-ion collisions at RHIC, from which it is concluded that the H-dibaryon does not exist in the mass region $2.136 < M_{\rm H} < 2.231$ GeV [5], effectively eliminating the possibility of a loosely-bound H-dibaryon at the physical light-quark masses. Recent experiments at KEK suggest there is a resonance near threshold in the H-dibaryon channel [6].

The first study of baryon-baryon interactions with Lattice QCD was performed more than a decade ago [7, 8]. This calculation was quenched and with $m_\pi \gtrsim 550$ MeV. The NPLQCD collaboration performed the first $n_f = 2+1$ QCD calculations of baryon-baryon interactions [9, 10] at low-energies but at unphysical pion masses. Quenched and dynamical calculations were subsequently performed by the HALQCD collaboration [11, 12]. A number of quenched Lattice QCD calculations [13–18] have searched for the H-dibaryon, but to date no definitive results have been reported. Earlier work concluded that the H-dibaryon does not exist as a stable hadron in quenched QCD [17], while more recent work [18, 19] finds a hint of a bound state. By inserting energy- and sink-dependent potentials into the Schrödinger equation in the SU(3) limit, a hint of an H-dibaryon has been found in Ref. [20], however, this hint evaporates when SU(3)-breaking is included [21].

In this work, Lüscher's method [22–25] is employed to extract two-particle scattering amplitudes below inelastic thresholds from Lattice QCD calculations. In the situation where only a single scattering channel is kinematically allowed, the deviation of the energy eigenvalues of the two-hadron system in the lattice volume from the sum of the single-hadron masses is related to the scattering phase shift, $\delta(q)$, as is made explicit in eq. (1). The Euclidean time behavior of Lattice QCD correlation functions of the form $C_\chi(t) = \langle 0|\chi(t)\chi^\dagger(0)|0\rangle$, where χ represents an interpolating operator with the quantum numbers of the one-particle or two-particle systems under consideration, determines the ground state energies of the one-particle and two-particle systems, $E_1 = m$ and E_2, respectively (we focus only on the ground state of the two-particle system in this work). The form of the interpolating operators, and the methodology used for extracting the energy shift are discussed in detail in Ref. [26]. For gauge-field configurations that have different lattice spacings in the temporal and spatial directions (anisotropic lattices), the two-particle energy is given by $E_2 = 2\sqrt{q^2/\xi^2 + m^2}$, where $\xi = b_s/b_t$ is the lattice anisotropy. By computing the mass of the particle and the ground-state energy of the two-particle system, the squared momentum, q^2 (in spatial lattice units (s.l.u)), which can be either positive or negative, is determined by this relation. For s-wave scattering below inelastic thresholds, q^2 is related to the real part of the inverse scattering amplitude through the eigenvalue equation [23] (neglecting phase-shifts in $l \geq 4$ partial-waves)

$$q \cot \delta(q) = \frac{1}{\pi\ L} S\left(q^2 \left(\frac{L}{2\pi}\right)^2\right) , \tag{1}$$

where the S-function is given by

$$S(x) = \lim_{\Lambda\to\infty} \sum_{\mathbf{j}}^{|\mathbf{j}|<\Lambda} \frac{1}{|\mathbf{j}|^2 - x} - 4\pi\ \Lambda\, . \tag{2}$$

This relation provides a Lattice QCD determination of the value of the phase shift at the momentum $\sqrt{q^2}$.

Determining energy-levels with the same quantum numbers in multiple volumes allows for bound states to be distinguished from scattering states. A bound state corresponds to a pole in the S matrix, and in the case of a single scattering channel, is signaled by $\cot \delta(q) \to +i$ in the large volume limit. Writing $q = i\kappa$ for two-particle states that are negatively shifted in energy, $E_2 < 2m$, in the lattice volume, the volume dependence of the binding momentum in the large volume limit follows directly from eq. (1) and is of the form [25]

$$\kappa \ = \ \gamma \ + \ \frac{g_1}{L} \left(\ e^{-\gamma L} \ + \ \sqrt{2}\ e^{-\sqrt{2}\gamma L} \ \right) \ + \ \ldots\ , \tag{3}$$

where γ is the infinite-volume value of the binding momentum, under the assumption that $\gamma \ll m_\pi$, and g_1 is treated as a fit parameter. With calculations in two or more lattice volumes that both have $q^2 < 0$ and $q \cot \delta(q) < 0$ it is possible to perform an extrapolation with eq. (3) to the infinite-volume limit to determine the binding energy of the bound state, $B_\infty = \gamma^2/m$. The range of nuclear interactions is set by the pion mass, and therefore the use of Lüscher's method requires that $m_\pi L \gg 1$ in order to strongly suppress the contributions that depend upon the volume as $e^{-m_\pi L}$ [27].

Our present results are from calculations on four ensembles of $n_f = 2{+}1$ anisotropic clover gauge-field configurations at a pion mass of $m_\pi \sim 389$ MeV, a spatial lattice spacing of $b_s \sim 0.123(1)$ fm, an anisotropy [28, 29] of $\xi = 3.50(3)$, and with spatial-extents of $16, 20, 24, 32$ lattice sites, corresponding to spatial dimensions of $L \sim 2.0$, 2.5, 3.0 and 3.9 fm respectively, and temporal extents of 128, 128, 128, and 256 lattice sites, respectively. The precision of the calculations is sufficiently high that the exponential volume dependence of the single baryon masses can be cleanly quantified. The Λ mass, unlike that of

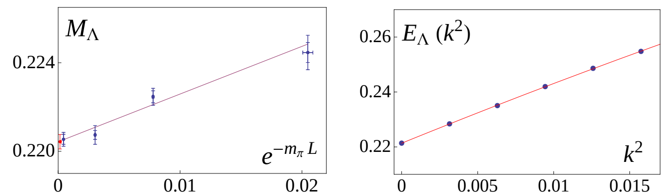

FIG. 1: Left panel: the mass of the Λ as a function of $e^{-m_\pi L}$ where L is the spatial extent of the lattice. The left-most (red) point and uncertainty is the infinite-volume extrapolation of the other (blue) points calculated in lattice volumes with spatial extents of, from left-to-right, $L = 32, 24, 20$, and 16. The curve corresponds to the best straight-line fit. Right panel: the energy-momentum relation of the Λ calculated on the $32^3 \times 256$ ensemble. The points (and uncertainties) are the results of lattice calculations and the (red) curve corresponds to the best fit (see text). The units of the vertical axes in both plots are t.l.u., and of the horizontal axis of the right plot are (t.l.u.)^2

the π and kaon, is found to have statistically significant volume-dependence, as shown in the left panel of fig. 1. It is clear that the Λ mass on the $16^3 \times 128$ ensemble ($m_\pi L \ = \ 3.9$) is significantly higher than its infinite-volume value and, more importantly, is shifted by an amount that is comparable to the two-baryon energy shifts. The deviation found in calculations on the $20^3 \times 128$ ensemble ($m_\pi L \ = \ 4.8$) is much less than that of the $16^3 \times 128$ ensemble, but we choose to use only calculations on the $24^3 \times 128$ ensemble ($m_\pi L \ = \ 5.8$) and on the $32^3 \times 256$ ensemble ($m_\pi L \ = \ 7.7$) in the bound-state analysis.

Lüscher's method assumes that the continuum single-hadron energy-momentum relation is satisfied over the range of energies used in the eigenvalue equation in eq. (1). In order to verify that this is the case, single hadron correlation functions were formed with well-defined lattice spatial momentum, $\mathbf{k} = \frac{2\pi}{L}\mathbf{n}$ for $|\mathbf{n}|^2 \leq 5$. As the low-lying states in the lattice volume have energies that are small compared with the Λ mass, it is sufficient to determine the non-relativistic energy-momentum relation,

$$E_\Lambda \ = \ M_0 \ + \ \frac{|\mathbf{k}|^2}{2M_1} \ - \ \frac{|\mathbf{k}|^4}{8M_2^3} \ + \ \ldots \quad . \tag{4}$$

The Λ energy as a function of momentum calculated on the $32^3 \times 256$ ensemble is shown in the right panel of fig. 1, and yields M_0, M_1, M_2 of 0.22135(10)(05), 0.2231(34)(13), 0.261(26)(04) t.l.u, respectively. Clearly the special-relativity limit of $M_0 = M_1 = M_2$ is satisfied, but an uncertainty of $\sim 2\%$ is introduced into q^2 from the uncertainties in the energy-momentum relation. The use of relativistic or lattice dispersion relations leads to similar conclusions.

In the absence of interactions, the $\Lambda\Lambda$-ΞN-$\Sigma\Sigma$ system is expected to exhibit three low-lying states as the mass-splittings between the single-particle states are (on the $32^3 \times 256$ ensemble)

$$2(M_\Sigma - M_\Lambda) \ = \ 0.01317(13)(19) \text{ t.l.u}$$
$$M_\Xi + M_N - 2M_\Lambda \ = \ 0.003397(61)(65) \text{ t.l.u} \quad . \quad (5)$$

However, if interactions generate a bound state, it is expected that the splitting between the ground-state and the two additional states will be larger than estimates based upon the single baryon rest masses. The effective mass plot (EMP) for the Λ calculated on the $24^3 \times 128$ and $32^3 \times 256$ ensembles that have been optimized for the ground-states using the matrix-Prony method [19] are shown in the left panels of fig. 2, and clear plateaus are identified. The calculated EMP for the energy-splittings between the $\Lambda\Lambda$-ΞN-$\Sigma\Sigma$ coupled-channels (optimized using the matrix-Prony method) and twice the energy of the Λ (formed from the ratio of correlation functions) on the $24^3 \times 128$ and $32^3 \times 256$ ensembles are shown in the right panels of fig. 2. The finite-volume binding energies calculated in the $L = 16$, 20, 24 and 32 lattice volumes are 12.3(1.1)(4.0), 4.5(1.1)(1.3), 16.3(1.2)(1.4), and 16.6(1.4)(3.1) MeV, respectively. In each lattice volume, the results are consistent with a single isolated ground-state with an energy that is below the $\Lambda\Lambda$ threshold (and considerably below the ΞN and $\Sigma\Sigma$ thresholds). The energy splittings and their uncertainties extracted from both ensembles lead to negative values of $q \cot\delta$ indicating that they both lie on the bound-state branch of the S-function (eq. (2)), and thus leads us to identify the

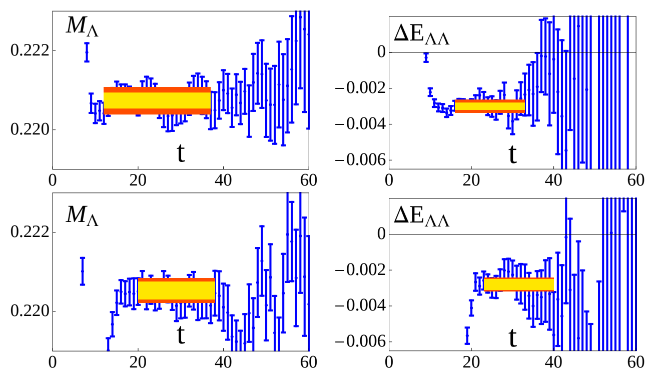

FIG. 2: The EMPs for the Λ (left panels) and the splitting between the $\Lambda\Lambda$ system and twice the Λ mass (right panels) calculated on the $24^3 \times 128$ (upper) and $32^3 \times 256$ (lower) ensembles. The units of both axes are t.l.u.

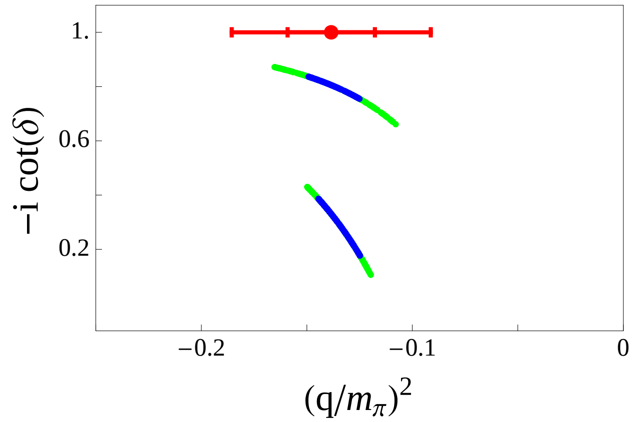

FIG. 3: The results of the Lattice QCD calculations of $-i\cot\delta$ versus q^2/m_π^2 obtained using eq. (1), along with the infinite-volume extrapolation using eq. (3). The dark (blue) (light (green)) lines correspond to the statistical (systematic and statistical uncertainties combined in quadrature) 68% confidence intervals calculated on the $24^3 \times 128$ ensemble (lower) and $32^3 \times 256$ (upper) ensembles. The (red) point and its uncertainty at $-i\cot\delta = +1$ corresponds to the infinite-volume extrapolation, the inner uncertainty being statistical and the outer being the systematic and statistical combined in quadrature.

H-dibaryon. The extracted values of $-i\cot\delta$ from the $24^3 \times 128$ and $32^3 \times 256$ ensembles and their uncertainties are shown in fig. 3, along with the infinite-volume extrapolation implicit in eq. (1), and made explicit in eq. (3). The H-dibaryon binding energy at this pion mass is found to be

$$B_\infty^{\rm H} \ = \ 16.6 \pm 2.1 \pm 4.5 \pm 1.0 \pm 0.6 \text{ MeV} \quad , \qquad (6)$$

where the first uncertainty is statistical, the second systematic, the third is an estimate of the uncertainty in the infinite-volume extrapolation, and the fourth is the uncertainty from the energy-momentum relation. Combining the various systematic uncertainties in quadrature gives $B_\infty^{\rm H} = 16.6 \pm 2.1 \pm 4.6$ MeV. A Monte-Carlo propagation of the uncertainties indicates that there is a probability greater than 0.98 that the H-dibaryon is bound at this pion mass.

In conclusion, we have presented strong evidence for the existence of a bound H-dibaryon from Lattice QCD calculations at a pion mass of $m_\pi \sim 389$ MeV. Our calculations were performed in four lattice volumes, and a negatively shifted ground-state was found in all four volumes. In order to avoid contamination from finite-volume modifications to the Λ mass and interactions, only the results obtained in the larger two volumes were used in the infinite-volume extrapolation. Within the uncertainties, the ground-state energies in the largest two volumes are the same, indicating that both volumes are large compared with the H-dibaryon size. This is consistent with the calculated binding energy. Calculations were performed at only one lattice spacing. However, given that lattice-spacing artifacts in these calculations are expected to scale as $\mathcal{O}(b_s^2)$, we expect such contributions to be small. Moreover, general arguments based on the low-energy effective theory of the Symanzik action suggest that $\mathcal{O}(b_s^2)$ effects largely cancel in forming the energy difference. Consequently, we expect the observation of the H-dibaryon to survive the continuum extrapolation. However, the quark-mass dependence of the H-dibaryon binding energy is presently unknown, so a direct comparison of our result with experiment is not yet possible. As with all such lattice calculations, we cannot rule out the possibility of an additional deeper bound state of the same quantum numbers in this channel that couples weakly to the interpolating operators.

The results of the Lattice QCD calculations presented in this letter provide the first clear evidence for a bound-state of two baryons directly from QCD. This is further strong motivation for pursuing Lattice QCD calculations in larger volumes, at smaller lattice spacings, and over a range of light-quark masses including those of nature, as the present calculations demonstrate that the study of light (hyper-) nuclei directly from QCD is feasible.

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