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We describe a method to enhance the sensitivity of precision measurements that takes advantage of a quantum sensor's environment to amplify the sensor's response to weak external perturbations. An individual qubit is used to sense the dynamics of surrounding ancillary qubits, which are in turn affected by the external field to be measured. The resulting sensitivity enhancement is determined by the number of ancillas that are coupled strongly to the sensor qubit; it does not depend on the exact values of the coupling strengths and is resilient to many forms of decoherence. The method achieves nearly Heisenberg-limited precision measurement, using a novel class of entangled states. We discuss specific applications to improve clock sensitivity using trapped ions and magnetic sensing based on electronic spins in diamond.

Precision measurement is among the most important applications of resonance methods in physics. For example, quantum control of atomic systems forms the physical basis of the world's best clocks. Ideas from quantum information science have been used to demonstrate that quantum entanglement can enhance these measurements [1, 2]. At the same time a wide range of quantum systems have been recently developed aimed at novel realizations of solid state qubits. Potentially such systems can be used as quantum measurement devices, such as magnetic sensors with a unique combination of sensitivity and spatial resolution [3–6]. In this Letter, we describe a novel technique that makes use of the sensor's local environment as a resource to amplify its response to weak perturbations. We will use solid state sensors and ion clocks as examples.

The purpose of quantum metrology is to detect a small external field coupled to the sensor by a Hamiltonian: $H_b^S = b(t)\kappa S_z$, where S_z is the spin operator of the quantum sensor. Here, $b(t)$ can be an external magnetic field or the detuning of a laser from a clock transition while κ is the spin's coupling to the field. The working principle of almost any quantum metrology scheme can be reduced to a Ramsey experiment [7, 8], where the field is measured via the induced phase difference between two states of the quantum sensor. The figure of merit for quantum sensitivity is the smallest field δb_{min} that can be read out during a total time T. For a single spin 1/2 the sensitivity is given by $\delta b_{min} \simeq \frac{1}{\kappa\sqrt{T\tau}}$, if the sensing time is limited to τ (e.g. by environmental decoherence).

In many cases the external field also acts on the sensor's environment, which normally only induces decoherence and limits the sensitivity. Here we show that in some cases the environment can instead be used to enhance the sensitivity. For generality we will illustrate the key ideas using the *central spin model* (Fig. 1(a)). In this model a central spin (which can be prepared in a well defined initial state, coherently manipulated and read out) is coupled to a bath of *dark* spins that can be polarized and collectively controlled, but cannot be directly detected. The system is described by the Hamiltonian $H = H_b + H_{int}$, with

$$H_b = b(t)\left(\kappa S_z + \xi \sum I_z^i\right),\ H_{int} = |1\rangle\langle 1| \sum \lambda_i I_z^i, \quad (1)$$

where λ_i are the couplings between sensor and environment spins, while κ and ξ are couplings to the external field of the central and dark spins respectively. Here $|0\rangle$, $|1\rangle$ and S_z refer to the central spin while $|\uparrow\rangle$, $|\downarrow\rangle$, I_z^i to the dark spins (and we set $\hbar = 1$). We consider two cases. In the first one, H_{int} can be turned on and off at will and is much larger than any other interaction in the system (e.g. a laser-mediated ion interaction). In the second case, H_{int} is intrinsic to the system and of the same order of magnitude as the inverse of the relevant sensing time (e.g. dipole-dipole interactions between solid state spins). In all cases we will assume collective control over the dark spins.

To illustrate the sensing method we consider first the idealized case where the couplings between the central and the dark spins can be turned on and off at will and the dark spins are initialized in a pure state $|\uparrow\uparrow ... \uparrow\rangle$. Consider the circuit in Fig. 1(b). First, the central spin is prepared in an equal superposition of the two internal states $\sim |0\rangle + |1\rangle$. Then H_{int} is rotated to the x-axis and applied for a time τ. This induces controlled rotations of

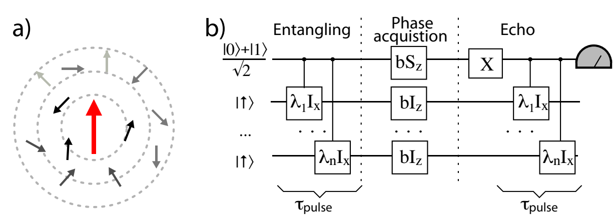

FIG. 1: (a) A central spin coupled to a spin bath, with varying couplings at different distances. (b) Ideal measurement circuit. Gates labeled $\lambda_i I_x$ represent controlled rotations $e^{-i\lambda_i I_x^i}$ of the dark spins, obtained via H_{int}. The gates bI_z represent rotations e^{-ibI_z} due to the external field b. At the end, only the state of the central spin is measured.

the dark spins, resulting in an entangled state

$$\left(|0\rangle|\uparrow\ldots\uparrow\rangle+|1\rangle|\varphi_1\ldots\varphi_N\rangle\right)/\sqrt{2}, \tag{2}$$

where $|\varphi_i\rangle\equiv\cos(\varphi_i)\,|\uparrow\rangle-i\,\sin(\varphi_i)\,|\downarrow\rangle$, with $\varphi_i=\lambda_i\tau$. This state is then used to sense the magnetic field. For now we neglect the coupling of the central spin to the external field (i.e. we set $\kappa=0$) and define $\theta_d=\xi\int_0^\tau dt\,b(t)$.

Under the action of the magnetic field, ignoring an overall phase, the state evolves to $\frac{1}{\sqrt{2}}\left(|0\rangle\,|\uparrow\ldots\uparrow\rangle+|1\rangle\,|\psi_1\ldots\psi_N\rangle\right)$, with

$$\begin{aligned}&|\psi_i\rangle=\cos\varphi_i\,|\uparrow\rangle_i-i\sin\varphi_i e^{i\theta_d/2}\,|\downarrow\rangle_i=\\&\left[\cos^2(\varphi_i)+e^{i\theta_d}\sin^2(\varphi_i)\right]|\varphi_i\rangle+\sin(\tfrac{\theta_d}{2})\sin(2\varphi_i)\left|\varphi_i^\perp\right\rangle\\&\quad\approx e^{i\theta_d\sin^2\varphi_i}\,|\varphi_i\rangle+\tfrac{1}{2}\theta_d\sin(2\varphi_i)\left|\varphi_i^\perp\right\rangle,\end{aligned}$$

where $\left\langle\varphi_i|\varphi_i^\perp\right\rangle=0$ and the last equation holds for small fields (to first order in θ_d). The central spin is then flipped by a π-pulse and another control operation with H_{int} along x is applied, yielding the final state $|\Psi_f\rangle=\frac{1}{\sqrt{2}}\left(|1\rangle|\varphi_1...\varphi_N\rangle+|0\rangle\,|\psi_1\ldots\psi_N\rangle\right)$. If the field were zero ($\theta_d=0$) the net result of the protocol would be to decouple the sensor spin from the environment—as in a spin-echo—since in this case $|\psi_i\rangle=|\varphi_i\rangle$. At the end of the sequence, the signal is read out by a single measurement of the S_y component of the central spin. For nonzero field the signal is given by $S=\langle\Psi_f|\left(S_y\otimes\mathbb{1}\right)|\Psi_f\rangle=\mathrm{Im}\left\{\prod_i\langle\varphi_i|\psi_i\rangle\right\}\approx\mathrm{Im}\left\{\prod_i e^{i\theta_d\sin^2\varphi_i}\right\}$. The effect of a small field is to introduce a phase difference $\Phi\approx\theta_d\sum_i\sin^2(\varphi_i)$ between the states of the dark spins, depending on the central spin state. This phase yields a non-zero signal $S\approx\Phi+O(b^2)$, while terms $\propto\left|\varphi_i^\perp\right\rangle$ lead to a second order (in $b\,(t)$) contribution to the signal. We can further add to the measured signal the phase acquired by the sensor spin alone, $\theta_s=\kappa\int_0^\tau dt\,b(t)$, (since H_b^S commutes with the rest of the Hamiltonian) to obtain $S\approx\Phi+\theta_s$.

While the signal is enhanced by a factor $\propto\Phi$, the quantum projection noise remains the same as we still read out one spin only. The minimum field that can be measured in a total time T is then:

$$\delta b_{min}=\sqrt{\frac{\tau}{T}}\frac{1}{\Phi+\theta_s}\approx\frac{1}{n\xi\sqrt{T\tau}}, \tag{3}$$

where n is the total number of dark spins. The linear scaling in n of the phase Φ can be achieved in principle for any distribution of λ_i's, since we can always choose a duration τ such that $\left\langle\sin^2(\lambda_i\tau)\right\rangle\geq\frac{1}{2}$, leading to order one contribution from each spin [It is still necessary to calibrate the quantum sensor by measuring a known external magnetic field to obtain an estimate of the sum $\sum_i\sin^2(\lambda_i\tau)$] . Thus we are able to perform Heisenberg-limited spectroscopy despite the fact that the precise form of the entangled state (2) is uncontrolled, and may not even be known to us [9]. This considerably relaxes the requirements for entanglement enhanced spectroscopy as compared to known strategies involving squeezed or GHZ-like states (for examples of non-symmetric and generalized entanglement applications in metrology see [10, 11]). Specifically, we note that in contrast to prior work [11], our scheme does not involve creating, or projecting onto, the GHZ or NOON states at any point during the protocol.

We now present two experimental implementations that approximate this idealized scheme: quantum clocks with trapped ions and spin-based magnetometry.

To reach high precision in quantum clocks, the ions must posses several characteristics: a stable clock transition, a cooling cycling transition, good initial state preparation and reliable state detection. In practice it is convenient to use two species of ions in the same Paul trap [12, 13]. The *spectroscopy* ions (e.g. $^{27}Al^+$) provide the clock transition while the *logic* ion (e.g. $^9Be^+$) fulfills the other requirements. Although inspired by a similar idea as the one proposed here, experiments using two ion species have so far been limited to just one spectroscopy ion [12, 13]; with our method the number of spectroscopy (*dark*) ions can be increased.

Specifically, we can implement the Hamiltonian H_{int} in Eq. (1) using multichromatic gates [14, 15]. These gates are known to be much more robust to heating noise than Cirac-Zoller gates used so far [16]. We can then use the method presented here to transfer the phase difference due to the detuning of several spectroscopy ions onto a single logic ion, which can be read out by fluorescence. This achieves Heisenberg-limited sensing of the clock transition without individual addressability of the spectroscopy ions and without creating GHZ states. Importantly, we accomplish this even if the spectroscopy ions have different couplings to the logic ion, as it is the case in a trap with different ion species due to the absence of a common center of mass mode.

We next briefly discuss the effects of decoherence on this method. It has been argued (see e.g. [17]) that the coherence time reduction for a n-spin entangled state, when each spin undergoes individual Markovian dephasing [17], reduces the sensitivity to roughly that of spectroscopy performed on n individual spins. The sensitivity scales then as $\sqrt{n}$, as opposed to the ideal scaling $\propto n$ derived here (see Ref. [18] for a different scaling in the case of atomic clocks). Since our method allows to read out n spectroscopy ions using only a single logic ion, even in this unfavorable case we would obtain an improvement $\propto\sqrt{n}$ with respect to current experimental realizations where only a single ancillary ion is available [12, 13]. Furthermore, this decoherence model is not so relevant in present setups, as technical noise during the gates and imperfect rotations are dominant for traps with many ions. Our method is highly robust to static repetitive imperfections during the gates, leading to further improvement depending on the exact noise model [15].

In many physical situations short bursts of controlled rotations, as used above, are not available. Instead, the couplings between the central and dark spins are always on and their exact strength is unknown. Examples of such systems are solid-state spin systems used for magnetometry [3–6]. It is still possible to achieve nearly Heisenberg-limited metrology even for these systems.

Specifically for this second case we consider magnetic sensing using a single Nitrogen Vacancy (NV) center in diamond [19], surrounded by *dark* spins associated with Nitrogen electronic impurities [20, 21]. We focus on NV centers since their electronic spins (S=1) can be efficiently initialized into the $S_z = 0$ state by optical pumping and measured via state selective fluorescence. By applying an external (static) magnetic field that splits the degeneracy between $S_z = \pm 1$ states and working on resonance with the $(0 \leftrightarrow 1)$ transition, the NV center can be reduced to an effective two-level system [4]. Then, the system comprising one NV center and several Nitrogen spins is well described by Hamiltonian (1).

In Fig. (2) we introduce a control sequence that yields an effect equivalent to the circuit in Fig. 1(b). The action of the pulse sequence can be best understood using the well known equivalence between Ramsey spectroscopy and Mach-Zehnder interferometry [7, 8], where the interferometer arms describe the central spin state. It is sufficient to consider the evolution of each arm separately, replacing S_z by its eigenvalues $m_s = \{0, 1\}$ and describing the evolution in the interaction frame defined by the control pulses [23]. Hamiltonian (1) becomes time-dependent, with dark spins alternating between I_z^i and I_x^i as shown in Fig. (2). Then, for different halves of the spin echo sequence, the coupling Hamiltonian in each arm is zero ($m_s = 0$) while for the other halves it has identical forms. In the absence of a magnetic field the evolution is thus the same along each arm of the interferometer. Adding an external field creates a phase shift between the two arms. For small field strengths we can then evaluate the phase difference acquired between the two arms as a perturbation. For a finite polarization P of the dark spins we find

$$\Phi = \overline{\theta_s}\left[1 + 2P\sum \frac{\overline{\theta_d}}{\overline{\theta_s}}\sin^2(\varphi_i/4)\right], \tag{4}$$

with $\overline{\theta_s} = \frac{\kappa}{\tau}\left[\int_0^{\frac{\tau}{2}} dt\, b(t) - \int_{\frac{\tau}{2}}^{\tau} dt\, b(t)\right]$, $\overline{\theta_d} = \frac{\xi}{\tau}\int_{\frac{\tau}{2}}^{\frac{3\tau}{4}} dt\, b\,(t)$. Compared to spin-echo based magnetometry [3], the signal is increased by the factor in the square bracket while the measurement noise is the same, as we still read out one spin only. Note that all the dark spins contribute positively. For values of the couplings such that $|\lambda_i\tau| \geq \pi$, or *strongly coupled* environment spins, we obtain a contribution $\propto\ 2n_{sc}\,\langle\sin^2(\frac{\lambda_i\tau}{4})\rangle \approx n_{sc}$. Each of the n_{sc} strongly coupled spins thus gives a contribution of order one, irrespective of the sign or exact value of the coupling. *Weakly coupled* dark spins ($\lambda_i\tau \leq 1$) contribute instead with a factor $(\lambda_i\tau)^2/8$ and we obtain a total phase $\Phi \approx \overline{\theta_s}\left[1 + \frac{\overline{\theta_d}}{\overline{\theta_s}}P\left(n_{sc} + \frac{1}{8}\sum{}^{'}\,(\lambda_i\tau)^2\right)\right]$, where the primed sum is on the weakly coupled spins only. In general the sensitivity enhancement scales at least as $\sim P\,n_{sc}$ [22]. We thus achieve nearly Heisenberg-limited sensing of the external field [17].

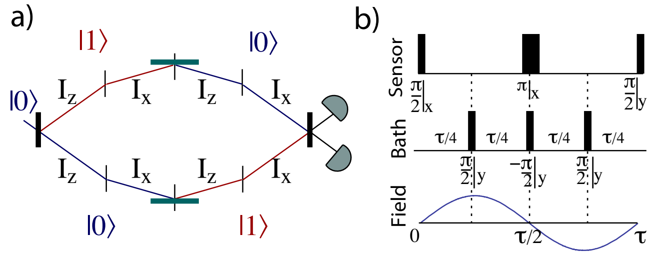

FIG. 2: Environment assisted magnetometry with NV centers. a) Mach-Zehnder interferometer, showing the central spin state and the orientation of the dark spins in the toggling frame of the external pulses. b) Pulse sequence acting on the sensor and dark spins.

We next take into account decoherence resulting from the interaction with the environment spins as well as decoherence of the dark spins themselves. To evaluate these effects, we compare the sensitivity achievable with the proposed pulse sequence to that obtained with a spin-echo sequence, considering the same system (a sensor spin surrounded by the same spin bath) and including the effects of decoherence (external perturbations and couplings between the dark spins). Once the central spin looses phase coherence due to interactions with the bath, it is no longer possible to use it for magnetometry. This limits the sensing time and consequently the magnetometer sensitivity. Spin echo (as well as more sophisticated decoupling techniques [15, 23, 24]) can be used to prolong the phase coherence of the central spin. Under realistic assumptions the coherence time for the pulse sequence presented here is on the same order of the sensor coherence time under spin-echo, T_2^s. In essence, the decoherence rate of an entangled state of the form (2) is dictated by the internal evolution of the strongly coupled spins, and the relevant decoherence time is thus the time it takes before any of these spins have decohered. The same is, however, the case for spin echo sequences: if a central spin is strongly coupled to n_{sc} dark spins, a spin flip of a single dark spin will lead to different evolutions in the two halves of the spin echo sequence, thus decohering the state of the central spin. As the source of both decoherence processes is the dipole-dipole interaction among dark spins, the coherence times of spin-echo and our procedure are on the same order and the signal amplification obtained with our strategy (Eq. 4) yields a sensitivity enhancement [25].

To be more quantitative, we analyze the evolution due to dipole-dipole couplings in the bath, described by the Hamiltonian

$$H = |1\rangle\langle 1|\sum \lambda_i I_z^i + \sum_{i<j}\kappa_{ij}\left(3I_z^i I_z^j - \overrightarrow{I_i}\cdot\overrightarrow{I_j}\right) \tag{5}$$

A short time expansion shows that the state fidelity at the end of the pulse sequence is given by $1 - \frac{\tau^4}{8}\cdot\sum_{i<j}\left(1-P^2\right)\kappa_{ij}^2\left(\lambda_i - \lambda_j\right)^2 + \ldots$, and $1 - \frac{\tau^4}{128}\sum_{i<j}\kappa_{ij}^2\,\times$

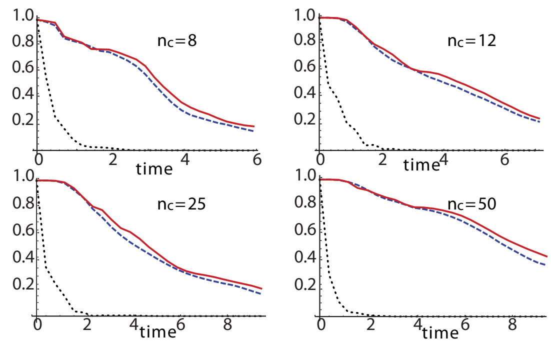

FIG. 3: **Simulations.** Normalized signal decay for the proposed sequence (red, solid lines), spin-echo (blue, dashed lines) and no control (black, dotted lines). We assumed perfect delta-pulses and set $P = 0$ for simplicity. A leading order cluster expansion was used [27]. 20 dark spins were randomly placed in a cube of side-length $\sqrt[3]{20}$ with the sensor spin at the center. We set $g\mu_B \equiv 1\,[m]^{3/2}\,[s]^{-1/2}$ to use dimensionless quantities. WAHUHA sequences [23] with n_c=8, 12, 25 and 50 cycles per echo interval were simulated. Each curve is an average over 10 Monte Carlo simulations.

$\big(7(\lambda_i^2+\lambda_j^2)+12\lambda_i\lambda_j+P^2\,(\lambda_i+2\lambda_j)\,(2\lambda_i+\lambda_j)\big)+...$, for the spin-echo and modified pulse sequences, respectively. For experimentally relevant values of polarizations, similar reductions in fidelity occur for both pulse sequences [26]. To go beyond the short time expansions we have simulated the signal decay for both spin-echo and the proposed pulse sequence (see Fig. 3). We compared these results to the signal decay when no control sequence is applied (for the same environment the decay is now described by the dephasing time T_2^*). We simulated a spin bath composed of spin 1/2 paramagnetic impurities, undergoing a WAHUHA sequence (which is designed to refocus the dipole-dipole coupling of the environment spins, but does not cancel out the coupling to the external field [15, 23]).

A different limitation on the sensing time τ is set by the fact that the orientation of the dark spin will not be static as assumed. Dipole-dipole couplings among dark spins during each $\tau/4$ period of free evolution rotate each spin away from the initial direction. This rotation means that the spins cease to build up a phase difference between the two arms of the interferometer for time scales comparable to the correlation time of the dark spin bath τ_c^d. The optimum sensing time is thus $T \sim \min\left\{T_2^s,\, \tau_c^d\right\}$. Since in most systems $\tau_c^d \geq T_2^s$ [23], the optimum sensing time of this pulse sequence is comparable to that of spin-echo based magnetometry and the sensitivity enhancement is roughly the same as the signal strength enhancement.

In conclusion, we proposed a scheme to enhance precision measurement by exploiting the possibility to coherently control the ancillary qubits. In solid state implementations we are able to exploit dark spins in the bath while preserving roughly the same coherence times as in spin-echo based magnetometry. Thus signal enhancement leads directly to sensitivity enhancement. For trapped ion implementations we can use imperfect phase gates and still achieve Heisenberg-limited sensitivity. Our method has the potential to be applied more generally, using different systems and more sophisticated pulse sequences [15]. It opens the possibility to use a broad class of partially entangled states to achieve Heisenberg limited metrology, even in the presence of disordered couplings, partial control and decoherence.

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