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Resistivity of a non-Galilean–invariant Fermi Liquid near Pomeranchuk Quantum Criticality

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We analyze the effect of the electron-electron interaction on the resistivity of a metal near a Pomeranchuk quantum phase transition (QPT). We show that Umklapp processes are not effective near a QPT, and one must consider both interactions and disorder to obtain finite and T dependent resistivity. By power counting, the correction to the residual resistivity at low T scales as $AT^{(D+2)/3}$ near a $Z = 3$ QPT. We show, however, that $A = 0$ for a simply connected, convex Fermi surface in 2D, due to hidden integrability of the electron motion. We argue that $A > 0$ in a two-band $(s-d)$ model and propose this model as an explanation for the observed $T^{(D+2)/3}$ behavior.

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A T^2 scaling of the resistivity ρ is the main signature of the Fermi liquid (FL) behavior in metals. Although this scaling is usually associated with the T^2 behavior of the quasiparticle scattering rate, $1/\tau_{\rm ee}$, the relation between $1/\tau_{\rm ee}$ and ρ is not straightforward because one has to specify a momentum relaxation mechanism. For example, even though $1/\tau_{\rm ee} \propto T^2$ in a Galilean-invariant FL (GIFL), its resistivity is zero (although the heat conductivity and viscosity are finite). In clean systems and at low T (when scattering on phonons can be neglected), such a mechanism is Umklapp electron-electron (ee) scattering [1, 2], which conserves the quasimomentum up to a reciprocal lattice vector: $\mathbf{k}+\mathbf{p} = \mathbf{k}'+\mathbf{p}'+\mathbf{b}$. An Umklapp process is allowed if the electron momenta $\mathbf{k}$ and $\mathbf{p}$, as well as the momentum transfer $\mathbf{q} = \mathbf{k}-\mathbf{k}'$, are all of order $\mathbf{b}$; these two conditions are usually satisfied in conventional metals. If this is the case, Umklapps occur at a rate comparable to $1/\tau_{\rm ee}$, and $\rho \propto 1/\tau_{\rm ee}$.

Even a conventional metal, however, can be tuned to a Pomeranchuk-type quantum phase transition (QPT) lowering the symmetry of the Fermi surface (FS).

A Pomeranchuk QPT is the $q = 0$ instability, manifested by the divergence of long wavelength fluctuations of the order parameter [3]. The FL near a Pomeranchuk QPT differs from that in a conventional metal in that the effective ee interaction is of a long-range. In the Hertz-Millis model,

$$U_{\rm eff}(\mathbf{q},\omega) = \frac{U_0}{q^2 + \xi^{-2} - i\gamma\omega/q}, \qquad (1)$$

where $\xi \gg b^{-1}$ is the correlation length (we omit the spin symbols for brevity). Conventional reasoning for this case (see, e.g., Ref. [4]) is that Umklapp scattering is accounted for if $\tau_{\rm ee}$ is replaced by the transport time $\tau_{\rm ee}^{\rm tr}$. For the interaction in Eq. (1), $1/\tau_{\rm ee} \propto T^2$ for $T \ll T_{\rm FL} \equiv 1/\gamma\xi^3$ and $1/\tau_{\rm ee} \propto T^{D/3}$ for $T \gg T_{\rm FL}$, while $1/\tau_{\rm ee}^{\rm tr} \sim (1/\tau_{\rm ee})\,(\bar{q}/k_F)^2$, where $\bar{q} = \max\{\xi^{-1}, (\gamma T)^{1/3}\}$. One then obtains $\rho \propto T^\alpha$, where $\alpha = 2$ in the FL regime and $\alpha = (D+2)/3$ in the non-FL regime. In 3D, $\alpha = 5/3$ in the non-FL regime, which is close to what has been observed in a number of itinerant ferromagnets [6].

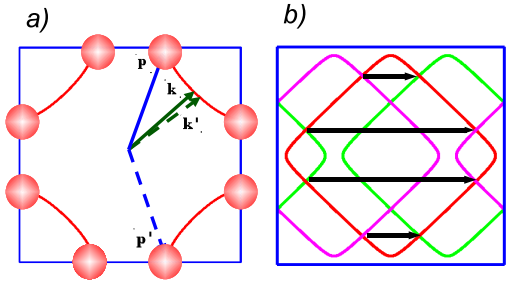

FIG. 1: (COLOR ONLINE) *a*) Umklapp process for a long-range electron-electron interaction. *b*) Umklapp processes for large q. The original FS is shown in red.

In this Letter we re-examine the role of Umklapp scattering and also analyze the interplay between ee and electron-impurity (ei) interactions near a Pomeranchuk instability. We argue that Umklapp processes do not give rise to the T^α behavior of the resistivity near a QPT in a clean system. In a dirty system, the *correction* to the residual resistivity scales nominally as AT^α at low T, but whether A is finite depends on dimensionality of the FS (2D vs 3D), its topology (simply connected vs multiply connected), and its shape (convex vs concave). For a simply-connected, convex FS in 2D, $A = 0$, and the first non-vanishing term in ρ scales as $T^{\alpha+2}$, which is always subleading to a T^2 contribution from non-critical scattering channels. The reason for the vanishing of A is hidden integrability: the constraint that all electrons involved in scattering must be on the FS lowers the effective dimensionality of scattering events from 2D to 1D, where the motion is integrable and hence no relaxation

is possible. Moving away from the FS breaks integrability but at the price of an extra T^2 factor. For a 3D, or multiply-connected, or concave FS, integrability is broken, and $A \neq 0$. However, if any of these three features is weak, i.e., the FS is quasi-2D, there is a crossover between integrable-like scaling (with exponent $\alpha+2$) at higher T to non-integrable–like scaling (with exponent α) at lower T. We also show that the resistivity saturates at high T, when *ee* scattering dominates, and that a true scaling regime can be achieved in a two-band ($s-d$) system with substantially different masses of charge carriers.

The $q=0$ nature of the QPT makes our case different from the one near an antiferromagnetic QPT [7]. There, Umklapps in both "hot" and "cold" parts of the FS do lead to finite resistivity, while disorder changes the balance of hot and cold contributions. For the same reason, the interplay between normal and Umklapp processes in a 2D Hubbard model [8] is also different from our case.

Umklapp scattering. The relation $\rho \propto 1/\tau_{\rm ee}^{\rm tr}$ is based on the assumption that Umklapp events are as frequent as normal ones. We argue that this assumption breaks down near Pomeranchuk criticality. Indeed, for small-angle scattering, one of the final momenta has to be close to the initial one, e.g., $|\mathbf{k}-\mathbf{k}'| \lesssim \bar{q} \ll b$. This implies that $\mathbf{b}$ is to be absorbed almost entirely by $\mathbf{p}-\mathbf{p}'$, which is only possible if $\mathbf{p}$ and $\mathbf{p}'$ are at the edges of the Brillouin zone (and the FS is open), see Fig. 1. As a result, the Umklapp rate is proportional to the phase space of "Umklapp hot spots" and is small by a factor of $\bar{q}^D$ compared to $1/\tau_{\rm ee}^{\rm tr}$. The conditions for the Umklapp hot spots to occur are rather stringent, e.g, they do not exist in a particle-hole symmetric system: if Umklapps are forbidden for a closed FS (at less-than-half filling), they are also forbidden for an open FS (at more-than-half filling). If particle-hole symmetry is broken, Umklapp hot spots do appear. However, one has to distinguish between real and pseudo Umklapp processes. For example, the process in Fig. 2*a* is a pseudo-Umklapp process because it can be viewed either as an Umklapp event on the open (electron) FS or as a normal event on the closed (hole) FS. Since normal scattering does not give finite resistivity, the same is true for this type of Umklapp scattering.

To emphasize the difference between the real and pseudo Umklapp processes, we relax the assumption of small q for a moment. A graphical construction for a closed FS is shown in Fig. 1*b*. If q is larger than some critical value, the Bragg replicas of shifted FSs (magenta and green) intersect the original FS (red) at more than four points. These points represent the initial and final states of Umklapp processes (shown by arrows), which cannot be mapped onto normal ones. These are real Umklapp processes which do give rise to finite ρ. For the case in Fig. 2*b*, real Umklapps occur if $b-k_{\max}<q_x<b$, where $k_{\max}$ is the maximal diameter of the FS in the x direction. For small q, this can happen only near half-filling, when $|k_{\max}-b| \lesssim \bar{q}$. However, half-filling favors a finite-q instability, e.g., antiferromagnetism, over the $q=0$ one.

The conclusion of this analysis is that Umklapp scattering cannot give rise to $\rho(T) \propto T^\alpha$ with $\alpha<2$ in a clean system with a long-range interaction.

A combination of normal ee and ei interactions. We now neglect Umklapp processes but invoke impurity scattering as a mechanism of momentum relaxation. Our analysis is based on the Boltzmann equation near equilibrium (its validity is discussed later in this Letter)

$$e\mathbf{v_k}\cdot\mathbf{E}n'_{\mathbf{k}} = -I_{\rm ei} - I_{\rm ee}, \tag{2}$$

where $\mathbf{E}$ is the electric field, $n_{\mathbf{k}}$ is the Fermi funtion, and $I_{\rm ei}$ and $I_{\rm ee}$ describe the *ei* and *ee* scatterings. Although all of our results are valid for the most general form of $I_{\rm ei}$, we will restrict our attention to δ function impurities, when $I_{\rm ei} = (f_{\mathbf{k}}-n_{\mathbf{k}})/\tau_{\rm i}$ with $\tau_{\rm i}=$ const. The *ee* collision integral for the non-equilibrium part of $f_{\mathbf{k}}$ defined by $f_{\mathbf{k}} = n_{\mathbf{k}} + n_{\mathbf{k}}(1-n_{\mathbf{k}})\,g_{\mathbf{k}}$ can be written as [9]

$$I_{\rm ee} = \sum_{\mathbf{p},\mathbf{q}} |M_{\mathbf{k},\mathbf{p}}(\mathbf{q},\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k}-\mathbf{q}})|^2\,(g_{\mathbf{k}}+g_{\mathbf{p}}-g_{\mathbf{k}-\mathbf{q}}-g_{\mathbf{p}+\mathbf{q}})$$
$$\times n_{\mathbf{k}}n_{\mathbf{p}}\,(1-n_{\mathbf{k}-\mathbf{q}})(1-n_{\mathbf{p}+\mathbf{q}})\delta\,(\epsilon_{\mathbf{k}}+\epsilon_{\mathbf{p}}-\epsilon_{\mathbf{k}-\mathbf{q}}-\epsilon_{\mathbf{p}+\mathbf{q}}) \tag{3}$$

where $M_{\mathbf{k},\mathbf{p}}(\mathbf{q},\omega) = U_{\rm eff}(\mathbf{q},\omega)S_{\mathbf{k},\mathbf{p}}$ is the matrix element of the effective *ee* interaction on the Bloch wave functions, and $S_{\mathbf{k},\mathbf{p}}$ is the structure factor for a given lattice. Normal *ee* collisions conserve the momentum, i.e., $\sum_{\mathbf{k}}\mathbf{k}I_{\rm ee}=0$. For a GIFL with $\mathbf{v_k}=\mathbf{k}/m$, the conductivity is obtained by multiplying Eq. (2) by $\mathbf{v_k}$ and summing over $\mathbf{k}$, upon which $I_{\rm ee}$ drops out, so that the resulting relation between the electrical current and $\mathbf{E}$ is independent of the *ee* interaction. For a non-GIFL with $\mathbf{v_k}=\nabla_{\mathbf{k}}\epsilon_{\mathbf{k}}\neq\mathbf{k}/m$, normal collisions, in general, affect the conductivity.

Low temperatures. The first question is whether the correction to the residual conductivity due to normal *ee* scattering scales as AT^α at low T, when $\tau_{\rm ee}\gg\tau_{\rm i}$. Solving Eq. (2) to first order in $I_{\rm ee}$, we obtain

$$\delta\sigma_{ii} = -\frac{e^2\tau_{\rm i}^2}{2T}\int\frac{d^Dq}{(2\pi)^D}\int\int\int d\omega d\epsilon_{\mathbf{k}}d\epsilon_{\mathbf{p}}\oint\oint\frac{da_{\mathbf{k}}}{v_{\mathbf{k}}}\frac{da_{\mathbf{p}}}{v_{\mathbf{p}}}|M_{\mathbf{k},\mathbf{p}}(\mathbf{q},\omega)|^2\Delta\mathbf{v}_i^2$$
$$\times n(\epsilon_{\mathbf{k}})\,n(\epsilon_{\mathbf{p}})\,[1-n(\epsilon_{\mathbf{k}}-\omega)]\,[1-n(\epsilon_{\mathbf{p}}+\omega)]\,\delta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k}-\mathbf{q}}-\omega)\,\delta(\epsilon_{\mathbf{p}}-\epsilon_{\mathbf{p}+\mathbf{q}}+\omega), \tag{4}$$

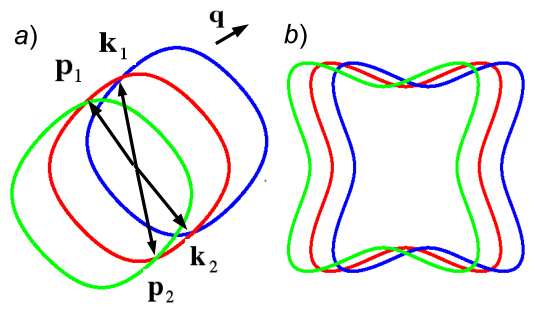

FIG. 2: (COLOR ONLINE) *a*) Normal scattering on a convex, simply connect, 2D Fermi surface. The blue and green FSs are obtained by shifting the original one (red) by momenta $\mathbf{q}$ and $-\mathbf{q}$, respectively. Process $(\mathbf{k}_1, \mathbf{p}_2) \leftrightarrows (\mathbf{k}_2, \mathbf{p}_1)$ is a Cooper channel scattering. Processes $(\mathbf{k}_{1,2}, \mathbf{p}_{1,2}) \leftrightarrows (\mathbf{p}_{1,2}, \mathbf{k}_{1,2})$ are momentum swaps. Neither of these processes affect the resistivity. *b*) Mormal scattering on a concave 2D FS. The initial and final states can be chosen from a set of twelve points where the shifted FSs intersect the original one.

where $\Delta\mathbf{v} \equiv \mathbf{v}_\mathbf{k} + \mathbf{v}_\mathbf{p} - \mathbf{v}_{\mathbf{k}-\mathbf{q}} - \mathbf{v}_{\mathbf{p}+\mathbf{q}}$, and $da_\mathbf{l}$ is the FS element. For a GIFL, $\Delta\mathbf{v} = 0$ and thus $\delta\sigma_{ii}$ vanishes identically. We will see, however, that the leading term in $\delta\sigma_{ii}$ may vanish even on a lattice. The crucial point is that the leading T dependence of $\delta\sigma_{ii}$ is obtained by neglecting ω in both δ functions, i.e., by projecting electrons onto the FS. Integrating over $\epsilon_\mathbf{k}$ and $\epsilon_\mathbf{p}$, we obtain

$$\delta\sigma_{ii} = -\frac{e^2\tau_\mathrm{i}^2 T^2}{2}\int \frac{d^D q}{(2\pi)^D}\oint\oint \frac{da_\mathbf{k}}{v_\mathbf{k}}\frac{da_\mathbf{p}}{v_\mathbf{p}} R_{\mathbf{k},\mathbf{p}}(\mathbf{q}) \times \Delta\mathbf{v}_i^2 \delta(\epsilon_\mathbf{k} - \epsilon_{\mathbf{k}-\mathbf{q}})|_{\epsilon_\mathbf{k}=0}\delta(\epsilon_\mathbf{p} - \epsilon_{\mathbf{p}+\mathbf{q}})|_{\epsilon_\mathbf{p}=0}, \quad (5)$$

where $R_{\mathbf{k},\mathbf{p}}(\mathbf{q}) \equiv \int d\omega\left(\omega^2/T^3\right)|M_{\mathbf{k},\mathbf{p}}(\mathbf{q},\omega)|^2 N(\omega) \times [N(\omega)+1]$ and $N(\omega)$ is the Bose function. By power counting, $\delta\sigma_{ii} \propto AT^\alpha$; yet one has to verify if $A \neq 0$.

The prefactor A is given by the solution of a purely geometric problem: for a fixed momentum transfer $\mathbf{q}$, find the initial states $\mathbf{k}$ and $\mathbf{p}$ on the FS so that energy conservation is satisfied. For small q, energy conservation reduces to $\epsilon_\mathbf{k} - \epsilon_{\mathbf{k}-\mathbf{q}} \approx \mathbf{v}_\mathbf{k}\cdot\mathbf{q} = 0$ and $\epsilon_\mathbf{p} - \epsilon_{\mathbf{p}+\mathbf{q}} \approx -\mathbf{v}_\mathbf{p}\cdot\mathbf{q} = 0$. Therefore, $\mathbf{k}$ and $\mathbf{p}$ are the points where *all* tangents in the direction of $\mathbf{q}$ intersect the FS. Since a convex, simply-connected, and 2D FS has only two tangents, there are only two solutions for $\mathbf{k}$ and $\mathbf{p}$ [8]. Equivalently, if the FS of this type is shifted by a small momentum $\mathbf{q}$, there are only two intersection points ($\mathbf{k}_1$ and $\mathbf{k}_2$ in Fig. 2*a*). A shift by $-\mathbf{q}$ gives two more points: $\mathbf{p}_1$ and $\mathbf{p}_2$. However, these solutions are not independent. Indeed, since the equation $\epsilon_{\mathbf{k}-\mathbf{q}} = \epsilon_\mathbf{k}$ has only two roots, and the second equation $\epsilon_{\mathbf{p}+\mathbf{q}} = \epsilon_\mathbf{p}$ reduces to the first one upon $\mathbf{p} \to -\mathbf{k}$, we have $\mathbf{p}_1 = -\mathbf{k}_2, \mathbf{p}_2 = -\mathbf{k}_1$. Therefore, the process $(\mathbf{k}_1, \mathbf{p}_2) \leftrightarrows (\mathbf{p}_1, \mathbf{k}_2)$ corresponds to the Cooper channel of scattering with zero total momentum. But this implies that $\mathbf{v}_{\mathbf{k}_1} + \mathbf{v}_{\mathbf{p}_2} = \mathbf{v}_{\mathbf{k}_2} + \mathbf{v}_{\mathbf{p}_1} = 0$, so that $\Delta\mathbf{v} = 0$ and $A = 0$. In addition, since $-\mathbf{k}+\mathbf{q}$ also solves $\epsilon_{\mathbf{k}-\mathbf{q}} = \epsilon_\mathbf{k}$, we must have $\mathbf{k}_1 = -\mathbf{k}_2 + \mathbf{q}$ (or v.v.), which implies that $\mathbf{p}_a + \mathbf{q} = \mathbf{k}_a$ $(a = 1, 2)$. Therefore, the remaining process, $(\mathbf{k}_a, \mathbf{p}_a) \leftrightarrows (\mathbf{p}_a, \mathbf{k}_a)$, just swaps the initial and final states, and $\Delta\mathbf{v} = 0$ again. Therefore, $A = 0$ even though Galilean invariance is broken.

The first non-vanishing term in $\delta\sigma_{ii}$ is obtained by considering electrons slightly away from the FS, i.e., by expanding the product of the energy δ functions to second order in ω. The derivatives of the δ functions produce the same solutions for $\mathbf{k}$ and $\mathbf{p}$ as the δ functions themselves. These solutions nullify $\Delta\mathbf{v}$ but not its derivatives generated by integration by parts. As a result, $\delta\sigma_{ii}$ is finite but contains an extra factor of T^2 compared to the power-counting estimate, i.e., the "critical" contribution to the resistivity behaves as $\rho_{ii}(T) - \rho_{ii}(0) = BT^{\alpha+2}$. Because $\alpha + 2 > 2$, the "critical" contribution is subleading to a T^2 contribution from non-critical channels, e.g., a charge channel near a magnetic instability.

A FS of any other type (3D, multiply connected, concave) has more than two self-intersection points when shifted by a small momentum, so that each of the equations $\epsilon_\mathbf{k} = \epsilon_{\mathbf{k}\pm\mathbf{q}}$ has more than two solutions: infinite number for a 3D FS and finite but larger than two number for a multiply connected or concave 2D FS. [cf. Fig. 2*b*]. Thus integrability is broken, and $A \neq 0$. However, in a number of situations (quasi-2D or slightly concave FS), integrability is broken only weakly. Suppose, for example, that $\epsilon_\mathbf{k} = \epsilon_{xy}(k_x, k_y) + \epsilon_z(k_z)$, where $\epsilon_{xy}(k_x, k_y) = 0$ describes a simply connected, convex, and 2D FS, and $\epsilon_z(k_z) = t_\perp[(1 - \cos(k_z c)]$ with $t_\perp \ll \epsilon_F$. The δ functions can be expanded to second order in both ω and ϵ_z, which produces two types of terms: $\rho_{ii}(T) - \rho_{ii}(0) = B_1 T^{\alpha+2} + B_2 t_\perp^2 T^\alpha$. For $T \gg t_\perp$ $(T \ll t_\perp)$, the first (second) term dominates.

High temperatures We now show that the resistivity saturates in the opposite limit of high temperatures, when $\tau_\mathrm{ee} \ll \tau_\mathrm{i}$. The proof is similar to the one for normal phonon-phonon collisions [10]. Frequent normal *ee* collisions establish a quasi-equilibrium distribution with the drift velocity $\mathbf{u}$, fixed by rare *ei* collisions. Accordingly, $f_\mathbf{k} = n'_\mathbf{k}\mathbf{k}\cdot\mathbf{u} + f_\mathbf{k}^{\{\mathrm{i}\}}$, where the first term nullifies I_ee, and the second term is small. To first order in $1/\tau_\mathrm{i}$, the Boltzmann equation reads $e\mathbf{v}_\mathbf{k}\cdot\mathbf{E}n'_\mathbf{k} = -I_\mathrm{ee}[f_\mathbf{k}^{\{\mathrm{i}\}}] - n'_\mathbf{k}\mathbf{k}\cdot\mathbf{u}/\tau_\mathrm{i}$. Applying $\sum_\mathbf{k}\mathbf{k}$, we eliminate I_ee and solve for $\mathbf{u}$ in terms of $\mathbf{E}$. The current is determined primarily by the first term in $f_\mathbf{k}$, which is independent of the *ee* interaction. Hence the critical component of $\rho(T)$ saturates at high T. The low- and high-T limits differ only in how the averaging over the FS is performed: $\sigma_{ij}(\infty) = e^2\nu_F\tau_\mathrm{i}\sum_l\langle v_i k_l\rangle\langle v_j k_l\rangle/\langle k_l^2\rangle$, whereas $\sigma_{ij}(0) = e^2\nu_F\tau_\mathrm{i}\langle v_i v_j\rangle$ (ν_F is the density of states). For a generic case, $\sigma_{ij}(\infty) \lesssim \sigma_{ij}(0)$. This implies that $\rho_{ii}(T)$ cannot be much larger than $\rho_{ii}(0)$, i.e., there is no true

scaling regime. If, however, $\sigma_{ij}(\infty) \ll \sigma_{ij}(0)$, scaling exists in a wide T interval where $\rho_{ii}(0) \ll \rho_{ii}(T) \ll \rho_{ii}(\infty)$.

$s-d$ model near criticality. Finally, we show that a true T^α scaling of the resistivity near a QPT is possible in a dirty two-band metal with substantially different band masses ($s-d$ model [5]). The heavy (d) band is assumed to be near criticality, the light (s) band is not critical on its own, but the interband interaction becomes critical due to renormalization in the $s-d$ channel: $V_{sd}(\mathbf{q},\omega) = V_{sd}^0/(1-\chi_{dd}(\mathbf{q},\omega)V_{dd}^0)$, where $\chi_{dd}(\mathbf{q}\to 0,0)V_{dd}^0 \approx 1$. In the absence of Umklapps, we still need to couple each of the bands to impurities to render ρ finite. The electron-impurity times are such that $\tau_{\mathrm{i}s} \propto m_s^{-1} \gg \tau_{\mathrm{i}d} \propto m_d^{-1}$. Since a two-band FS is already non-integrable, we adopt the simplest model of two parabolic bands in 2D and neglect all other interactions except for $V_{sd}(\mathbf{q},\omega)$. An exact solution of two coupled Boltzmann equations gives

$$\rho(T) = \frac{\pi\hbar}{e^2\epsilon_F} \frac{\frac{1}{\tau_{\mathrm{i}s}\tau_{\mathrm{i}d}} + \frac{1}{\tau_{sd}(T)}\left(\frac{1}{\tau_{\mathrm{i}s}}\frac{m_s}{m_d} + \frac{1}{\tau_{\mathrm{i}d}}\frac{m_d}{m_s}\right)}{\frac{1}{\tau_{\mathrm{i}s}} + \frac{1}{\tau_{\mathrm{i}d}} + \frac{1}{\tau_{sd}(T)}\left(2 + \frac{m_s}{m_d} + \frac{m_d}{m_s}\right)}, \quad (6)$$

where $\tau_{sd}^{-1}(T) = \frac{(m_s m_d)^{1/2}}{2T\epsilon_F^2}\iint d\omega dq q |V_{sd}|^2 \omega^2 N(\omega)\left[N(\omega)+1\right]$. At criticality, $1/\tau_{sd}(T) \propto T^{4/3}$. The low- and high-$T$ limits are controlled by the s and d electrons, respectively: $\rho(0) \approx \pi\hbar/e^2\epsilon_F\tau_{\mathrm{i}s} \ll \rho(\infty) \approx \pi\hbar/e^2\epsilon_F\tau_{\mathrm{i}d}$. The scaling regime corresponds to the interval $T_1 \ll T \ll T_2$, where $\tau_{sd}(T_1) = \tau_{\mathrm{i}s}m_d/m_s$ and $\tau_{sd}(T_2) = \tau_{\mathrm{i}d}m_d/m_s$. In this regime, $\rho(T)$ is independent of disorder and behaves as $\rho(T) = \left(\pi\hbar/e^2\right)(m_d/m_s)\left(1/\epsilon_F\tau_{sd}\right) \propto T^{4/3}$ (similarly, $\rho \propto T^{5/3}$ in 3D). Since quantum-critical metals typically have light and heavy bands, it is quite possible that the $s-d$ physics is responsible for the observed critical scaling of the resistivity. Equation (6) also applies to a ferromagnetic metal with only band in the paramagnetic phase. In this case, "s" and "d" refer to spin-up and spin-down electrons. The $T^{(D+2)/3}$ term in ρ is, however, non-zero only in the symmetry-broken phase.

Limitations of the Boltzmann-equation approach. An obvious deficiency of the semiclassical Boltzmann equation is that it neglects both quantum [13] and viscous [14] corrections to resistivity. Both effects are, in general, relevant but, in wide T intervals, they are smaller than the direct contribution of the *ee* interaction to the resistivity, $\delta\rho_{\mathrm{d}}$ discussed in this paper, if the latter is not suppressed by integrability. The quantum-interaction correction $\delta\rho_{\mathrm{QI}}$ is smaller than $\delta\rho_{\mathrm{d}}$ in the ballistic regime, where $\bar{q}v_F\tau_{\mathrm{i}} \gg 1$: in a 2D FL, $|\delta\rho_{\mathrm{QI}}|/\rho(0) \sim T/\epsilon_F$ [15] while $|\delta\rho_{\mathrm{d}}|/\rho(0) \sim T^2\tau_{\mathrm{i}}/\epsilon_F$, so that $\delta\rho_{\mathrm{d}}/|\delta\rho_{\mathrm{Q}I}| \sim T\tau_{\mathrm{i}} \gg 1$; the same is true also in the non-FL regime [16]. In the diffusive regime (where $\bar{q}v_F\tau_{\mathrm{i}} \ll 1$), $|\delta\rho_{\mathrm{QI}}| \gg \delta\rho_d$. The viscous correction is also smaller than $\delta\rho_{\mathrm{d}}$, if the impurity scattering length is smaller than $v_F\tau_{\mathrm{i}}$.

The Boltzmann approach may also fail because quasiparticles are not well-defined in the non-FL regime. However, if the critical *ee* interaction can be treated within the Eliashberg approximation, the validity of the Boltzmann equation does not rely on the assumption of well-defined quasiparticles – the proof follows the Prange-Kadanoff reasoning for an electron-phonon system [11]. Although recent findings [12] indicate that the Eliashberg approximation for the self-energy is not controlled for $D=2, Z=3$ criticality, it is possible that transport properties, which are less sensitive to infrared singularities, can still be described within this approximation.

As a final remark, we note that some of our results are applicable beyond the model with interaction in Eq. (1). In particular, all results for the FL regime do not depend on a particular form of the interaction, as long as it remains long-ranged. Moreover, integrability exists for any interaction on a small yet anisotropic Fermi surface.

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