Surprises in the Evaporation of 2D Black Holes

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Surprises in the Evaporation of 2-dimensional Black Holes

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Quantum evaporation of Callan-Giddings-Harvey-Strominger (CGHS) black holes is analyzed in the mean field approximation. This semi-classical theory incorporates back reaction. Detailed analytical and numerical calculations show that, while some of the assumptions underlying the standard evaporation paradigm are borne out, several are not. Furthermore, if the black hole is initially macroscopic, the evaporation process exhibits remarkable universal properties (which are distinct from the features observed in the simplified, exactly soluble models). Although the literature on CGHS black holes is quite rich, these features had escaped previous analyses, in part because of lack of required numerical precision, and in part because certain properties and symmetries of the model were not fully recognized. Finally, our results provide support for the full quantum scenario recently developed by Ashtekar, Taveras and Varadarajan.

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I. Introduction. Since the early nineties, a number of 2-dimensional (2D) black hole models have been studied to gain further insight into the quantum dynamics of black hole evaporation. Physically, the most interesting among them is due to Callan-Giddings-Harvey-Strominger (CGHS) [1]. Simplified versions of this model are exactly soluble but also have important limitations discussed, e.g., in [2, 3]. Therefore results obtained in those models are not reliable indicators of what happens in the full CGHS dynamics. In this letter we present key results from a new analysis of CGHS black holes using a mean-field or semi-classical approximation. These findings are surprising in two respects. First, several features of the standard CGHS paradigm [2] of quantum evaporation are not realized. Second, black holes resulting from a prompt collapse of a large Arnowitt-Deser-Misner (ADM) mass exhibit rather remarkable behavior: after an initial transient phase, dynamics of various physically interesting quantities at right future null infinity $\mathcal{I}_{\rm R}^{+}$ flow to *universal curves*, independent of the details of the initial collapsing matter distribution. This universality strongly suggests that information in the collapsing matter on $\mathcal{I}_{\rm R}^{-}$ can *not* in general be recovered at $\mathcal{I}_{\rm R}^{+}$. However, we also find strong evidence supporting the scenario of [4] in which the S-matrix from (left past infinity) $\mathcal{I}_{\rm L}^{-}$ to $\mathcal{I}_{\rm R}^{+}$ is unitary. This distinction between unitarity and information recovery is a peculiarity of 2D.

In this letter we summarize the main results. An extensive treatment can be found in [5]; details of the numerics in [6]; and a thorough investigation of the full quantum issues in [7].

II. Model. In the CGHS model, geometry is encoded in a physical metric g and a dilaton field ϕ, and coupled to N massless scalar fields f_i. Since we are in 2D with $\mathbb{R}^2$ topology, we can fix a fiducial flat metric η and write g as $g^{ab} = \Omega\eta^{ab}$. Then it is convenient to describe geometry through $\Phi := e^{-2\phi}$ and $\Theta := \Omega^{-1}\Phi$. The model has 2 constants, κ with dimensions $[L]^{-1}$ and G with dimensions $[ML]^{-1}$.

Our investigation is carried out within the mean field approximation (MFA) of [4, 7] in which one ignores quantum fluctuations of geometry but not of matter. To ensure a sufficiently large domain of validity, we must have large N and we assume that each scalar field f_i has the same profile.

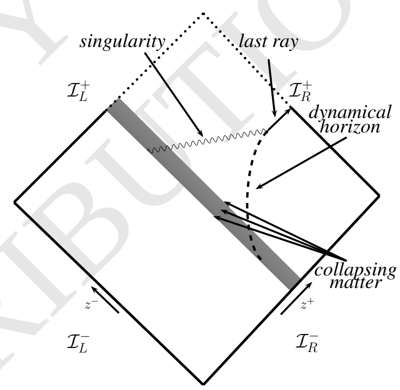

Figure 1. A Penrose diagram of an evaporating CGHS black hole in the mean field approximation (MFA). The incoming state is the vacuum on $\mathcal{I}_{\rm L}^{-}$, and left moving matter distribution on $\mathcal{I}_{\rm R}^{-}$. The collapse creates a generalized dynamical horizon (GDH), which subsequently evaporates. Quantum radiation fills the spacetime to the causal future of matter. Inside the GDH, a singularity forms in the geometry. It meets the GDH when the latter shrinks to zero area. The "last ray" emanating from this meeting point is a future Cauchy horizon.

Black hole formation and evaporation is described entirely in terms of non-linear partial differential equations. Denote by $z^{\pm}$ the advanced and retarded null coordinates of η so that $\eta_{ab} = 2\partial_{(a}z^{+}\,\partial_{b)}z^{-}$. We will set $\partial_{\pm} \equiv \partial/\partial z^{\pm}$. Then we have the evolution equations

$$\Box_{(\eta)}\, f_i = 0 \quad \Leftrightarrow \quad \Box_{(g)} f_i = 0. \qquad (1)$$

for matter fields, and

$$\partial_+\,\partial_-\,\Phi + \kappa^2\Theta = G\,\langle\hat{T}_{+-}\rangle \equiv \bar{N}G\hbar\,\partial_+\,\partial_-\,\ln(\Phi\Theta^{-1})$$
$$\Phi\partial_+\,\partial_-\ln\Theta = -G\,\langle\hat{T}_{+-}\rangle \equiv -\bar{N}G\hbar\,\partial_+\,\partial_-\,\ln(\Phi\Theta^{-1}) \qquad (2)$$

for geometric fields Θ, Φ. The terms on the right side are quantum corrections to the classical equations due to conformal anomaly and encode the back reaction of quantum radiation. As in 4D general relativity there are constraints which are preserved by the evolution equations:

$$-\partial_-^2\,\Phi + \partial_-\,\Phi\partial_-\,\ln\Theta \;=\; G\,\langle\hat{T}_{--}\rangle$$
$$-\partial_+^2\,\Phi + \partial_+\,\Phi\partial_+\,\ln\Theta \;=\; G\,\langle\hat{T}_{++}\rangle\,. \qquad (3)$$

Here, $\bar{N} := N/24$ and $\langle \hat{T}_{ab} \rangle$ denotes the expectation value of the stress-energy tensor of the N fields f_i.

We solve this system of equations as follows. As is standard in the CGHS literature, we assume that prior to $z^+ = 0$ the space-time is given by the classical vacuum solution and matter falls in from $\mathcal{I}^-_{\rm R}$ after that (see Fig. 1). Therefore, to specify consistent initial data, it suffices to choose a matter profile $f_+(z^+)$ on $\mathcal{I}^-_{\rm R}$, and solve for the initial (Θ, Φ) using (3). We then evolve (Θ, Φ) to the future of the initial data surfaces using (2). Trivially, $f_i(z^+, z^-) = f_+(z^+)$ from (1).

We now discuss the interpretation of solutions via horizons, singularities and the Bondi mass. Note first that in analogous 4-dimensional (4D) spherically symmetric reductions, Φ is related to the radius r by $\Phi = \kappa^2 r^2$ [2, 5]. Therefore, a point in the CGHS space-time (M, g) is said to be *future marginally trapped* if $\partial_+\Phi$ vanishes and $\partial_-\Phi$ is negative there [2, 8]. The quantum corrected "area" of a trapped point is given by $\mathbf{a} := (\Phi - 2\bar{N}G\hbar)$. The world-line of these marginally trapped points forms a *generalized dynamical horizon* (GDH). As time evolves, this area *shrinks* because of quantum radiation , and finally goes to zero. The MFA equations are formally singular where $\Phi = 2\bar{N}G\hbar$; thus at the end-point of evaporation the GDH meets a space-like singularity. The 'last ray' —the null geodesic from this point to $\mathcal{I}^+_{\rm R}$— is the future Cauchy horizon of the semi-classical space-time. See Fig. 1.

We assume (and this is borne out by the simulations) that the semi-classical space-time is asymptotically flat at $\mathcal{I}^+_{\rm R}$ in the sense that, as $z^+ \to \infty$, the field Φ has the following behavior along z^- = const lines

$$\Phi = A(z^-)\, e^{\kappa z^+} + B(z^-) + O(e^{-\kappa z^+}), \tag{4}$$

where A and B are smooth functions of z^-. A similar expansion holds for Θ. The physical semi-classical metric g_{ab} admits an *asymptotic* time translation t^a. Its affine parameter y^- is given by $e^{-\kappa y^-} = A(z^-)$. Up to an additive constant, y^- serves as the unique physical time parameter at $\mathcal{I}^+_{\rm R}$. The MFA equations imply that there is a balance law at $\mathcal{I}^+_{\rm R}$ [4, 7], motivating new definitions of a Bondi mass $M^{\rm ATV}_{\rm Bondi}$ and a manifestly positive energy flux $F^{\rm ATV}$:

$$M^{\rm ATV}_{\rm Bondi} = \frac{{\rm d}B}{{\rm d}y^-} + \kappa B \,+\, \bar{N}\hbar G\, \big(\frac{{\rm d}^2 y^-}{{\rm d}z^{-2}}\, (\frac{{\rm d}y^-}{{\rm d}z^-})^{-2}\,\big) \tag{5}$$

$$F^{\rm ATV} = \frac{\bar{N}\hbar G}{2}\, \big[\frac{{\rm d}^2 y^-}{{\rm d}z^{-2}}\, (\frac{{\rm d}y^-}{{\rm d}z^-})^{-2}\, \big]^2, \tag{6}$$

so that ${\rm d}(M^{\rm ATV}_{\rm Bondi})/{\rm d}y^- = -F^{\rm ATV}$. In the classical theory ($\hbar = 0$), there is no energy flux at $\mathcal{I}^+_{\rm R}$, and $M^{\rm ATV}_{\rm Bondi}$ reduces to the standard Bondi mass formula, which includes only the first two terms in (5). Previous literature [1, 2, 8–10] on the CGHS model used this classical expression also in the semi-classical theory. But we will see that this traditionally used Bondi mass, $M^{\rm Trad}_{\rm Bondi}$, is physically unsatisfactory.

III. Scaling and the Planck Regime. It turns out that the mean field theory admits a scaling symmetry. To express it explicitly, let us fix $z^\pm$ and regard all fields as functions of $z^\pm$. Then, given any solution (Θ, Φ, N, f_+) to all the field equations and a positive number λ, $(\lambda\Theta, \lambda\Phi, \lambda N, f_+)$ is also a solution (once z^- is shifted to $z^- + (\ln\lambda)/\kappa$) [5, 12]. Under this transformation, we have

$$g^{ab} \to g^{ab}, \;\; (M, F^{\rm ATV}, \mathbf{a}_{\rm GDH}) \to \lambda(M, F^{\rm ATV}, \mathbf{a}_{\rm GDH})$$

where $\mathbf{a}_{\rm GDH}$ denotes the area of the GDH, and M is either the Bondi mass $M^{\rm ATV}_{\rm Bondi}$ or the ADM mass $M_{\rm ADM}$. This symmetry implies that, *as far as space-time geometry and energetics are concerned, only the ratio M/N matters.* Thus, whether a black hole is 'macroscopic' or 'Planck size' depends on the ratios M/N and $\mathbf{a}_{\rm GDH}/N$ rather than on the values of M or $\mathbf{a}_{\rm GDH}$ themselves. Hence we are led to define

$$(M^\star, M^\star_{\rm Bondi}, F^\star) = (M_{\rm ADM}, M^{\rm ATV}_{\rm Bondi}, F^{\rm ATV})/\bar{N}, \quad \text{and} \;\; m^\star = M^\star_{\rm Bondi}|_{\text{last ray}}\,. \tag{7}$$

To compare these quantities to the Planck scale, note that there are subtleties as $G\hbar$ is dimensionless in 2D; careful considerations lead us to set $M^2_{\rm Pl} = \hbar\kappa^2/G$, and $\tau^2_{\rm Pl} = G\hbar/\kappa^2$ [5]. We can regard a black hole as macroscopic if its evaporation time is much larger than the Planck time. Using the fact that, in the external field approximation, the energy flux is given by $F_{\rm Haw} = (\bar{N}\hbar\kappa^2/2)$, this condition leads us to say that *a black hole is macroscopic if $M^\star \gg G\hbar\; M_{\rm Pl}$*. Note that the relevant quantity is $M^\star$ rather than M. The precise nature of this scaling property was not appreciated until recently. For example, in [13] it was noted that N could be "scaled out" of the problem and that the results are "qualitatively independent of N", whereas in fact for a given M they can vary significantly as N changes. Similarly, the condition that a macroscopic black hole should have large M/N appears in [9]. But it was arrived at by physical considerations involving static solutions rather than an exact scaling property of the full equations.

IV. Results. Here we describe some key results from numerical solution of the CGHS equations (1)-(2). We consider two families of initial data, most conveniently described in a "Kruskal-like" coordinate $\kappa x^+ = e^{\kappa z^+}$. The first is a collapsing shell used extensively in the CGHS literature,

$$\left(\partial f_+/\partial x^+\right)^2 = \frac{M^\star}{12}\, \delta\left(x^+ - 1/\kappa\right), \tag{8}$$

parameterized by $M^\star$. The other is a smooth ($f_+(x^+)$ is $\mathcal{C}^4$), two parameter $(\tilde{M}^\star, w)$ profile defined by

$$\int_0^{x^+} {\rm d}\bar{x}^+\, (\tfrac{\partial f_+}{\partial \bar{x}^+})^2 \,=\, \tfrac{\tilde{M}^\star}{12} \left(1 - e^{\left(\kappa x^+ - 1\right)^2/w^2}\right)^4 \theta(x^+ - 1/\kappa), \tag{9}$$

where θ is the unit step function, w characterizes the width of the matter distribution, and $\tilde{M}^\star$ is related to the ADM mass via $M^\star \approx \tilde{M}^\star(1 + 1.39\, w)$. Unraveling of the unforeseen behavior required high precision numerics [6], which is crucial in the macroscopic mass limit that is of primary importance. Numerical solutions from both classes of initial data were obtained for a range of masses $M^\star$ from 2^{-10} to 16, a range of widths from $w = 0$ to $w = 4$, and $\bar{N}$ varying from 0.5 to 1000. Since we are primarily interested in black holes which are initially macroscopic, here we will focus on $M^\star \geq 1$ and, since the computations did bear out the scaling behavior, on the case $\bar{N} = 1$. We set $\hbar = G = \kappa = 1$.

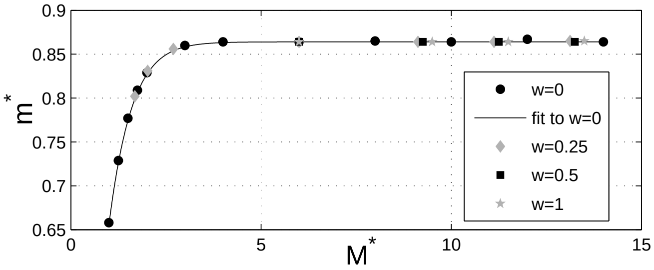

Figure 2. The final mass $m^\star$ versus the initial mass $M^\star$ (7) for a variety of initial data (8-9). The curve fit to the data is $m^\star = \alpha\,(1 - e^{-\beta(M^\star)^\gamma})$, with $\alpha \approx 0.864$, $\beta \approx 1.42$, and $\gamma \approx 1.15$.

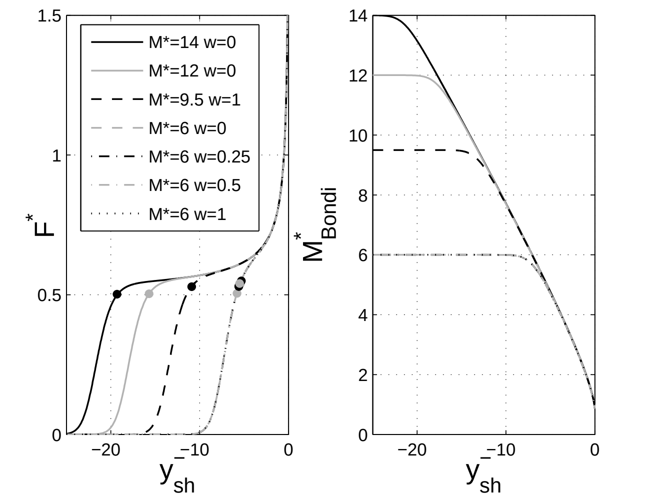

Figure 3. $F^\star$ and $M^\star_{\rm Bondi}$ of Eq (7) plotted against $y^-_{\rm sh}$ for solutions with several values of parameters $M^\star$ and w of Eqs (8)-(9). In all cases $F^\star$ starts at 0 in the distant past ($\kappa y^-_{\rm sh} \ll -1$), and then joins a universal curve at a time that depends on the initial mass. The time when the dynamical horizon first forms is marked on each flux curve (which is later for larger w, though note that the mass and flux curves for all the $M^\star = 6$ cases are indistinguishable in the figure). We have not yet found an extrapolation of the flux to the last ray $y^-_{\rm sh} = 0$ that conclusively answers whether it is finite there. However, all functions we tried that fit the data well have a *finite integrated* flux. Moreover, when the flux starts rising rapidly, we are still well within the regime where the numerical solution converges, and we can follow the solution clearly into a regime where the mass has reached its final value of $m^\star \approx 0.864$.

Our numerical simulations show that, as expected, the semi-classical space-time is asymptotically flat at $\mathcal{I}^+_{\rm R}$ but, in contrast to the classical theory, $\mathcal{I}^+_{\rm R}$ is incomplete, i.e., y^- has a finite value at the last ray. However, dynamics also exhibits some surprising features which can be summarized as follows.

First, the traditionally used Bondi mass $M^{\rm Trad}_{\rm Bondi}$ can become *negative and large even when the* GDH *is macroscopic.* For CGHS black holes, negative $M^{\rm Trad}_{\rm Bondi}$ was known to occur [11] but only for black holes which are of Planck size even before evaporation begins. For initially macroscopic black holes, the standard paradigm assumed that $M^{\rm Trad}_{\rm Bondi}$ is positive and tends to zero as the GDH shrinks (so that one can attach a 'flat corner' of Minkowski space to the future of the last ray). Second, while the improved Bondi mass, $M^{\rm ATV}_{\rm Bondi}$, does remain positive throughout evolution, at the last ray it can be large. In fact this 'end state' exhibits a universality shown in Fig 2 where $m^\star$, the final value of $(M^\star_{\rm Bondi})$, is plotted against the rescaled ADM mass $M^\star$ for a range of initial data. It is clear from the plot that there is a qualitative difference between $M^\star \gtrsim 4$ and $M^\star \lesssim 4$. In the first case the value of the end point Bondi mass is universal, $m^\star \approx 0.864$. For $M^\star < 4$ on the other hand, the value of $m^\star$ depends sensitively on $M^\star$. Thus in the MFA it is natural to regard CGHS black holes with $M^\star \gtrsim 4$ as *macroscopic*, and those with $M^\star \lesssim 4$ as *microscopic*. Numerical studies have been used in the past to clarify properties of the CGHS model [3, 10, 11, 13], such as dynamics of the GDH. However, they could not uncover universal behavior because, in the present terminology, they covered only *microscopic* cases ($M^\star \le 2.5$ in all prior studies).

Third, for macroscopic ($M^\star \gtrsim 4$) black holes that form *promptly*, after early transient behavior, dynamics of physical quantities at the GDH and at $\mathcal{I}^+_{\rm R}$ approach *universal curves.* By promptly, we mean the characteristic width of the ingoing pulse is less than that of the initial GDH (more precisely, $w/M^\star \lesssim 0.1$). This is most clearly demonstrated in the behavior of the flux $F^\star$, or equivalently the Bondi mass $M^\star_{\rm Bondi}$, measured at $\mathcal{I}^+_{\rm R}$. An appropriately shifted affine parameter $y^-_{\rm sh} = y^- + const$ provides an invariantly defined time coordinate and Fig. 3 shows the universality of evolution of $F^\star$ and $M^\star_{\rm Bondi}$ with respect to it. The shift aligns the y^- coordinates amongst the solutions, which we are free to do as y^- is only uniquely defined to within a (physically irrelevant) additive constant. Finally, note that this universality is qualitatively different from the known uniqueness results for solutions of certain simplified soluble models [14]. It occurs only if the black hole is initially macroscopic, formed by a prompt collapse. And in this case, after the transient phase, the behavior of physical quantities at $\mathcal{I}^+_{\rm R}$ does not even depend on the mass.

The situation with universality bares parallels to the discovery of critical phenomena at the threshold of gravitational collapse in classical general relativity [15] where universal properties were discovered in a system that, at the time, seemed to have been already explored exhaustively. Of course, numerical investigations cannot *prove* universality; here we only studied two families of initial data. However, since these families, in particular the distribution, are not 'special' in any way, we believe this is strong evidence that universality is a feature of the 'pure' quantum decay of a GDH, pure in that the decay is not contaminated by a continued infall from $\mathcal{I}^-_{\rm R}$.

Finally, along the last ray, our simulations show that curvature remains finite. Thus, contrary to wide spread belief, based in part on [3], and in contrast to simplified and soluble models, there is no 'thunderbolt singularity' in the metric.

V. Conclusions. In the external field approximation, the energy flux is initially zero and, after the transient phase, quickly asymptotes to the Hawking value $F_{\rm Haw} = \bar{N}\hbar\kappa^2/2 \equiv 0.5$ for the constants used in the simulations shown here. In the

MFA calculation, on the other hand, at the end of the transient phase the energy flux is *higher* than this value, keeps monotonically increasing and is about 70% greater than $F_{\rm Haw}$ when $M_{\rm Bondi} \sim 2\bar{N}M_{\rm Pl}$ (see Fig 3). One might first think that the increase is because, as in 4D, the black hole gets hotter as it evaporates. This is *not* so: For CGHS black holes, $T_{\rm Haw} = \kappa\hbar/2\pi$ and κ is an absolute constant. Rather, the departure from $F_{\rm Haw} = 0.5$ shows that, once the back reaction is included, the flux fails to be thermal at the late stage of evaporation, *even while the black hole is macroscopic*. This removes a widely quoted obstacle against the possibility that the outgoing quantum state is pure in the full theory.

In the classical solution, $\mathcal{I}_{\rm R}^+$ is *complete* and its causal past covers only a part of space-time; there is an event horizon. But $\mathcal{I}_{\rm R}^+$ is smaller than $\mathcal{I}_{\rm L}^-$ in a precise sense: z^-, the affine parameter along $\mathcal{I}_{\rm L}^-$ is finite at the future end of $\mathcal{I}_{\rm R}^+$. This is why pure states on $\mathcal{I}_{\rm L}^-$ of a *test* quantum field $\hat{f}_-$ on the classical solution evolve to mixed states on $\mathcal{I}_{\rm R}^+$ [4, 7], i.e., why the S matrix is non-unitary. In the MFA, by contrast, our analysis shows that as expected y^- is *finite* at the last ray on $\mathcal{I}_{\rm R}^+$. Thus, $\mathcal{I}_{\rm R}^+$ is incomplete whence we cannot even ask if the semi-classical space-time admits an event horizon; *what forms and evaporates is, rather, the GDH.* However, this incompleteness also opens the possibility that $\bar{\mathcal{I}}_{\rm R}^+$, the right null infinity of the full quantum space-time, may be larger than $\mathcal{I}_{\rm R}^+$ and unitarity may be restored. Indeed, since there is no thunderbolt, space-time can be continued beyond the last ray. In the mean field theory the extension is ambiguous. But it is reasonable to expect that the ambiguities will be removed by full quantum gravity [16]. Indeed, since we only have $(0.864/24)M_{\rm Pl}$ of Bondi mass left over at the last ray *per evaporation channel* (i.e., per scalar field), it is reasonable to assume that this remainder will quickly evaporate after the last ray and $M_{\rm Bondi}^{\rm ATV}$ and $F^{\rm ATV}$ will continue to be zero along the quantum extension $\bar{\mathcal{I}}_{\rm R}^+$ of $\mathcal{I}_{\rm R}^+$. The form of $F^{\rm ATV}$ now implies that $\bar{\mathcal{I}}_{\rm R}^+$ is 'as long as' as $\mathcal{I}_{\rm L}^-$ and hence the S-matrix is unitary: The vacuum state on $\mathcal{I}_{\rm L}^-$ evolves to a many-particle state with *finite* norm on $\bar{\mathcal{I}}_{\rm R}^+$ [4, 7]. Thus unitarity of the S matrix follows from rather mild assumptions on what transpires beyond the last ray.

Note, however, this unitarity of the S-matrix from $\mathcal{I}_{\rm R}^-$ to the extended $\mathcal{I}_{\rm R}^+$ does *not* imply that all the information in the infalling matter on $\mathcal{I}_{\rm R}^-$ is imprinted in the outgoing state on $\bar{\mathcal{I}}_{\rm R}^+$. Indeed, the outgoing quantum state is completely determined by the function $y^-(z^-)$ and our universality results imply that, on $\mathcal{I}_{\rm R}^+$, this function only depends on $M_{\rm ADM}$ and not on further details of the matter profile [5]. Since only a tiny fraction of Planck mass is radiated per channel in the portion of $\bar{\mathcal{I}}_{\rm R}^+$ that is not already in $\mathcal{I}_{\rm R}^+$, it seems highly unlikely that the remaining information can be encoded in the functional form of $y^-(z^-)$ in that portion. Thus, information in the matter profile on $\mathcal{I}_{\rm R}^-$ will not all be recovered at $\bar{\mathcal{I}}_{\rm R}^+$ even in the full quantum theory of the CGHS model. This contradicts a general belief; indeed, because the importance of $y^-(z^-)$ was not appreciated and its universality was not even suspected, there have been attempts at constructing mechanisms for recovery of this information [9].

To summarize, in 2D there are two distinct issues: i) unitarity of the S-matrix from $\mathcal{I}_{\rm L}^-$ to $\bar{\mathcal{I}}_{\rm R}^+$; and ii) recovery of the infalling information on $\mathcal{I}_{\rm R}^-$ at $\bar{\mathcal{I}}_{\rm R}^+$. The distinction arises because right and left pieces of $\mathcal{I}^\pm$ do not talk to each other. In 4D, by contrast, we only have one $\mathcal{I}^-$ and only one $\mathcal{I}^+$. Therefore if the S-matrix from $\mathcal{I}^-$ to $\mathcal{I}^+$ is unitary, all information in the ingoing state at $\mathcal{I}^-$ is automatically recovered in the outgoing state at $\mathcal{I}^+$. To the extent that the CGHS analysis provides guidance for the 4D case, it suggests that unitarity of the S-matrix should continue to hold also in 4D [7].

Acknowledgements

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[1] C. G. Callan, S. B. Giddings, J. A. Harvey and A. Strominger, Phys. Rev. **D45**, R1005-R1009 (1992).
[2] S. B. Giddings, *arXiv:Hep-th/9412138*; A. Strominger, *arXiv:Hep-th/9501071*
[3] S. W. Hawking and J. M. Stewart, Nucl. Phys. **B 400**, 393-415 (1993)
[4] A. Ashtekar, V. Taveras and M. Varadarajan, Phys. Rev. Lett. **100**, 211302 (2008)
[5] A. Ashtekar, F. Pretorius and F. M. Ramazanoğlu, arXiv:1011.1024
[6] F. M. Ramazanoğlu and F. Pretorius, *arXiv:1009.1440*
[7] M. Varadarajan and A. Ashtekar (pre-print, 2010)
[8] S. Hayward, Class. Quant. Grav. **10**, 985-994 (1993).
[9] L. Susskind and L. Thorlacius, Nucl. Phys. **B 382**, 123-147 (1992)
[10] D. A. Lowe, Phys. Rev. **D 47**, 2446-2453 (1993)
[11] T. Tada and S. Uehara, Phys. Rev. D **51** 4259-4264 (1995)
[12] A. Ori, Phys. Rev. D **82**, 104009 (2010)
[13] T. Piran and A. Strominger, Phys. Rev. D**48**, 4729-4734 (1993)
[14] J. Rousso, L. Susskind and L. Thorlacius, Phys. Rev. D**46**, 3444 (1992)
[15] M. Choptuik, Phys. Rev. Lett. **70**, 9 (1993)
[16] S. W. Hawking, Commun. Math. Phys. **43**, 199-220 (1975)