Interface between Topological and Superconducting Qubits

Liang Jiang, Charles L. Kane, and John Preskill

Phys. Rev. Lett. **106**, 130504 — Published 28 March 2011

DOI: 10.1103/PhysRevLett.106.130504

Interface between Topological and Superconducting Qubits

Liang Jiang,[1] Charles L. Kane,[2] John Preskill[1]
[1] *Institute for Quantum Information, California Institute of Technology, Pasadena, CA 91125, USA and*
[2] *Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA*

We propose and analyze an interface between a topological qubit and a superconducting flux qubit. In our scheme, the interaction between Majorana fermions in a topological insulator is coherently controlled by a superconducting phase that depends on the quantum state of the flux qubit. A controlled phase gate, achieved by pulsing this interaction on and off, can transfer quantum information between the topological qubit and the superconducting qubit.

Introduction Topologically ordered systems are intrinsically robust against local sources of decoherence, and therefore hold promise for quantum information processing. There have been many intriguing proposals for topological qubits, using spin lattice systems [1], p+ip superconductors [2], and fractional quantum Hall states with filling factor 5/2 [3]. The recently discovered topological insulators [4] can also support topologically protected qubits [5]. Meanwhile, conventional systems for quantum information processing (*e.g.*, ions, spins, photon polarizations, superconducting qubits) are steadily progressing; recent developments include high fidelity operations using ions [6] and superconducting qubits [7], long-distance entanglement generation using single photons [8, 9], and extremely long coherence times using nuclear spins [10].

Interfaces between topological and conventional quantum systems have also been considered recently [11, 12]. Hybrid systems [13, 14] may allow us to combine the advantages of conventional qubits (high fidelity readout, universal gates, distributed quantum communication and computation) with those of topological qubits (robust quantum storage, protected gates). In this paper, we propose and analyze an interface between a topological qubit based on Majorana fermions (MFs) at the surface of a topological insulator (TI) [5] and a conventional superconducting (SC) flux qubit based on a Josephson junction device [15]. The flux qubit has two basis states, for which the SC phase of a particular SC island has two possible values. In our scheme, this SC phase coherently controls the interaction between two MFs on the surface of the TI. This coupling between the MFs and the flux qubit provides a coherent interface between a topological and conventional quantum system, enabling exchange of quantum information between the two systems.

Topological Qubit The topological qubit can be encoded with four Majorana fermion operators $\{\gamma_i\}_{i=1,2,3,4}$, which satisfy the Majorana property $\gamma_i^\dagger = \gamma_i$ and fermionic anti-commutation relation $\{\gamma_i, \gamma_j\} = \delta_{ij}$. A Dirac fermion operator can be constructed from a pair of MFs $\Gamma_{ij}^\dagger = (\gamma_i - i\gamma_j)/\sqrt{2}$, defining a two dimensional Hilbert space labeled by $n_{ij} = \Gamma_{ij}^\dagger \Gamma_{ij} = 0, 1$. The two basis states for the topological qubit, each with an even number of Dirac fermions, are $|0\rangle_{\rm topo} = |0_{12}0_{34}\rangle$ and $|1\rangle_{\rm topo} = |1_{12}1_{34}\rangle$.

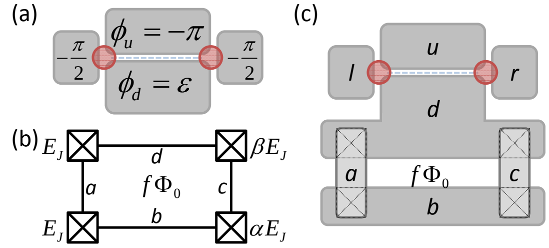

FIG. 1: (color online). On the surface of TI, patterned SC islands can form (a) STIS quantum wire, (b) flux qubit, and (c) hybrid system of topological and flux qubits. (a) Two MFs (red dots) are localized at two SC tri-junctions, connected by an STIS quantum wire (dashed blue line). The coupling between the MFs is controlled by the SC phases $\phi_d = \varepsilon$ and $\phi_u = -\pi$. (b) A flux qubit consists of four JJs connecting four SC islands (a,b,c,d) in series, enclosing an external magnetic flux $f\Phi_0$. (c) The hybrid system consists of an STIS wire and a flux qubit. The STIS wire (between islands d and u) couples the MFs, with coupling strength controlled by the flux qubit. The SC phase ϕ_c can be tuned by a phase-controller (not shown), and $\phi_d = \phi_c \pm \theta_4^*$ with the choice of $\pm$ sign depending on the state of the flux qubit.

The MFs can be created on the surface of a TI patterned with s-wave superconductors [5]. Due to the proximity effect [16], Cooper pairs can tunnel into the TI; hence the effective Hamiltonian describing the surface includes a pairing term, which has the form $V = \Delta_0 e^{i\phi}\psi_\uparrow^\dagger \psi_\downarrow^\dagger + h.c.$ (where $\psi_\uparrow^\dagger$, $\psi_\downarrow^\dagger$ are electron operators), assuming that the chemical potential is close to the Dirac point [17]. Here ϕ is the SC phase of the island. Each MF is localized at an SC vortex that is created by an SC tri-junction (i.e., three separated SC islands meeting at a point, see Fig. 1(a)). The MFs can interact via a superconductor-TI-superconductor (STIS) wire (Fig. 1(a)) that separates the SC islands d and u with $\phi_d = \varepsilon$ and $\phi_u = -\pi$, respectively. For a narrow STIS wire with width $W \ll v_F/\Delta_0$, the effective Hamiltonian is

$$H^{\rm STIS} = -iv_F\tau^x\partial_x + \delta_\varepsilon\tau^z, \tag{1}$$

where v_F is the effective fermi velocity, $\delta_\varepsilon = \Delta_0 \cos(\phi_d - \phi_u)/2 = -\Delta_0 \sin\varepsilon/2$, and $\tau^{x,z}$ are Pauli matrices acting on the wire's two zero energy modes [5].

As shown in Fig. 1(a), the STIS wire connects two localized MFs (indicated by two red dots at the tri-junctions) separated by distance L; these are two of the four MFs comprising the topological qubit. The coupling between the MFs (denoted as γ_1 and γ_2) via the STIS wire can be characterized by the Hamiltonian $\tilde{H}_{12}^{\mathrm{MF}} = iE(\varepsilon)\gamma_1\gamma_2/2$, with an induced energy splitting $E(\varepsilon)$ depending on the SC phase ε. The effective Hamiltonian for the topological qubit is

$$H_{12}^{\mathrm{MF}} = -\frac{E(\varepsilon)}{2}\mathbf{Z}_{\mathrm{topo}}. \tag{2}$$

where $Z_{\mathrm{topo}} = (|0\rangle\langle 0| - |1\rangle\langle 1|)_{\mathrm{topo}}$.

In Fig. 2(a), we plot $E(\varepsilon)$ as a function of a dimensionless parameter $\Lambda_\varepsilon \equiv \frac{\Delta_0 L}{v_F}\sin\frac{\varepsilon}{2}$. For $\Lambda_\varepsilon \gg 1$ and $0 < \varepsilon < \pi/2$ [5], the energy splitting $E(\varepsilon) \approx 2|\delta_\varepsilon| e^{-\Lambda_\varepsilon} \sim 0$ is negligibly small for localized MFs at the end of the wire, as the wavefunctions are proportional to $e^{-\Lambda_\varepsilon x/L}$ and $e^{-\Lambda_\varepsilon (L-x)/L}$. On the other hand, for $\Lambda_\varepsilon \lesssim 1$, the two MFs are delocalized and $E(\varepsilon)$ becomes sensitive to ε. We emphasize that $E(\varepsilon)$ is a non-linear function of ε [18], which enables us to switch the coupling on and off.

Flux Qubit The SC island d can also be part of an SC flux qubit (Fig. 1(b)), with $\phi_d = \varepsilon = \varepsilon^0$ or ε^1 depending on whether the state of the flux qubit is $|0\rangle_{\mathrm{flux}}$ or $|1\rangle_{\mathrm{flux}}$ as shown in Fig. 2(b,c). Therefore, the Hamiltonian H_{12}^{MF} couples the flux qubit and the topological qubit. Assuming a small *phase separation* $\Delta\varepsilon \equiv \varepsilon^0 - \varepsilon^1 \ll \pi/2$, we can switch off the coupling H_{12}^{MF} by tuning $\varepsilon^{0,1}$ to satisfy $v_F/L\Delta_0 \ll \varepsilon^{0,1} < \pi/2$ [5], so that the MFs are localized and uncoupled with negligible energy splitting $E(\varepsilon^0) \approx E(\varepsilon^1) \sim 0$. We can also switch on the coupling H_{12}^{MF} by adiabatically ramping to the parameter regime $\varepsilon^{0,1} \lesssim v_F/L\Delta_0$ to induce a nonnegligible $\left|E(\varepsilon^0) - E(\varepsilon^1)\right| \sim \Delta_0\Delta\varepsilon$. Because flux qubit designs with three Josephson junctions (JJs) [15, 19] are not amenable to achieving a small phase separation $\Delta\varepsilon \ll \pi/2$ [18], we are motivated to modify the design of the flux qubit by adding more JJs.

As shown in Fig. 1(b), our proposed flux qubit consists of a loop of four Josephson junctions in series that encloses an applied magnetic flux $f\Phi_0$ ($f \approx 1/2$ and $\Phi_0 = h/2e$ is the SC flux quantum). The Hamiltonian for the flux qubit is

$$H^{\mathrm{flux}} = T + U, \tag{3}$$

with Josephson potential energy $U = \sum_{i=1,2,3,4} E_{J,i}(1-\cos\theta_i)$, and capacitive charging energy $T = \frac{1}{2}\sum_{i=1,2,3,4} C_i V_i^2$. For the i-th JJ, $E_{J,i}$ is the Josephson coupling energy, θ_i is the gauge-invariant phase difference, C_i is the capacitance, and V_i is the voltage across the junction [15, 19]. In addition, there are relations satisfied by the phase accumulation around the loop $\sum_i \theta_i + 2f\pi \equiv 0 \,(\mathrm{mod}\, 2\pi)$ and the voltage across each junction $V_i = \left(\frac{\Phi_0}{2\pi}\right)\dot{\theta}_i$ [16]. The parameters are chosen as follows: the first two JJs have equal Josephson coupling energy $E_{J,1} = E_{J,2} = E_J$, the third JJ has $E_{J,3} = \alpha E_J$ with $0.5 < \alpha < 1$, and the fourth JJ has $E_{J,4} = \beta E_J$ with $\beta \gg 1$. For JJs with the same thickness but different junction area $\{A_i\}$, $E_{J,i} \propto A_i$ and $C_i \propto A_i$. The charging energies can be defined as $E_{C,1} = E_{C,2} = E_C = \frac{e^2}{2C_1}$, $E_{C,3} = \alpha^{-1}E_C$ and $E_{C,4} = \beta^{-1}E_C$. For these parameters and $f \approx 1/2$, the system has two stable states with persistent circulating current of opposite sign. We identify the flux qubit basis states with the two potential minima $|0\rangle_{\mathrm{flux}} = |\{\theta_i^*\}\rangle$ and $|1\rangle_{\mathrm{flux}} = |\{-\theta_i^*\}\rangle$ (modulo 2π), as illustrated in Fig. 2(b,c).

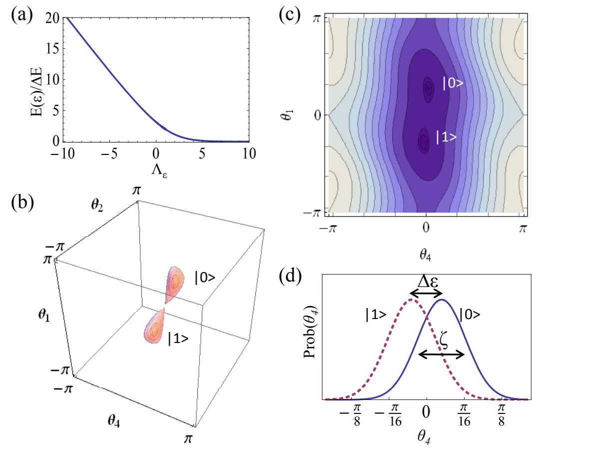

FIG. 2: (color online). (a) The energy splitting $E(\varepsilon)$ (in units of $\Delta E = v_F/L$) as a function of $\Lambda_\varepsilon = \frac{\Delta_0 L}{v_F}\sin\varepsilon/2$. (b) A contour plot of potential energy U as a function of $\{\theta_1, \theta_2, \theta_4\}$ with $\theta_3 = \pi - \theta_1 - \theta_2 - \theta_4$. There are two potential minima associated with flux qubit states $|0\rangle$ and $|1\rangle$. (c) A contour plot of U as a function of $\{\theta_1, \theta_4\}$ with $\theta_1 = \theta_2$ and $\theta_3 = \pi - 2\theta_1 - \theta_4$. (d) Marginal probability distributions of θ_4 associated with states $|0\rangle$ (blue solid line) and $|1\rangle$ (red dashed line). The parameters are $E_J/E_C = 80$ and $\{E_{J,i}/E_J\}_{i=1,2,3,4} = \{1, 1, \alpha = 0.8, \beta = 10\}$.

When $\beta \to \infty$, we may neglect the fourth junction and this system reduces to the previous flux qubit design with three JJs [15, 19]. For $\beta \gg 1$, there is a small phase difference across the fourth JJ [18], $\theta_4 = \pm\theta_4^* \approx \pm\frac{\sqrt{4\alpha^2-1}}{2\alpha}\frac{1}{\beta}$, where the choice of $\pm$ sign depends on the direction of the circulating current. We may write $\theta_4 = \mathbf{Z}_{\mathrm{flux}}\theta_4^*$, with $\mathbf{Z}_{\mathrm{flux}} = (|0\rangle\langle 0| - |1\rangle\langle 1|)_{\mathrm{flux}}$. The fourth JJ connects SC islands c and d, and if we fix ϕ_c relative to ϕ_u with a phase-controller [25], then ϕ_d will be $\varepsilon^0 = \phi_c + \theta_4^*$ or $\varepsilon^1 = \phi_c - \theta_4^*$ depending on the state of the flux qubit. The separation

$$\Delta\varepsilon \approx \frac{\sqrt{4\alpha^2-1}}{\alpha}\frac{1}{\beta} \tag{4}$$

between the two possible values of ϕ_d becomes small, as we desired, when β is large.

Aside from this small phase separation, there are also *quantum fluctuations* in θ_4 due to the finite capacitance.

Near its minimum at $\pm\{\theta_i^*\}$, the potential energy is approximately quadratic; therefore, for $\beta \gg 1$, the dynamics of θ_4 can be well described by a harmonic oscillator (HO) Hamiltonian

$$H^{\mathrm{HO}} = \frac{p_{\theta_4}^2}{2M_4} + \frac{E_{J,4}}{2}\left(\theta_4 - \mathbf{Z}_{\mathrm{flux}}\theta_4^*\right)^2, \qquad (5)$$

where the effective mass is $M_4 = \frac{1}{8E_{C,4}}$ and the canonical momentum p_{θ_4} satisfies $[\theta_4, p_{\theta_4}] = i$ (with $\hbar \equiv 1$). We may rewrite $H^{\mathrm{HO}} = \left(a^\dagger a + 1/2\right)\omega$ and $\theta_4 = \mathbf{Z}_{\mathrm{flux}}\theta_4^* + \zeta\left(a^\dagger + a\right)/\sqrt{2}$, where the oscillator frequency is $\omega = \sqrt{8E_J E_C}$ and the magnitude of quantum fluctuations is $\zeta = \left(\frac{8E_C}{E_J}\right)^{1/4}\beta^{-1/2}$. Fig. 2(d) shows the probability distribution functions $p_{0/1}(\theta_4) \approx \frac{1}{\zeta\sqrt{\pi}}e^{-(\theta_4 \mp \theta_4^*)^2/\zeta^2}$ associated with $|0\rangle_f$ and $|1\rangle_f$. The magnitude of the quantum fluctuations ζ is comparable to the phase separation $\Delta\varepsilon$; indeed $\zeta \propto \beta^{-1/2}$ may even dominate the phase separation $\Delta\varepsilon \propto \beta^{-1}$ for large β (Fig. 3). [26] Therefore, we should consider both the phase separation and the quantum fluctuations.

Hybrid System The Hamiltonian for the hybrid system of topological and flux qubits (Fig. 1(c)) is:

$$H = H^{\mathrm{HO}} + H_{12}^{\mathrm{MF}} = \left(a^\dagger a + 1/2\right)\omega - \frac{1}{2}E(\varepsilon)\,\mathbf{Z}_{\mathrm{topo}} \qquad (6)$$

where $\varepsilon = \phi_c + \theta_4 = \phi_c + \mathbf{Z}_{\mathrm{flux}}\theta_4^* + \zeta\left(a^\dagger + a\right)/\sqrt{2}$. In both flux qubit basis states, the oscillator is in its ground state with $\langle a^\dagger a\rangle = 0$. To first order in the small parameter $\delta \equiv \frac{\zeta}{\omega}\frac{dE(\phi)}{d\phi}\Big|_{\phi=\phi_c} \ll 1$, the Hamiltonian becomes

$$H = H^{\mathrm{HO}} - \frac{1}{2}\left(\langle E_0\rangle|0\rangle\langle 0| + \langle E_1\rangle|1\rangle\langle 1|\right)_{\mathrm{flux}} \otimes \mathbf{Z}_{\mathrm{topo}} + O\left(\delta^2\right)$$

where $\langle E_{0/1}\rangle \equiv \int d\theta_4 E(\phi_c + \theta_4)\,p_{0/1}(\theta_4)$.

Up to a single-qubit rotation, the effective Hamiltonian coupling the flux and topological qubits is

$$H_I = \frac{g}{4}\,\mathbf{Z}_{\mathrm{flux}}\mathbf{Z}_{\mathrm{topo}} \qquad (7)$$

with coupling strength $g = \langle E_1\rangle - \langle E_0\rangle \approx \left(E\left(\varepsilon^1\right) - E\left(\varepsilon^0\right)\right) + \frac{1}{4}\left(E''\left(\varepsilon^1\right) - E''\left(\varepsilon^0\right)\right)\zeta^2 + O\left(\zeta^3\right)$.The first term arises from the phase separation and the second term from the quantum fluctuations; corrections higher order in $\zeta \ll 1$ are small.

Because the energy splitting function $E(\varepsilon)$ is highly non-linear, we may tune ϕ_c to ϕ_{off} such that $v_F/L\Delta_0 \ll \varepsilon^{0,1} = \phi_{\mathrm{off}} \pm \Delta\varepsilon/2 < \pi/2$ and switch off the coupling $g \approx \Delta_0\Delta\varepsilon e^{-|\phi_{\mathrm{off}}|\Delta_0 L/2v_F} \sim 0$. On the other hand, we may adiabatically ramp ϕ_c to $\phi_{\mathrm{on}} \lesssim v_F/L\Delta_0$, which effectively switches on the coupling $g \approx \Delta_0\Delta\varepsilon$. By adiabatically changing ϕ_c from $\phi_{\mathrm{off}} \to \phi_{\mathrm{on}} \to \phi_{\mathrm{off}}$ with $\int g(t)\,dt = \pi$, we can implement the controlled-phase ($\mathrm{CP}_{t,f}$) gate between the topological (t) and flux (f) qubits. With Hadamard gates Had_f, we can achieve $\mathrm{CNOT}_{t,f} = \mathrm{Had}_f \cdot \mathrm{CP}_{t,f} \cdot \mathrm{Had}_f$, which flips the flux qubit conditioned on $|1\rangle_t$ and can be used for quantum non-demolition measurement of the topological qubit [11, 20]. Furthermore, with Hadamard gates Had_t (implemented by exchanging two MFs [3, 5]), we can achieve the swap operation $\mathrm{SWAP}_{t,f} = \left(\mathrm{Had}_t \cdot \mathrm{Had}_f \cdot \mathrm{CP}_{t,f}\right)^3$. Finally, with $\mathrm{CP}_{t,f}$, Had_t and single-qubit rotations U_f, we can achieve arbitrary unitary transformations for the two-qubit hybrid system of flux and topological qubits [13, 14].

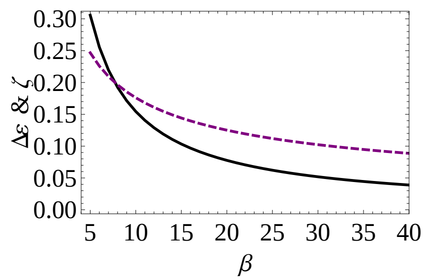

FIG. 3: (color online). Comparison between the phase separation $\Delta\varepsilon \propto \beta^{-1}$ (dark solid line) and the magnitude of quantum fluctuations $\zeta \propto \beta^{-1/2}$ (purple dashed line), assuming $E_J/E_C = 80$.

Imperfections There are four relevant imperfections for the coupled system of flux and topological qubits [27]. The first imperfection is related to the tunneling between $|0\rangle_{\mathrm{flux}}$ and $|1\rangle_{\mathrm{flux}}$ of the flux qubit, with tunneling rate $t \sim \omega\exp\left(-\sqrt{E_J/E_C}\right)$. The coupling between flux and topological qubits should be strong enough, $g \gg t$, to suppress the undesired tunneling probability $\eta_{tunnel} \approx (t/g)^2$.

The next imperfection comes from undesired excitations of the oscillators. According to the Hamiltonian H for the hybrid system, the oscillators may be excited via interaction $E(\phi_c + \theta_4) = E\left(\phi_c + \mathbf{Z}_{\mathrm{flux}}\theta_4^*\right) + \frac{dE}{d\varepsilon}\zeta\frac{\hat{a}_1^\dagger + \hat{a}_1}{\sqrt{2}} + \cdots$. The excitation probability can be estimated as $\eta_{excite} \approx \left(\frac{\zeta}{2\omega}\frac{dE}{d\varepsilon}\right)^2$. Since $\left|\frac{dE}{d\varepsilon}\right| \lesssim \Delta_0$, $\zeta \approx \left(\frac{8E_C}{E_J}\right)^{1/4}\beta^{-1/2}$, and $\omega = \sqrt{8E_J E_C}$, we estimate $\eta_{excite} \lesssim \frac{1}{20\beta}\left(\frac{\Delta_0}{E_J}\right)^2\sqrt{\frac{E_J}{E_C}}$.

The third imperfection is due to the finite length of the STIS wire, which limits the fidelity for the topological qubit itself. When we switch off the coupling between the flux and topological qubits by having $\phi_c = \phi_{\mathrm{off}}$ and $\Lambda_{\phi_{\mathrm{off}}} \gg 1$ for the STIS wire, there is an exponentially small energy splitting $E \sim \Delta_0 e^{-\Lambda_{\phi_{\mathrm{off}}}}$.

The last relevant imperfection is associated with the excitation modes of the quantum wire, with excitation energy $E' \approx v_F/L$ [5]. Occupation of these modes can potentially modify the phase separation of the flux qubit. Therefore, we need sufficiently low temperature to exponentially suppress the occupation of these modes by the factor $e^{-E'/k_B T}$.

Physical Parameters We may choose the following design parameters for the flux qubit: $\alpha = 0.8$, $\beta = 10$, $E_J/E_C = 80$, and $E_J = 200\,(2\pi)$ GHz. Both phase separation and quantum fluctuations depend sensitively on β (see Fig. 3), with $\Delta\varepsilon \approx 0.16$ and $\zeta \approx 0.18$. Meanwhile, the flux qubit has plasma oscillation frequency $\omega \approx 60\,(2\pi)$ GHz, energy barrier $\Delta U \approx 0.26 E_J$, tunneling rate $t \approx 1.8\sqrt{E_J E_C}\exp\left[-0.7\,(E_J/E_C)^{1/2}\right] \approx 70\,(2\pi)$ MHz; these parameters only marginally depend on β [18].

For mesoscopic aluminum junctions with critical current density 500 A/cm^2, the largest junction ($E_{J,4} = \beta E_J$) has an area of about 1 μm^2 [15]. For the topological qubit, it is feasible to achieve the parameters $\Delta_0 \sim 0.1$meV$\approx 25\,(2\pi)$GHz, $v_F \sim 10^5$m/s, $L \sim 5\mu m$, and $T = 20$mK. For the interface, the effective coupling is $g \sim \Delta_0 \Delta\varepsilon \sim 2\,(2\pi)$GHz. Therefore, we have imperfections $\eta_{tunnel} \sim 10^{-3}$, $\eta_{excite} \lesssim 10^{-3}$, $e^{-\Lambda_{\phi_{\rm off}}} \approx e^{-20|\sin\phi_{\rm off}/2|} < 10^{-3}$ (assuming $\phi_{\rm off} \approx \pi/4$), and $e^{-E'/k_B T} < 10^{-3}$ [28].

Phase Qubit A similar interface can be constructed to couple the SC phase qubit [7, 21] and the topological qubit. A phase qubit is just a JJ with a fixed DC-current source I. The phase qubit Hamiltonian is $H^{\rm phase} = T + U^{\rm phase}$, where $T = \frac{1}{2} E_C V^2$ and $U^{\rm phase} = -I\Phi_0\phi - I_0\Phi_0\cos\phi$. The qubit can be encoded in the two lowest energy states, $|0\rangle_{\rm phase}$ and $|1\rangle_{\rm phase}$, with magnitude of quantum fluctuations ζ_0 and ζ_1, respectively. The coupling strength between phase and topological qubits can be estimated as $g^{\rm phase} \approx E''(\varepsilon)\left(\zeta_1^2 - \zeta_0^2\right)$.

Notes added It was recently proposed to use the Aharonov-Casher (AC) effect for quantum non-demolition measurement of a topological qubit [11, 12]. This proposal, which applies in the parameter regime $\alpha > 1$ where the flux qubit has two possible tunneling pathways, exploits the observation that whether two tunneling paths interfere destructively or constructively can be controlled by the state of the topological qubit. In contrast, our proposal, which applies in the parameter regime $\alpha < 1$ where the flux qubit has only one tunneling pathway, exploits the non-linearity of the energy splitting $E(\varepsilon)$ to achieve a controlled-phase coupling between the topological and flux qubits. After this Letter was finished, the related work [22] appeared.

Conclusion We have proposed and analyzed a feasible interface between flux and topological qubits. Our proposal uses a flux qubit design with four JJs, such that the two basis states of the qubit have a small phase separation $\Delta\varepsilon$ on a particular superconducting island, enabling us to adiabatically switch on and off the coupling between the flux and topological qubits. Such interfaces may enable us to store and retrieve quantum information using the topological qubit, to repetitively readout the topological qubit with a conventional qubit, or to switch between conventional and topological systems for various quantum information processing tasks.

We are especially indebted to Mikhail Lukin for inspiring discussions. We also thank Anton Akhmerov, Jason Alicea, Erez Berg, David DiVincenzo, Garry Goldstein, Netanel Lindner and Gil Refael for helpful comments. This work was supported by the Sherman Fairchild Foundation, by NSF grants DMR-0906175 and PHY-0803371, by DOE grant DE-FG03-92-ER40701, and by NSA/ARO grant W911NF-09-1-0442.

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[25] An SC phase-controller can fix the phase difference between two SC islands. It can be implemented by controlling either the external flux or the current [18].
[26] Despite the large quantum fluctuations in θ_4 ($\zeta > \Delta\varepsilon$), the two quantum states of the flux qubit are well localized because the two potential minima are widely separated in the θ_1 direction (see Fig. 2(c)).
[27] We assume that flux qubit parameters can be accurately measured, and hence ignore fabrication uncertainties.
[28] The SQUID circuit that measures the flux qubit is another potential source of error, as it may introduce an additional plasma mode with low frequency ω'. Assuming $\omega' \sim 2\,(2\pi)$GHz [23]), we may choose $g = 200(2\pi)$MHz and $t = 10(2\pi)$MHz, so that $\omega' \gg g \gg t \gg 1/T_2$. If the flux qubit's spin-echo coherence time T_2 is long ($\approx 4\mu$s [24]), a 1% error rate can be achieved.