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Observable T_7 Lepton Flavor Symmetry at the Large Hadron Collider

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More often than not, models of flavor symmetry rely on the use of nonrenormalizable operators (in the guise of flavons) to accomplish the phenomenologically successful tribimaximal mixing of neutrinos. We show instead how a simple renormalizable two-parameter neutrino mass model of tribimaximal mixing can be constructed with the non-Abelian discrete symmetry T_7 and the gauging of $B-L$. This is also achieved without the addition of auxiliary symmetries and particles present in almost all other proposals. Most importantly, it is verifiable at the Large Hadron Collider.

In 2001, the non-Abelian discrete symmetry A_4 was shown for the first time [1] to allow for the seemingly incompatible pattern that charged-lepton masses are all very different and yet a symmetry exists to predict the neutrino mixing matrix without knowing the individual neutrino masses. In 2004, it was shown for the first time [2] that A_4 could also predict neutrino tribimaximal mixing with $\sin^2 2\theta_{atm} = 1$ and $\tan^2\theta_{sol} = 1/2$. Since early 2005, when the solar angle in neutrino oscillations was revised by SNO [3] to $\tan^2\theta_{sol} = 0.45 \pm 0.05$, this idea became widely accepted and the use of non-Abelian discrete symmetries [4] for understanding flavor has appeared in very many publications [5]. The two earliest papers [6, 7] after the SNO revision in 2005 both used A_4 and suggested two different two-parameter neutrino mass matrices, whereas the original proposal [2] of 2004 had three parameters.

If the 3×3 Majorana neutrino mass matrix $\mathcal{M}_\nu$ is rotated by the Cabibbo-Wolfenstein unitary matrix [8, 9]

$$U = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \tag{1}$$

where $\omega = \exp(2\pi i/3) = -1/2 + i\sqrt{3}/2$, then the tribimaximal form was shown [2] to be

$$U\mathcal{M}_\nu U^T = \begin{pmatrix} a+2b & 0 & 0 \\ 0 & a-b & d \\ 0 & d & a-b \end{pmatrix}. \tag{2}$$

The two examples mentioned above are then [6] $b=0$ and [7]

$$U\mathcal{M}_\nu U^T = \begin{pmatrix} a-d^2/a & 0 & 0 \\ 0 & a & d \\ 0 & d & a \end{pmatrix}. \tag{3}$$

However, these forms are only obtained at the expense of additional auxiliary symmetries and particles, and with the use of nonrenormalizable operators [6].

On the other hand, it has been shown recently [10] that A_4 alone is sufficient to obtain $b=0$, if the alternative A_4 lepton assignments of Ref. [11] are used instead of the original proposal of Ref. [1] and that neutrinos become massive through Higgs triplets [12] in a renormalizable model. Here we show how Eq. (3) may be obtained by the canonical seesaw mechanism for neutrino mass, using the non-Abelian discrete symmetry T_7 [13] and gauging $B-L$ [14], without the addition of auxiliary symmetries and particles or the use of nonrenormalizable operators.

Most importantly, our proposal is verifiable at the Large Hadron Collider (LHC). The predicted Z' gauge boson will decay into scalars which support the T_7 symmetry. Their subsequent decays into charged leptons will then reveal the predicted T_7 flavor structure used in obtaining neutrino tribimaximal mixing.

Since there are three families, non-Abelian discrete symmetries with irreducible three-dimensional representations are of special interest. The smallest group with a real $\underline{3}$ representation is A_4 which has 12 elements. The smallest group with a complex $\underline{3}$ representation is T_7 which has 21 elements. The group $\Delta(27)$ [15] is slightly bigger (27 elements) and also has a complex $\underline{3}$ representation. They are all subgroups of $SU(3)$. The various irreducible representations of the three groups are

$$A_4 \;:\; \underline{1}_i \;(i=1,2,3), \quad \underline{3}; \tag{4}$$

$$T_7 \;:\; \underline{1}_i \;(i=1,2,3), \quad \underline{3}, \quad \underline{\bar{3}}; \tag{5}$$

$$\Delta(27) \;:\; \underline{1}_i \;(i=1,2,3,4,5,6,7,8,9), \quad \underline{3}, \quad \underline{\bar{3}}. \tag{6}$$

Their crucial differences are in the following group multiplications

$$A_4 \;:\; \underline{3}\times\underline{3} = \sum_i \underline{1}_i + \underline{3} + \underline{3}; \tag{7}$$

$$\begin{aligned} T_7 \;:\; & \underline{3}\times\underline{3} = \underline{3} + \underline{\bar{3}} + \underline{\bar{3}}, \quad \underline{\bar{3}}\times\underline{\bar{3}} = \underline{\bar{3}} + \underline{3} + \underline{3}, \\ & \underline{3}\times\underline{\bar{3}} = \sum_i \underline{1}_i + \underline{3} + \underline{\bar{3}}; \end{aligned} \tag{8}$$

$$\Delta(27) \;:\; \underline{3}\times\underline{3} = \underline{\bar{3}} + \underline{\bar{3}} + \underline{\bar{3}}, \quad \underline{\bar{3}}\times\underline{\bar{3}} = \underline{3} + \underline{3} + \underline{3},$$

$$\underline{3} \times \underline{\bar{3}} = \sum_i \underline{1}_i. \quad (9)$$

We will show that our T_7 model assignments cannot be replaced by either those of A_4 or $\Delta(27)$.

The finite group T_7 is generated by two noncommuting 3×3 matrices:

$$a = \begin{pmatrix} \rho & 0 & 0 \\ 0 & \rho^2 & 0 \\ 0 & 0 & \rho^4 \end{pmatrix}, \quad b = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad (10)$$

where $\rho = \exp(2i\pi/7)$, so that $a^7 = 1$, $b^3 = 1$, and $ab = ba^4$. Let $\underline{3} = (x_1, x_2, x_3)$, and $\underline{\bar{3}} = (\bar{x}_1, \bar{x}_2, \bar{x}_3)$, then their possible multiplications form the following $\underline{3}$ representations: $(x_3 y_3, x_1 y_1, x_2 y_2)$, $(x_2 \bar{y}_1, x_3 \bar{y}_2, x_1 \bar{y}_3)$, $(\bar{x}_2 \bar{y}_3 \pm \bar{x}_3 \bar{y}_2, \bar{x}_3 \bar{y}_1 \pm \bar{x}_1 \bar{y}_3, \bar{x}_1 \bar{y}_2 \pm \bar{x}_2 \bar{y}_1)$, and the following $\underline{\bar{3}}$ representations: $(\bar{x}_3 \bar{y}_3, \bar{x}_1 \bar{y}_1, \bar{x}_2 \bar{y}_2)$, $(x_1 \bar{y}_2, x_2 \bar{y}_3, x_3 \bar{y}_1)$, $(x_2 y_3 \pm x_3 y_2, x_3 y_1 \pm x_1 y_3, x_1 y_2 \pm x_2 y_1)$. The combinations $x_1 \bar{y}_1 + \omega^{k-1} x_2 \bar{y}_2 + \omega^{2k-2} x_3 \bar{y}_3$ form the representations $\underline{1}_k$ for $k = 1, 2, 3$ respectively.

Under T_7, let $L_i = (\nu, l)_i \sim \underline{3}$, $l^c_i \sim \underline{1}_i$, $i = 1, 2, 3$, $\Phi_i = (\phi^+, \phi^0)_i \sim \underline{3}$, which means that $\tilde{\Phi}_i = (\bar{\phi}^0, -\phi^-)_i \sim \underline{\bar{3}}$. The Yukawa couplings $L_i l^c_j \tilde{\Phi}_k$ generate the charged-lepton mass matrix

$$m_l = \begin{pmatrix} f_1 v_1 & f_2 v_1 & f_3 v_1 \\ f_1 v_2 & \omega^2 f_2 v_2 & \omega f_3 v_2 \\ f_1 v_3 & \omega f_2 v_3 & \omega^2 f_3 v_3 \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} \begin{pmatrix} f_1 & 0 & 0 \\ 0 & f_2 & 0 \\ 0 & 0 & f_3 \end{pmatrix} v, \quad (11)$$

if $v_1 = v_2 = v_3 = v/\sqrt{3}$, as in the original A_4 proposal [1].

Let $\nu^c_i \sim \underline{\bar{3}}$, then the Yukawa couplings $L_i \nu^c_j \Phi_k$ are allowed, with

$$m_D = f_D \begin{pmatrix} 0 & v_1 & 0 \\ 0 & 0 & v_2 \\ v_3 & 0 & 0 \end{pmatrix} = \frac{f_D v}{\sqrt{3}} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad (12)$$

for $v_1 = v_2 = v_3 = v/\sqrt{3}$ which is already assumed for m_l. Note that Φ and $\tilde{\Phi}$ have $B - L = 0$.

Now add the neutral Higgs singlets $\chi_i \sim \underline{3}$ and $\eta_i \sim \underline{\bar{3}}$, both with $B - L = -2$. Then there are two Yukawa invariants: $\nu^c_i \nu^c_j \chi_k$ and $\nu^c_i \nu^c_j \eta_k$ (which has to be symmetric in i, j). Note that $\chi^*_i \sim \underline{\bar{3}}$ is not the same as $\eta_i \sim \underline{\bar{3}}$ because they have different $B - L$. This means that both $B - L$ and the complexity of the $\underline{3}$ and $\underline{\bar{3}}$ representations in T_7 are required for this scenario. The heavy Majorana mass matrix for ν^c is then

$$M = h \begin{pmatrix} u_2 & 0 & 0 \\ 0 & u_3 & 0 \\ 0 & 0 & u_1 \end{pmatrix} + h' \begin{pmatrix} 0 & u'_3 & u'_2 \\ u'_3 & 0 & u'_1 \\ u'_2 & u'_1 & 0 \end{pmatrix} = \begin{pmatrix} A & 0 & B \\ 0 & A & 0 \\ B & 0 & A \end{pmatrix}, \quad (13)$$

where $A = hu_1 = hu_2 = hu_3$ and $B = h'u'_2$, $u'_1 = u'_3 = 0$ have been assumed, i.e. χ_i breaks in the (1,1,1) direction, whereas η_i breaks in the (0,1,0) direction. This is the $Z_3 - Z_2$ misalignment also used in A_4 models.

The seesaw neutrino mass matrix is now

$$m_\nu = -m_D M^{-1} m_D^T = \frac{-f_D^2 v^2}{3A(A^2 - B^2)} \begin{pmatrix} A^2 - B^2 & 0 & 0 \\ 0 & A^2 & -AB \\ 0 & -AB & A^2 \end{pmatrix}, \quad (14)$$

which has only two parameters and is identical to Eq. (3). Detailed numerical analysis of this form was already done in Ref. [7]. Here we achieve the same result without the auxiliary $Z_4 \times Z_3$ symmetry and extra particles assumed there. The key is that $\underline{\bar{3}} \times \underline{\bar{3}} \times \underline{3}$ is an invariant in T_7, but not in $\Delta(27)$, whereas A_4 cannot distinguish this from $\underline{3} \times \underline{3} \times \underline{3}$ which yield two other invariants in T_7.

To realize the misalignment of $\langle \chi \rangle \sim (1, 1, 1)$ and $\langle \eta \rangle \sim (0, 1, 0)$, we need to choose the soft breaking terms in the Higgs potential consistent with these different residual symmetries [16]. However, the quartic terms $\chi^*_i \chi_j \eta^*_k \eta_l$ have several T_7 invariants, and most of them will destroy this pattern. To maintain the desired misalignment, this model has to be supersymmetrized.

Consider $\chi \sim \underline{3}$ and $\eta \sim \underline{\bar{3}}$ as superfields with $B - L = -2$. Add $\chi' \sim \underline{\bar{3}}$ and $\eta' \sim \underline{3}$ with $B - L = 2$. Then the superpotential contains the terms

$$W = f^\chi_{ijk} \nu^c_i \nu^c_j \chi_k + f^\eta_{ijk} \nu^c_i \nu^c_j \eta_k + m_\chi \chi_i \chi'_i + m_\eta \eta_i \eta'_i, \quad (15)$$

from which the F terms of the Higgs potential are

$$V_F = |m_\chi \chi_i|^2 + |m_\eta \eta_i|^2 + |f^\chi_{ijk} \nu^c_i \nu^c_j + m_\chi \chi'_k|^2 + |f^\eta_{ijk} \nu^c_i \nu^c_j + m_\eta \eta'_k|^2, \quad (16)$$

whereas the D terms from $U(1)_{B-L}$ are

$$V_D = 2 g^2_{B-L} |\chi^*_i \chi_i + \eta^*_i \eta_i - {\chi'_i}^* \chi'_i - {\eta'_i}^* \eta'_i|^2. \quad (17)$$

With the addition of bilinear soft terms $\chi^*_i \chi_i$, ${\chi'_i}^* \chi'_i$, $\chi_i \chi'_i + H.c.$, $\eta^*_2 \eta_2$, ${\eta'_2}^* \eta'_2$, $\eta_2 \eta'_2 + H.c.$, $\eta^*_1 \eta_1 + \eta^*_3 \eta_3$, ${\eta'_1}^* \eta'_1 + {\eta'_3}^* \eta'_3$, $\eta_1 \eta'_1 + \eta_3 \eta'_3 + H.c.$, and $(\chi_1 + \chi_2 + \chi_3) \eta'_2 + (\chi'_1 + \chi'_2 + \chi'_3) \eta_2 + H.c.$, which preserve $U(1)_{B-L}$ as they must, T_7 is broken with the desired pattern.

Flavor-changing leptonic interactions through Higgs exchange are present in this model, but they are suppressed by lepton masses, as in the original A_4 proposal [1]. The set of three Higgs doublets Φ_i transforming as $\underline{3}$ under T_7 is rotated by U of Eq. (1) to form mass eigenstates $\phi_{0,1,2} \sim 1, \omega, \omega^2$ under the residual Z_3, where ϕ_0 is identified as the one Higgs doublet (with $\langle \phi^0_0 \rangle = v$) of the Standard Model, with Yukawa couplings $v^{-1}[m_e \bar{e}_L e_R + m_\mu \bar{\mu}_L \mu_R + m_\tau \bar{\tau}_L \tau_R]$, which is of course flavor-conserving. The "flavor-changing" interactions of $\phi_{1,2}$ are then given by

$$\mathcal{L}_{int} = v^{-1}[m_\tau \overline{L}_{\mu L} \tau_R + m_\mu \overline{L}_{eL} \mu_R + m_e \overline{L}_{\tau L} e_R] \phi_1 + v^{-1}[m_\tau \overline{L}_{eL} \tau_R + m_\mu \overline{L}_{\tau L} \mu_R + m_e \overline{L}_{\mu L} e_R] \phi_2 + H.c. \quad (18)$$

However, if the neutrino sector is ignored, a lepton flavor triality (Z_3 symmetry) [17] exists here, under which $e, \mu, \tau \sim 1, \omega^2, \omega$, implying thus the decays $\tau^+ \to \mu^+\mu^+e^-$ and $\tau^+ \to e^+e^+\mu^-$, but no others. In particular, $\mu \to e\gamma$ is forbidden. Using

$$B(\tau^+ \to \mu^+\mu^+e^-) = \frac{m_\tau^2 m_\mu^2 (m_1^2+m_2^2)^2}{m_1^4 m_2^4} B(\tau \to \mu\nu\nu),$$

the experimental upper limit of 2.3×10^{-8} yields the bound [17] $m_1 m_2/\sqrt{m_1^2+m_2^2} > 22$ GeV (174 GeV$/v$) on the masses of $\psi^0_{1,2} = (\phi^0_1 \pm \bar{\phi}^0_2)/\sqrt{2}$.

Since the Higgs singlets χ and η which support the neutrino tribimaximal mixing under T_7 also transform under $U(1)_{B-L}$, this model can be tested at the LHC by discovering the $Z'_{B-L}(\equiv Z')$ gauge boson. The partial decay rates of Z' to the usual quarks and leptons are easily calculated. Let $\Gamma_0 = g^2_{B-L} m_{Z'}/12\pi$, then $\Gamma_q = (6)(3)(1/3)^2\Gamma_0$, $\Gamma_l = (3)(-1)^2\Gamma_0$, $\Gamma_\nu = (3)(-1)^2(1/2)\Gamma_0$. As for $Z' \to \psi^0_{1,2}\bar{\psi}^0_{2,1}$, it has the effective partial rate $\Gamma_\psi \simeq (2)(-2)^2 \sin^4\theta (1/4)\Gamma_0$, where $\sin\theta$ is an effective parameter accounting for the mixing of $\psi^0_{1,2}$ to χ and η (with the help of a $B-L=0$ singlet $S_i \sim \underline{3}$). Using Eq. (18), we find their signature decays to be given by

$$\psi^0_{1,2} \to \tau^+\mu^-,\ \tau^-e^+, \quad \bar{\psi}^0_{1,2} \to \tau^-\mu^+,\ \tau^+e^-, \qquad (19)$$

resulting in Z' leptonic final states such as $\tau^-\tau^-\mu^+e^+$ for example. In addition to being crucial for neutrino tribimaximal mixing to work under T_7, the $U(1)_{B-L}$ gauge symmetry is seen to provide also the means of verifying its predicted interactions. If the singlet neutrinos ν^c_i are light enough, they can also be produced by Z' decay as discussed in Ref. [14]. The mass eigenstates of ν^c_i are given by Eq. (13). Their decays into $\phi_{1,2}$ and leptons, and the subsequent decays of $\phi_{1,2}$ to leptons (resulting in six leptons in the final state) will then give a complete picture of tribimaximal mixing in this model.

We now study in detail the process $q\bar{q} \to Z' \to \psi_1\bar{\psi}_2 + \psi_2\bar{\psi}_1$ (assuming $m_1 = m_2$) with the subsequent decays $\psi \to \tau^-e^+$ and $\bar{\psi} \to \tau^-\mu^+$ at the LHC with $E_{cm} = 14$ TeV. We consider only the leptonic decay modes of the τ^-, with branching fraction 17.4% to either e^- or μ^-. The collider signature of such events is $e^+\mu^+\ell^-\ell^-$ plus missing energy, where $\ell = e, \mu$. The dominant backgrounds yielding the same signature are the processes (generated by MadEvent/MadGraph [18]):

$$\begin{aligned}
&WWZ : pp \to W^+W^-Z, W^\pm \to \ell^\pm\nu, Z \to \ell^+\ell^-,\\
&ZZ : pp \to ZZ, Z \to \ell^+\ell^-, Z \to \tau^+\tau^-, \tau^\pm \to \ell^\pm\nu\bar{\nu},\\
&t\bar{t} : pp \to t\bar{t} \to b(\to \ell^-)\bar{b}(\to \ell^+)W^+W^-,\ W^\pm \to \ell^\pm\nu,\\
&Zb\bar{b} : pp \to Zb(\to \ell^-)\bar{b}(\to \ell^+),\ Z \to \ell^+\ell^-, \qquad (20)
\end{aligned}$$

where $\ell = e, \mu$. Other SM backgrounds, e.g. ZZZ and $WWWW$, occur at a negligible rate after kinematic cuts, and are not shown here. We require no jet tagging and consider only events with both e^+ and μ^+ in the final state. The first two processes are the irreducible background, while the last two are reducible as they only contribute when some tagged particles escape detection, carrying away small transverse momentum (p_T) or falling out of the detector rapidity coverage.

TABLE I: Signal and background cross sections (fb) before and after cuts for four $(m_{Z'}, m_\psi)$ (GeV) benchmark points: (A) (1000,100), (B) (1500,100), (C) (1000,300), and (D) (1500,300). The "no cut" rates correspond to all leptonic decay modes of τ^- after $e^+\mu^+$ identification, the "basic cut" and "H_T cut" rates are obtained after imposing Eq. (21) and Eq. (22), respectively, whereas the "$x_{\tau_i} > 0$" rates are obtained after the τ^- reconstruction cuts. The bottom row shows the cut acceptance ($\mathcal{A}_{cut}$).

	(A)	(B)	(C)	(D)	$t\bar{t}$	WWZ	ZZ	$Zb\bar{b}$
no cut	5.14	0.98	2.57	0.72	1.22	0.21	27.11	2.99
basic cut	1.46	0.066	1.05	0.36	0.16	0.02	0.0052	0.024
H_T cut	1.41	0.065	1.04	0.36	0.08	0.006	0.0	0.0
$x_\tau > 0$	0.69	0.032	0.52	0.18	0.015	0.002	0.0	0.0
$\mathcal{A}_{cut}$	13.4%	3.2%	20%	25%	1.2%	1%	0.0	0.0

Our benchmark points are chosen as follows: $m_{Z'} = 1000$ (1500) GeV, $m_\psi = 100$ (300) GeV, $g_{B-L} = g = e/\sin\theta_W$, and $\sin^2\theta = 0.2$. In our analysis all events are required to pass the following *basic* acceptance cuts:

$$\begin{aligned}
&p^{(1,2)}_{T,\ell} \geq 50\,\text{GeV},\ p^{(3,4)}_{T,\ell} \geq 20\,\text{GeV},\ \ |\eta_\ell| \leq 2.5,\\
&\Delta R_{\ell\ell'} \geq 0.4,\ \ \not{E}_T > 30\ \text{GeV}, \qquad (21)
\end{aligned}$$

where (1-4) in the superscript index is the p_T order of the charged leptons. ΔR_{ij} is the separation in the azimuthal angle (ϕ) - pseudorapidity (η) plane between i and j, defined as $\Delta R_{ij} \equiv \sqrt{(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2}$. We also model detector resolution effects by smearing the final-state energy. To further suppress the SM backgrounds, we demand

$$H_T \equiv \sum_i p_{T,\,i} + \not{E}_T > 300\ \text{GeV}, \qquad (22)$$

where i denotes the visible particles. Figure 1(a) shows the normalized H_T distribution of both signal and background before the H_T cut. The signal spectrum exhibits an endpoint around the mass of Z', about 1 TeV, but with a long tail due to the Z' width and detector smearing effects. Table I displays the signal and background cross sections (fb) before and after cuts. The cut acceptance ($\mathcal{A}_{cut}$) increases with m_ψ as heavy scalar decay generates hard leptons and large H_T. For a light ψ and a heavy Z' (e.g. the benchmark B), $\mathcal{A}_{cut}$ decreases as the two charged leptons from the light scalar decay are very much parallel and fail the ΔR separation cuts.

To reconstruct the scalar ψ, we adopt the collinear approximation that the charged lepton and neutrinos from

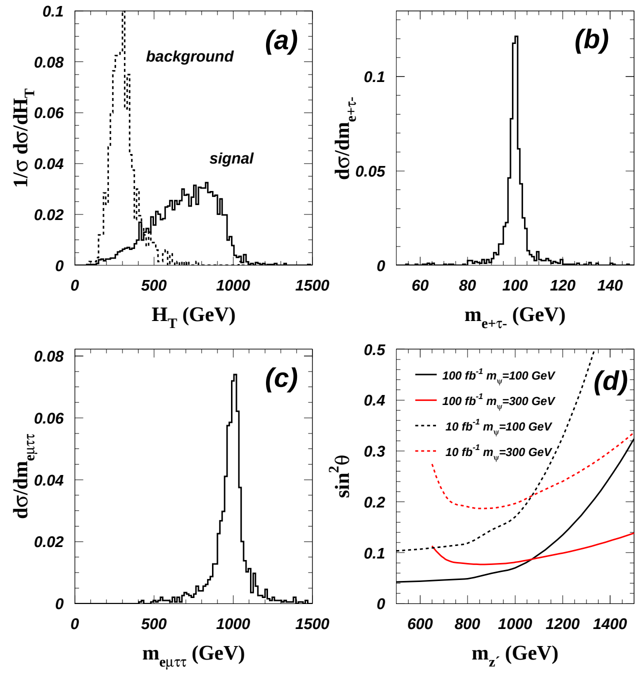

FIG. 1: (a) Normalized distribution of H_T; (b) Distribution of the invariant mass of the e^+ and reconstructed τ^-pair; (c) Distribution of the mass of the reconstructed Z'; (d) The 5σ significance contours in the plane of $m_{Z'}$ and $\sin^2\theta$.

τ decays are parallel due to the large boost of the τ. Such a condition is satisfied to an excellent degree because the τ leptons originate from a heavy scalar decay in the signal event. Denoting by x_{τ_i} the fraction of the parent τ energy which each observable decay particle carries, the transverse momentum vectors are related by [19]

$$\vec{\not{E}}_T = (1/x_{\tau_1} - 1)\,\vec{p}_1 + (1/x_{\tau_2} - 1)\,\vec{p}_2. \qquad (23)$$

When the decay products are not back-to-back, Eq. (23) gives two conditions for x_{τ_i} with the τ momenta as $\vec{p}_1/x_{\tau_1}$ and $\vec{p}_2/x_{\tau_2}$, respectively. We further require the calculated x_{τ_i} to be positive to remove the unphysical solutions. There are two possible combinations of $e^+\ell^-$ clusters for reconstructing the scalar ψ and gauge boson Z'. To choose the correct combination, we require the $e^+\ell^-$ pairing to be such that $\Delta R_{e^+\ell^-}$ is minimized. The mass spectra of the reconstructed ψ and Z' are plotted in Fig. 1(b) and (c), respectively, which clearly display sharp peaks around m_ψ and $m_{Z'}$. In Fig. 1(d), we show the 5σ discovery contours in the plane of $m_{Z'}$ and $\sin^2\theta$ by requiring 8.5 (5) signal events for an integration luminosity of 100 (10) fb^{-1} respectively. The regions above those curves are good for discovery.

In the quark sector, if we use $Q_i = (u,d)_i \sim \underline{3}$ and $u_i^c, d_i^c \sim \underline{1}_i, i = 1,2,3$ as we assume for the charged leptons, we again obtain arbitrary quark masses, but no mixing. To have realistic mixing angles, the residual Z_3 symmetry has to be broken.

Non-Abelian discrete symmetries have been successful in explaining the tribimaximal mixing of neutrinos, but not their masses. However, they are very difficult to test experimentally. In this paper, by combining T_7 and $U(1)_{B-L}$, we show how a simple renormalizable two-parameter neutrino mass model of tribimaximal mixing can be constructed, with verifiable experimental predictions. The key is the possible discovery of Z' at the TeV scale, which then decays into neutral Higgs scalars, whose subsequent exclusive decays into charged leptons have a distinct flavor pattern which may be observable at the LHC.

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