Variety of c-Axis Collective Excitations in Layered Multigap Superconductors

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A variety of c-axis collective excitations in layered multigap superconductors

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We present a dynamical theory for the phase differences along a stacked direction of intrinsic Josephson junctions (IJJ's) in layered multigap superconductors, motivated by the discovery of highly-anisotropic iron-based superconductors with thick perovskite-type blocking layers. The dynamical equations describing AC and DC intrinsic Josephson effects peculiar to multigap IJJ's are derived, and collective Leggett mode excitations in addition to the Josephson plasma established in single-gap IJJ's are predicted. The dispersion relations of their collective modes are explicitly displayed, and the remarkable peculiarity of the Leggett mode is demonstrated.

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Highly-anisotropic layered High-T_c superconductors are natural nano-scale stacks of Josephson junctions, i.e., intrinsic Josephson junction (IJJ) arrays, since superconducting and insulating layers with atomic thickness regularly alternate along the crystalline c-axis. Their high-quality single crystals clearly exhibit Josephson effects only in c-axis electromagnetic response, which are called intrinsic Josephson effects (IJE's). IJE's have been experimentally confirmed in various layered High-T_c copper oxide superconductors, such as $Bi_2Sr_2CaCu_2O_8$ [1–3]. An intriguing feature in IJE's is unique dynamics arising from couplings between the stacked junctions. Two types of inter-Junction couplings due to inductive [4, 5] and capacitive [6, 7] origins have been mainly proposed. The inductive coupling in IJJ's is very strong since the in-plane magnetic penetration depth characterizing the magnetic field screening range extends over several hundred junctions. Meanwhile, the capacitive one is not so strong since the charge screening length is comparable to the layer thickness. However, it has a significant role on IJE's due to the atomic-scale structure [8].

Most of High-T_c cuprate materials are identified as single-band superconductor. One then defines just a single phase difference between consecutive superconducting layers. The dynamics of the phase difference in single-gap IJJ's has been intensively studied after the discovery of High-T_c cuprate IJJ's [9]. In this paper, we extend the dynamical theory for the phase difference to multi-gap IJJ's, in which more than one phase differences are active through stacked all junctions. Our motivation comes from the discovery of highly-anisotropic iron-based supercondcutors, such as $(Fe_2As_2)(Sr_4V_2O_6)$, $(Fe_2P_2)(Sr_4Sc_2O_6)$ and $(Fe_2As_2)(Sr_4(Mg, Ti)_2O_6)$ [10–12]. These compounds contain thick perovskite-type blocking layers $Sr_4M_2O_6$ (M $=$ Sc, Cr, V) with a thickness of $\sim 15\,\text{Å}$, which clearly remind us of $Bi_2Sr_2CaCu_2O_8$. The first principles calculations on these materials indicate that they are multiband systems with strong two-dimensional character [13, 14] whose anisotropy is estimated to be comparable to $Bi_2Sr_2CaCu_2O_8$. In fact, the experimentally observed resistivity of their polycrystalline samples [10] exhibits a broad superconducting transition in the presence of external magnetic field, which is a clear sign of high anisotropy. We also note a direct report that single crystals of an iron-based superconductor, $PrFeAsO_{0.7}$, show the I-V characteristics peculiar to Josephson junctions in the c-axis [15].

What is the most fundamental issue in multi-gap IJJ's? Since the tunneling channel is also multiple, the number of collective modes in the phase oscillation is simply expected to be multiplied. Confining ourselves to the most simple two-gap systems, we study the multiple collective modes. First, we derive coupled equations of motion for the phase differences describing AC and DC multigap IJE's. Then, a mode analysis on them clarifies that the in-phase mode corresponding to the Josephson plasma is not significantly altered while the out-of-phase one, i.e., Leggett mode suggested in the presence of two superfluid orders by Leggett [16], emerges as a unique mode. An intriguing focus in this paper is that the c-axis Leggett mode is weakly dispersive and favors synchronous oscillations along c-axis. Such a behavior is striking contrast to the Josephson plasma.

Consider the two-gap IJJ's composed of N junctions as shown in Fig. 1(a). We assume the pairing interaction as $-\sum_{ij=1,2} g_{ij}\hat{\psi}^{\dagger}_{li\uparrow}\hat{\psi}^{\dagger}_{li\downarrow}\hat{\psi}_{lj\downarrow}\hat{\psi}_{lj\uparrow}$ on each superconducting layer, where $\hat{\psi}_{li\sigma}$ is the electron field operator with spin σ in the ith band on the lth superconducting layer. The coupling constants g_{11} and g_{22} ($g_{12} = g_{21}$) denote the intra-band (inter-band) pairing interaction constants. The inter-band pairing interaction generates the inter-band Josephosn coupling energy, $v_{\rm in}\cos(\varphi^{(1)}_{\ell} - \varphi^{(2)}_{\ell})$, in the effective action of superconducting phases [17]. Here, $\varphi^{(i)}_{l}$ is the phase of the superconducting gap in the ith band on the lth superconducting layer, and the coupling constant $v_{\rm in}$ is given as $v_{\rm in} = 4\kappa_{\rm in}|g_{12}/g||\Delta^{(1)}||\Delta^{(2)}|$, where $|\Delta^{(i)}|$ is the ith superconducting gap amplitude and $g = g_{11}g_{22} - (g_{12})^2$. The coefficient $\kappa_{\rm in}$ is the sign factor defined as $\kappa_{\rm in} = 1$ for $g_{12} > 0$ and $\kappa_{\rm in} = -1$ for $g_{12} < 0$. For the Josephson coupling between consecutive superconducting layers, one can derive the so-called Josephson coupling energy on two channels as $\sum_{i=1,2}(\hbar j_{c,i}/e^*)\cos[\varphi^{(i)}_{l+1} - \varphi^{(i)}_l - (e^*d/\hbar c)A^z_{l+1,l}]$, where $j_{c,1}$ and $j_{c,2}$ are the Josephson critical current densities, $A^z_{l+1,l} = (1/d)\int_{ld}^{(l+1)d} A^z(z)dz$ is the z-component of the vector potential, and $e^* = 2e$. Here, we neglect the inter-band crossing channel because it is the forth-order process in the coherent tunneling case. On the basis of this result, we propose an effective Lagrangian for the two-gap IJJ's as

$$
\begin{aligned}
L \;=\; & \sum_l \Bigg[\frac{sq^2_{l,1}}{8\pi\mu_1^2} + \frac{sq^2_{l,2}}{8\pi\mu_2^2} - \frac{sv^2_{l,1}}{8\pi\lambda^2_{ab,1}} - \frac{sv^2_{l,2}}{8\pi\lambda^2_{ab,2}} \\
& + \frac{\hbar j_{c,1}}{e^*}\cos\theta^{(1)}_{l+1,l} + \frac{\hbar j_{c,2}}{e^*}\cos\theta^{(2)}_{l+1,l} + \frac{\hbar J_{\rm in}}{e^*}\cos\psi_l \\
& + \frac{\epsilon d}{8\pi}(E^z_{l+1,l})^2 - \frac{d}{8\pi}(B^y_{l+1,l})^2\Bigg], \qquad (1)
\end{aligned}
$$

where $q_{l,i} = (\hbar/e^*)\partial_t\varphi^{(i)}_l + A^0_l$, $v_{l,i} = (\hbar c/e^*)\partial_x\varphi^{(i)}_l - A^x_l$, $\theta^{(i)}_{l+1,l} = \varphi^{(i)}_{l+1} - \varphi^{(i)}_l - (e^*d/\hbar c)A^z_{l+1,l}$, $J_{\rm in} = e^*v_{\rm in}s/\hbar$, and $\psi_l = \varphi^{(1)}_l - \varphi^{(2)}_l$. The parameters, s and d, are the thicknesses of the superconducting and insulating layers, respectively, μ_i ($\lambda_{ab,i}$) is the charge screening length (in-plane penetration depth) relevant to the ith-band electrons, ϵ is the dielectric constant in the insulating layers, and A^0_l and A^x_l are, respectively, the scalar potential and the x-component of the vector potential on the lth superconducting layer. Without losing generality, we consider only the z-component of the electric field and

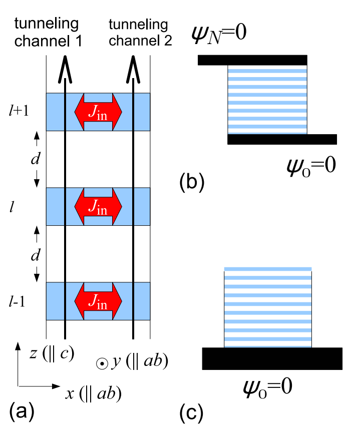

FIG. 1: (color online) (a) Schematic diagram for IJJ's with multiple gap superconducting layers. (b) Double-sided type junction and the associated boundary condition for ψ_l. (c) Mesa type junction and the associated boundary condition for ψ_l.

only the y-component of the magnetic field, which are expressed as $E^z_{l+1,l} = -(1/c)\partial_t A^z_{l+1,l} - (A^0_{l+1} - A^0_l)/d$ and $B^y_{l+1,l} = (A^x_{l+1} - A^x_l)/d - \partial_x A^z_{l+1,l}$. As in the single-gap IJJ's one can define the inductive [4, 5] and capacitive [6, 7] coupling constants in the dimensionless form as $\eta_i = \lambda^2_{ab,i}/sd$ and $\alpha_i = \epsilon\mu_i^2/sd$ for each channel in this system. The effective action (1) describes low energy dynamics of the superconducting phases in the two-gap IJJ's. In the derivation of Eq. (1), all junction parameters (e.g. $j_{c,i}$ and $\lambda_{{\rm ab},i}$) are approximated as local quantities for brevity, although they are originally nonlocal ones. As for the nonlocal electromagnetic effects, see Ref. [18].

Now, let us derive the coupled equations of motion of the superconducting phase differences. First, one obtains the so-called Josephson relations associated with time and spatial variations of the superconducting phase differences as,

$$
\partial_t\theta_{l+1,l} - \frac{\xi}{2}\partial_t\psi_{l+1,l} = \frac{e^*d}{\hbar}(1 - \tilde{\alpha}\triangle^{(2)})E^z_{l+1,l}, \quad (2a)
$$

$$
\partial_x\theta_{l+1,l} - \frac{\zeta}{2}\partial_t\psi_{l+1,l} = \frac{2\pi d}{\Phi_0}(1 - \tilde{\eta}\triangle^{(2)})B^y_{l+1,l}, \quad (2b)
$$

where $\tilde{\alpha}$ and $\tilde{\eta}$ are, respectively, the reduced capacitive and inductive coupling constants given as $\tilde{\alpha}^{-1} = \alpha_1^{-1} + \alpha_2^{-1}$ and $\tilde{\eta}^{-1} = \eta_1^{-1} + \eta_2^{-1}$, $\triangle^{(2)}$ the second-order finite difference defined as $\triangle^{(2)}f_{l+1,l} = f_{l+2,l+1} - 2f_{l+1,l} + f_{l,l-1}$ (for $^\forall f_{l+1,l}$), $\Phi_0(= 2\pi\hbar c/e^*)$ the unit flux, and $\xi = (\alpha_1 - \alpha_2)/(\alpha_1 + \alpha_2)$ and $\zeta = (\eta_1 - \eta_2)/(\eta_1 - \eta_2)$. Here, we introduce $\theta_{l+1,l} = (\theta^{(1)}_{l+1,l} + \theta^{(2)}_{l+1,l})/2$ and $\psi_{l+1,l} = \psi_{l+1} - \psi_l$. Equations (2a) and (2b) are interpreted as the generalized Josephson relations in the two-gap IJJ's. These relations are reduced to the conventional ones in the single-gap IJJ's when $\xi = \zeta = 0$. On the variation of $A^z_{l+1,l}$, the Lagrangian (1) derives the Maxwell equation as

$$
\partial_x B^y_{l+1,l} - \frac{\epsilon}{c}\partial_t E^z_{l+1,l} = \frac{4\pi}{c}(j^{\rm J}_{l+1,l} + j^{\rm QP}_{l+1,l}), \qquad (3)
$$

where $j^{\rm J}_{l+1,l} = \sum_{i=1}^2 j_{c,i}\sin\theta^{(i)}_{l+1,l}$ and $j^{\rm QP}_{l+1,l} = \sum_{i=1}^2 j^{{\rm QP}(i)}_{l+1,l}$. Here, we add the quasi-particle tunneling current $j^{\rm QP}_{l+1,l}$, which can be derived microscopically [9]. Furthermore, we have the continuity equations, which can be derived by the variation with respect to $\varphi^{(i)}_l$ [9]. From the continuity equations with Eq. (2) we also have the "pseudo" Maxwell equation, which describe the motion of the relative phase differences $\psi_{l+1,l}$, as

$$
\begin{aligned}
\partial_x\widetilde{B}^y_{l+1,l} - \frac{\epsilon}{c}\partial_t\widetilde{E}^z_{l+1,l} = & \frac{4\pi}{c}2J_{\rm in}(\sin\psi_{l+1} - \sin\psi_l) \\
& + \frac{4\pi}{c}\triangle^{(2)}(d^{\rm J}_{l+1,l} + d^{\rm QP}_{l+1,l}), \quad (4)
\end{aligned}
$$

where $d^{\rm J}_{l+1,l} = -j_{c,1}\sin\theta^{(1)}_{l+1,l} + j_{c,2}\sin\theta^{(2)}_{l+1,l}$, $d^{\rm QP}_{l+1,l} = -j^{{\rm QP}(1)}_{l+1,l} + j^{{\rm QP}(2)}_{l+1,l}$. The "pseudo" electromagnetic fields

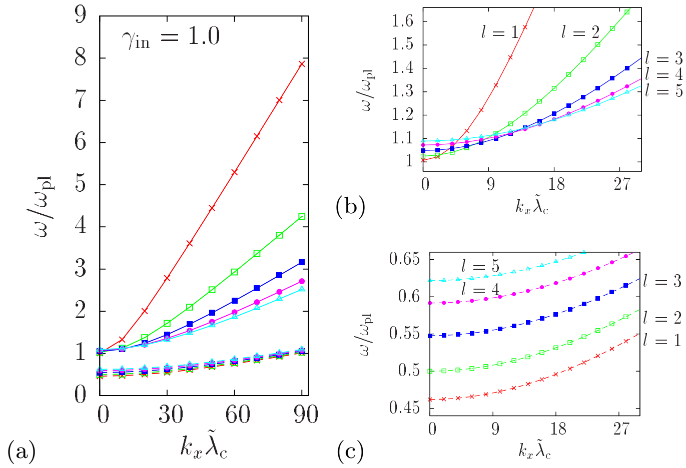

FIG. 2: (color online) (a) Dispersion relations for the Josephson-plasma (solid lines) and the Leggett's modes (dash lines) when $N = 5$ and $\gamma_{\rm in} = 1.0$. We set $\alpha_1 = \alpha_2 = 0.1$, $\eta_1 = \eta_2 = 10^3$, and $j_{c,1} = j_{c,2}$. Enlarged views of the dispersion relations at small in-plane wavenumbers for the five eigenmodes of the Josephson-plasma (b) and the Leggett's modes (c).

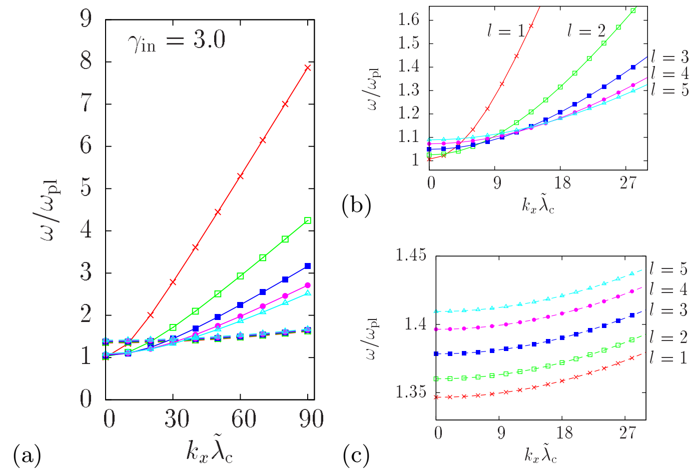

FIG. 3: (color online) (a) Dispersion relations for the Josephson-plasma (solid lines) and the Leggett's modes (dash lines) when $\gamma_{\rm in} = 3.0$. The other parameter values are the same as in Fig. 2. Enlarged viewe at small wave numbers are shown in (b) and (c).

$\widetilde{E}^z_{l+1,l}$ and $\widetilde{B}^y_{l+1,l}$ are defined as

$$\widetilde{E}^z_{l+1,l} = \frac{\hbar}{e^*}\frac{1-\xi^2}{\tilde{\alpha}d}\partial_t\psi_{l+1,l} + \xi\triangle^{(2)}E^z_{l+1,l},$$

$$\widetilde{B}^y_{l+1,l} = \frac{\Phi_0}{2\pi}\frac{1-\zeta^2}{\tilde{\eta}d}\partial_x\psi_{l+1,l} + \zeta\triangle^{(2)}B^y_{l+1,l}.$$

Equations (2), (3), and (4) provide a set of equations of motion for the phase differences and the electromagnetic fields in the two-gap IJJ's, that is, the DC and AC Josephson effects in the two-gap IJJ's can be described by these coupled equations. To solve these equations it is convenient to use the relation defined as $\psi_l = \sum_{m=1}^{l}\psi_{m,m-1} + \psi_0$, where the value of ψ_0 is specified as the boundary condition [Figs. 1(b) and 1(c)].

Let us focus on the collective phase oscillation modes in the two-gap IJJ's. Consider the N junction system under the periodic boundary condition along the c-axis. For simplicity, the case of $\xi = \zeta = 0$ and $j_{c,1} = j_{c,2}$ is examined in the following. More general cases will be published elsewhere. Assuming small oscillations, we linearize Eqs. (3) and (4) around $\theta_{l+1,l} = 0$ and $\psi_l = 0$ with neglecting the dissipation currents $j^{\rm QP}_{\ell+1,\ell}$ and $d^{\rm QP}_{l+1,l}$ for the standard mode analysis [19]. The dynamical simulation taking into account the quasiparticle contributions was performed in Ref. [20]. Eliminating the electric and magnetic fields from the coupled linearized equations, we can derive the decoupled equations for $\theta_{l+1,l}$ and $\psi_{l+1,l}$ as follows,

$$\boldsymbol{C}\partial_x^2\vec{\theta} - \frac{\epsilon}{c^2}\boldsymbol{L}\partial_t^2\vec{\theta} = \frac{1}{\tilde{\lambda}_c^2}\boldsymbol{C}\boldsymbol{L}\vec{\theta}, \tag{5a}$$

$$\frac{1}{2\tilde{\eta}}\boldsymbol{I}\partial_x^2\vec{\psi} - \frac{1}{2\tilde{\alpha}}\boldsymbol{I}\partial_t^2\vec{\psi} = \frac{2}{\lambda_{\rm in}^2}\boldsymbol{N}\vec{\psi}, \tag{5b}$$

where $\tilde{\lambda}_c^{-2} = \lambda_{c,1}^{-2} + \lambda_{c,2}^{-2}$, $\lambda_{c,i}^{-2} = 4\pi e^* d j_{c,i}/\hbar c^2$, $\lambda_{\rm in}^{-2} = 4\pi e^* d|J_{\rm in}|/\hbar c^2$, $\vec{\theta} = {}^t(\theta_{2,1}, \theta_{3,2}, \ldots, \theta_{N-1,N})$, and $\vec{\psi} = {}^t(\psi_{2,1}, \psi_{3,2}, \ldots, \psi_{N-1,N})$. The coeffcients, $\boldsymbol{C}$ and $\boldsymbol{L}$, are $N \times N$ matrices given as $\boldsymbol{C} = (1 + 2\tilde{\alpha})\boldsymbol{I} - \tilde{\alpha}\boldsymbol{S}$ and $\boldsymbol{L} = (1 + 2\tilde{\eta})\boldsymbol{I} - \tilde{\eta}\boldsymbol{S}$, where $\boldsymbol{S}$ is an $N \times N$ tridiagonal matrix with the elements, $S_{l,l} = 0$ and $S_{l,l\pm1} = 1$, and $\boldsymbol{I}$ is the $N \times N$ unit matrix. We note that the matrices $\boldsymbol{C}$ and $\boldsymbol{L}$ represent, respectively, the capacitive and inductive couplings between junctions, which are the same as those in the single-gap IJJ's. Thus, the collective motion of the mean phase differences, which is described by Eq. (5a), is understood to be the Josephson plasma. Moreover, it is clearly found that its dispersion is brought about by the inductive and capacitive couplings between junctions. On the other hand, due to two-gap IJJ's, we have another collective oscillation mode in the relative phase channel, which is described by Eq. (5b). In the new mode, its origin, i.e., the coupling between junctions is found to be induced by the off-diagonal components of the matrix $\boldsymbol{N} = (1 + 2\nu)\boldsymbol{I} - \nu\boldsymbol{S}$ with

$$\nu = \frac{1}{4\gamma_{\rm in}^2}, \quad \gamma_{\rm in} = \frac{\tilde{\lambda}_c}{\lambda_{\rm in}} = \sqrt{\frac{|J_{\rm in}|}{j_{c,1} + j_{c,2}}}. \tag{6}$$

We note that the coupling constant $\gamma_{\rm in}$ (or ν) depends on not the inductive and capacitive coupling constants but just the inter-band Josephson coupling $J_{\rm in}$. Thus, this mode has its origin only in the inter-band pairing interaction. Hence, one understands that Eq. (5b) describes the Leggett mode in the two-gap IJJ's. From these results, it is concluded that the Josephson plasma mode is originated from the inductive and capacitive coupling arising from the electromagnetic field screening, while the Leggett mode is brought about by the intra-layer inter-band coupling.

The dispersion relations of these two eigen-modes are obtained from Eqs. (5a) and (5b), which are specified in terms of the wave numbers k_x (in-plane direction) and $k_z = l\pi/(N+1)d$ (c-direction) as

$$\omega_{\rm P}(k_x, l) = \omega_{\rm P}(0, l)\sqrt{1 + \frac{k_x^2\tilde{\lambda}_c^2}{1 + 2\tilde{\eta}(1 - s_l)}}, \tag{7}$$

with $\omega_{\rm P}(0, l) = \omega_{\rm pl}\sqrt{1 + 2\tilde{\alpha}(1 - s_l)}$ for the longitudinal Josephson plasma and

$$\omega_{\rm L}(k_x, l) = \omega_{\rm L}(0, l)\sqrt{1 + \frac{k_x^2\lambda_{\rm Leg}^2}{1 + 2\nu(1 - s_l)}}, \tag{8}$$

with $\lambda_{\rm Leg} = 2\sqrt{\nu}\tilde{\lambda}_c/\sqrt{\eta_1 + \eta_2}$ and $\omega_{\rm L}(0, l) = \omega_{\rm Leg}\sqrt{1 + 2\nu(1 - s_l)}$ for the longitudinal Leggett mode, where $s_l = \cos[l\pi/(N+1)]$ and $\omega_{\rm pl}$ and $\omega_{\rm Leg}$ are, respectively, the Josephson plasma and the Leggett mode frequencies, i.e., $\omega_{\rm pl} = c/\sqrt{\epsilon}\tilde{\lambda}_c$ and $\omega_{\rm Leg} = c\sqrt{\alpha_1 + \alpha_2}/\sqrt{\epsilon}\lambda_{\rm in}$. Here, it is clearly found that the origin of the Leggett mode is a fluctuation between two superfluids which is essential to neutral multiple superfluids.

We plot the dispersion relations of these eigen modes in the case of $N = 5$ with $j_{c,1} = j_{c,2}$ for $\gamma_{\rm in} = 1.0$ and 3.0, respectively, in Figs. 2 and 3. The values of the inductive and capacitive coupling constants are chosen as $\alpha_1 = \alpha_2 = 0.1$ and $\eta_1 = \eta_2 = 10^3$, which are the values applicable to the cuprate IJJ's. If the Leggett mode is a low-energy excitation mode and can lie inside the energy gaps as the Josephon plasma, then it is possible that both modes are closely located in the low energy range as shown in these figures. It should be also noted that the Josephson plasma mode with the largest c-axis wave number, i.e., $l = 5$, is the lowest energy one close to $k_x = 0$, but this mode changes to the highest one for larger values of k_x. This is because the large inductive coupling, which is predominant in a wide k_x range, favors π phase shift in the phase differences between consecutive junctions [21]. This discussion clearly leads to that π anit-phase synchronization is preferable in the Josephson plasma mode under the presence of the layer parallel magnetic field. In fact, strong synchronous electromagnetic-wave emission is observed only at the zero and weak field in layered High-$T_{\rm c}$ copper oxide superconductors [22]. On the other hand, the dispersions in the Leggett mode does not show such level crossing as seen in the figures, since the excitation mode is associated with only the density channel. This indicates that the Leggett excitation always prefers synchronous oscillations along junction stacked direction even in the presence of the magnetic field. If the Leggett mode is excited by the charge injection or other ways, then the synchronized Leggett oscillation emerges and a conversion into the synchronized Josephson plasma excitation due to inherent nonlinearity may occur.

Finally, we mention that when the difference between the two tunneling channels exist (i.e., $j_{\rm c,1} \neq j_{\rm c,2}$, $\alpha_1 \neq \alpha_2$, and $\eta_1 \neq \eta_2$) a mode coupling between the Josephson-plasma and the Leggett modes can occur. Such a coupling effect is an interesting future task.

In summary, we derived the coupled dynamical equations for the phase differences which can be utilized for the analysis of AC and DC Josephson effects in the multi-gap IJJ's. The equations revealed that multi-gap IJJ's have two collective phase oscillation modes, the Josephson plasma and the Leggett mode whose origins are different. Moreover, it is found the Josephson plasma and Leggett modes favor π anti-phase and in-phase synchronization along the junction stacking , respectively, in a wide wave-number range.

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