

This is the accepted manuscript made available via CHORUS. The article has been published as:

Current-Induced Decoherence in the Multichannel Kondo Problem

Aditi Mitra and Achim Rosch

Phys. Rev. Lett. **106**, 106402 — Published 8 March 2011

DOI: [10.1103/PhysRevLett.106.106402](https://doi.org/10.1103/PhysRevLett.106.106402)

Current induced decoherence in the multichannel Kondo problem

Aditi Mitra¹ and Achim Rosch²¹*Department of Physics, New York University, 4 Washington Place, New York, New York 10003, USA*²*Institute for Theoretical Physics, University of Cologne, 50937 Cologne, Germany*

The properties of a local spin $S = 1/2$ coupled to K independent wires is studied in the presence of bias voltages which drive the system out of thermal equilibrium. For $K \gg 1$, a perturbative renormalization group approach is employed to construct the voltage dependent scaling function for the conductance and the T-matrix. In contrast to the single-channel case, the Kondo resonance is split even by bias voltages small compared to the Kondo temperature T_K , $V \ll T_K$. Besides the applied voltage V , the current induced decoherence rate $\Gamma \ll V$ controls the physical properties of the system. While the presence of V changes the structure of the renormalization group considerably, decoherence turns out to be very effective in prohibiting the flow towards new nonequilibrium fixed points even in variants of the Kondo model where currents are partially suppressed.

PACS numbers: 71.10.-w, 71.27.+a, 73.63.-b, 05.70.Ln

The problem of a local spin S exchange coupled to K independent conduction electron reservoirs, inspite of its apparent simplicity shows a rich variety of phases. For example when $K=2S$, the local spin is completely screened and the system behaves as a Fermi-liquid [1]. In contrast for $K > 2S$, the spin is over-screened, and the system exhibits non-Fermi liquid behavior characterized by a zero temperature entropy $S = \ln g$ where g is a non-integer [2–5]. Recently the over-screened Kondo problem for $K=2$, $S=1/2$ was realized in a controlled experimental set-up [6], opening up the possibility of probing these exotic systems in the far out of equilibrium regime.

An important question in the study of any strongly correlated system is the possibility of realizing new fixed points and scaling behavior by driving the system out of equilibrium. In [7] it was proposed that the single channel Kondo model in the large bias limit should flow to a new fixed point which is characterized by a change in the number of independent screening channels as coherent scattering processes between leads are prohibited by the bias voltage. However subsequent studies [8–12] of the nonequilibrium single-channel Kondo model ruled out such a nonequilibrium regime due to current induced decoherence which qualitatively plays the role of an effective temperature (or an infra-red cutoff), and can be as large as the voltage in the single-channel Kondo model.

In this paper we study the over-screened Kondo problem when a local spin $S=1/2$ is coupled to K independent current carrying channels. In the limit $K \gg 1$, the intermediate coupling non-Fermi liquid fixed point can be studied within renormalized Keldysh perturbation theory in $1/K$. We show that the current induced decoherence in this model is $\mathcal{O}(1/K)$ and thus considerably suppressed. In spite of this we find that decoherence is highly effective in cutting off the flow to any nonequilibrium fixed points. We present results for the voltage dependence of the conductance and the T-matrix, the latter being related to the local density of states that can be probed in a tunneling experiment.

The Hamiltonian is $H = H_0 + V_X$ where

$$H_0 = \sum_{\substack{k\sigma\alpha=L,R \\ m=1\dots K}} (\epsilon_k - \mu_\alpha^m) c_{k\sigma m\alpha}^\dagger c_{k\sigma m\alpha} \quad (1)$$

represents K independent free electron reservoirs labeled by m . Each of the K reservoirs is split into a left (L) and a right (R) part which can be maintained at different chemical potentials $\mu_L^m \neq \mu_R^m$. The local spin is coupled to the spin density of each wire via the exchange interaction,

$$V_X = \frac{1}{2} \vec{S} \cdot \sum_{\substack{\alpha,\beta=L,R;k,k' \\ \sigma\sigma';m=1\dots K}} J_{\alpha\beta} c_{k\sigma m\alpha}^\dagger \vec{\sigma}_{\sigma\sigma'} c_{k'\sigma' m\beta} \quad (2)$$

Here, J_{LR} connects the L and R leads so that a net current can flow within each reservoir when $\mu_L^m \neq \mu_R^m$. However there is no flow of current from channel m to channel $m' \neq m$.

We evaluate two physical quantities, the current through the m -th lead $\hat{I}_m = e \frac{dN_{Lm}}{dt}$ which is given by $\hat{I}_m = -i \frac{e}{\hbar} J_{LR} \frac{\vec{S}}{2} \cdot \sum_{kk'\sigma\sigma'} \left(c_{k\sigma Lm}^\dagger \vec{\sigma}_{\sigma\sigma'} c_{k'\sigma' Rm} - h.c. \right)$, and the T -matrix of the electrons in the m -th lead defined by $G_R^{mkk'\alpha\alpha'} = \delta_{kk'} \delta_{\alpha\alpha'} G_R^{0mk} + G_R^{0mk\alpha} T_R^{m\alpha\alpha'} G_R^{0mk'\alpha'}$, where $G_R^{mkk'\alpha\alpha'}$ is the retarded propagator for the electrons in the m -th lead and the T -matrix is the expectation value of the composite operator $T_{ab}^{m\alpha\alpha'}(t, t') = -iT_C \langle O_a(t) O_b^\dagger(t') \rangle$ where $O_a(t) = \frac{1}{2} \sum_{\alpha_1 k_1 \sigma_1} J_{\alpha\alpha_1} \vec{S}(t) \cdot \vec{\sigma}_{\sigma\sigma_1} c_{m\alpha_1 k_1 \sigma_1 a}(t)$, $a, b = \pm$ denote Keldysh labeling and T_C denotes Keldysh time-ordering [13]. Formally, under assumptions that the interactions V_X was switched on at $t = -\infty$, the expectation value of any operator \hat{O} at time $t=0$ is [13] $\langle \hat{O}(0) \rangle = Tr \left[\left(\tilde{T} e^{i \int_{-\infty}^0 dt' V_X(t')} \right) \hat{O} \left(T e^{-i \int_{-\infty}^0 dt V_X(t)} \right) \rho \right]$ where ρ is the initial density matrix at $t = -\infty$ and $V_X(t) = e^{iH_0 t} V_X e^{-iH_0 t}$. We assume $\rho = \rho_{leads} \otimes \rho_S$ where ρ_S is the density matrix for the free spin, and ρ_{leads} is such that $\langle c_{k\sigma m\alpha}^\dagger c_{k\sigma m\alpha} \rho_{leads} \rangle = f(\epsilon_k - \mu_\alpha^m)$, f being the

Fermi function. We also assume that the leads have a uniform density of states ν and a bandwidth $2D$.

Identical voltage drops across the K-wires: Let us suppose that all the K wires have the same voltage drop $V = |\mu_L - \mu_R|$ applied across them. Then a perturbative treatment to two-loop order, where only the diagrams which are $\mathcal{O}(K)$ at two loop are kept gives the following result for the conductance $G_m = \partial I_m / \partial V$,

$$G_m = \frac{3\pi}{4} \frac{e^2}{h} (\nu J_{LR})^2 \left[1 + 2\nu (J_{LL} + J_{RR}) \ln \frac{D}{V} - K \ln \frac{D}{V} \sum_{\alpha\beta=L,R} \theta(D - |\mu_\alpha - \mu_\beta|) (\nu J_{\alpha\beta})^2 \right] \quad (3)$$

In addition the imaginary part of the T -matrix of the m -th wire is found to be,

$$\begin{aligned} -\pi\nu \text{Im} \left[T_R^{m\alpha\alpha'\sigma}(\Omega) \right] &= \frac{3\pi^2}{16} \nu^2 \sum_{\alpha_1} [J_{\alpha\alpha_1} J_{\alpha_1\alpha'}] \\ &+ 2\nu \sum_{\alpha_2} J_{\alpha\alpha_1} J_{\alpha_1\alpha_2} J_{\alpha_2\alpha'} \ln \left(\frac{D}{|\Omega - \mu_{\alpha_1}|} \right) \\ &- K J_{\alpha\alpha_1} J_{\alpha_1\alpha'} \ln \left(\frac{D}{|\Omega - \mu_{\alpha_1}|} \right) \\ &\times \sum_{\gamma\delta} \nu^2 J_{\gamma\delta}^2 \theta(D - |\mu_\gamma - \mu_\delta|) \end{aligned} \quad (4)$$

According to Eq. (3) all logarithmic singularities in the conductance are cutoff by the voltage. This reflects the fact that in a model where the cutoff D is much smaller than the voltage difference V no current will flow by energy conservation. In contrast, the calculation of the T -matrix shows that even in this regime resonant spin-flip processes lead to logarithmic renormalizations. For the renormalization group (RG) analysis it will be important, that some J_{LR} processes do not contribute for $D < V$ as described by the θ function terms in Eq. (4).

We will formulate the RG equations in terms of dimensionless couplings, $g_{ab} = \nu J_{ab}$ assuming symmetric couplings, $g_{LL} = g_{RR} = g_d$ but we will allow for $g_{LR} \neq g_d$. While for a simple Anderson model one always obtains $g_{LR} = g_d$, this is not valid for more complex models.

When deriving two-loop equations in full generality for a non-equilibrium situation using, e.g., functional renormalization group [14], flow equations [10] or real-time renormalization group approaches [12], it is necessary to take into account the full energy dependence of interaction vertices. Even at the one-loop level it is useful [15] to keep track of how the coupling constants g depend on the energy of the incoming electron. To leading order in $1/K$, however, and for the quantities considered in this paper, we believe that it is sufficient to use a simpler version guided by results from perturbation theory, concentrating on a few on-shell coupling constants. Already

in perturbation theory to order J^3 the structure of logarithmic corrections depends on which physical quantity is considered. The simplest case is the conductance where all logarithmic corrections up to the order considered are cutoff by the voltage. Thus for the conductance it is sufficient to use the well-known equilibrium RG equations [1]

$$\begin{aligned} \frac{dg_d}{d \ln D} &= - \left[(g_d^2 + g_{LR}^2) - \frac{K}{2} g_d (2g_d^2 + 2g_{LR}^2) \right] \\ \frac{dg_{LR}}{d \ln D} &= - \left[2g_d g_{LR} - \frac{K}{2} g_{LR} (2g_d^2 + 2g_{LR}^2) \right] \end{aligned} \quad (5)$$

supplemented by the condition that the RG flow is cutoff at $D=V$.

However, when one requires instead that the T -matrix $T^{\alpha\alpha}(\mu_\alpha)$, evaluated at the Fermi energy of the lead, remains invariant under RG, one obtains the RG equations

$$\begin{aligned} \frac{dg_d}{d \ln D} &= - \left[(g_d^2 + g_{LR}^2 \Theta_V) - \frac{K}{2} g_d (2g_d^2 + 2g_{LR}^2 \Theta_V) \right] \\ \frac{dg_{LR}}{d \ln D} &= - \left[g_d g_{LR} (1 + \Theta_V) - \frac{K}{2} g_{LR} (2g_d^2 + 2g_{LR}^2 \Theta_V) \right] \end{aligned} \quad (6)$$

Here the factors $\Theta_V = \Theta(D - V)$ take into account that part of the logarithms and therefore the RG-flow is cutoff by the voltage when the cutoff becomes smaller than the voltage. Therefore the equilibrium RG flow is modified at the scale V in a way which can be simply understood as a suppression of all resonant scattering processes from one to the other chemical potential by g_{LR} .

Note that Eq. (6) is not valid in the limit $D \rightarrow 0$ because the RG flow is ultimately cutoff by the decoherence processes as had been pointed out previously [8–11]. A careful analysis of the decoherence rates (including 3 loop diagrams) has been performed by Schoeller and Reininghaus in [12] by investigating the time evolution of the density matrix of the impurity. Their analysis shows that within the precision of a two-loop calculation (i.e. within our model to leading order in $1/K$), the decoherence rate is given by the simple formula used previously [8, 9] for one-loop calculations. Up to prefactors of order 1, irrelevant for our discussion, the RG flow is ultimately cutoff at the scale Γ ,

$$\Gamma = \pi K g_{LR}(V)^2 V \quad (7)$$

where $g_{LR}(V)$ is the running coupling constant at the scale $D=V$. The factor K arises as each of the K channels contributes to the dephasing. Γ is the Korringa spin-relaxation rate [8–11] and is proportional to the current.

Let us make the assumption $g_d = g_{LR}$ (we will relax this condition later). For the regime $D > V$ it is convenient to define $g = 2g_d$ which obeys the RG equation

$$\frac{dg}{d \ln D} = - \left[g^2 - \frac{K}{2} g^3 \right] = \beta(g) \quad (8)$$

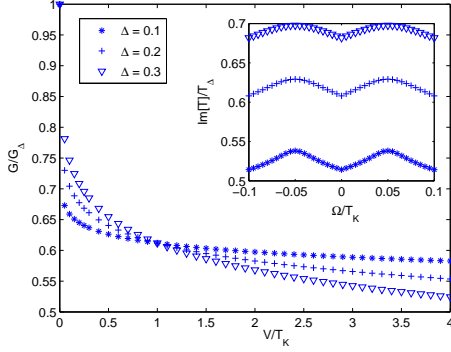


FIG. 1: (Color online) Main panel: The conductance in the scaling limit and for several different $\Delta=2/K$. $G_\Delta = \left(\frac{2e^2}{h}\right) \frac{3\pi^2}{16} \Delta^2$. Inset: $\text{Im}[T(\Omega)]$ for $V/T_K=0.1$ and the same Δ as in the main panel. $T_\Delta = 3\pi^2 \Delta^2 / 32$.

The above equation has the well known fixed point at $g^*=2/K$ [2], whereas the scaling dimension of the leading irrelevant operator is $\Delta = \beta'(g^*) = \frac{2}{K}$ valid for $K \gg 1$ (the exact result [4] is $\Delta = 2/(K+2)$). Integrating Eq. (8) upto an energy-scale D , and defining the Kondo temperature as $T_K = D_0 \left(\frac{g_0}{g^*}\right)^{K/2} e^{-1/g_0}$, (D_0, g_0 being the initial bandwidth and coupling constant respectively), one gets [5]

$$|g(D) - g^*| = |g_0 - g^*| \left[\frac{g(D)}{g^*} \right]^{K\Delta/2} \left(\frac{D}{T_K} \right)^\Delta e^{-\Delta/g(D)} \quad (9)$$

Eq. (9) is valid for arbitrary D/T_K . For $D \gg T_K$, i.e. for $g \ll g^*$, we obtain $g(D) \simeq \frac{1}{\ln D/T_K} + \frac{K}{2} \frac{\ln(2/K \ln D/T_K)}{(\ln D/T_K)^2} + \mathcal{O}(\frac{1}{\ln D/T_K})^3$. For $D \ll T_K$, setting $g(D)=g^*$ on the r.h.s of Eq. (9) one gets

$$g(D) = g^* \left(1 - \xi \left(\frac{D}{T_K} \right)^\Delta \right) \quad (10)$$

where $\xi = (1 - g_0/g^*) e^{-\Delta/g^*}$. In the scaling limit $g_0 \rightarrow 0$, and for large K , ξ takes the universal value $\xi = 1/e$.

Integrating upto the energy scale $D=V$, the logarithms in the conductance are resummed giving $G_m \approx \frac{3\pi}{4} \frac{e^2}{h} g_{LR}^2(V)$. In particular the result for $V \ll T_K$ is

$$G_m \approx \frac{3\pi}{4} \frac{e^2}{h} \frac{g^*{}^2}{4} \left[1 - 2\xi \left(\frac{V}{T_K} \right)^\Delta \right] \quad (11)$$

where we have dropped higher order terms $\sim \left(\frac{V}{T_K}\right)^{2\Delta}$. Note that the conductance near the fixed point is a quantity of $\mathcal{O}(1/K^2)$. For arbitrary V/T_K and in the scaling limit, the conductance is given by the universal function $G/G_\Delta = 1/[1 + W(e^{-1}(V/T_K)^\Delta)]^2$ where

$G_\Delta = \left(\frac{2e^2}{h}\right) \frac{3\pi^2}{16} \Delta^2$ and $W(x)$ is the Lambert W function. The conductance for several different Δ is plotted in Fig. 1. At the fixed point $g_{LR}^*=1/K$. Thus the decoherence rate (defined in Eq. (7)) for $V \ll T_K$ is,

$$\Gamma = \frac{\pi V}{K} \quad (12)$$

and thus apparently small for $K \gg 1$. However, as we show below, this decoherence rate plays an important role in cutting off the logarithms in the T -matrix.

For $V=T=0$ the T matrix has a powerlaw cusp, $\text{Im}T(\omega) \approx c_1 - c_2|\omega|^\Delta$ [4, 5]. Eqn. (4) suggests that this peak will split by V and we also expect it to be broadened by Γ . As $\Gamma \ll V$ this splitting will be observable even for $V \ll T_K$. In the following we will use RG to calculate this split Kondo resonance for $V \ll T_K$ (results for $V \gg T_K$ follow from the literature of the single-channel Kondo model [15]). We assume the symmetrical application of voltages $\mu_L = -\mu_R = V/2 > 0$. For $|\Omega - V/2| \ll V$, and for $V \gg D > |\Omega - V/2|, \Gamma$, Eq. (6) gives,

$$g_d(D) \approx \frac{1}{2} \left[g^* - (g^* - g(V)) \left(\frac{D}{V} \right)^{\Delta/2} \right] \quad (13)$$

$$\approx \frac{g^*}{2} \left[1 - \xi \left(\frac{V}{T_K} \right)^\Delta \left(\frac{D}{V} \right)^{\Delta/2} \right] \quad (14)$$

The resummed T -matrix is then given by,

$$-\pi\nu \text{Im}[T_R^{\alpha\alpha'\sigma}(\Omega)] \approx \frac{3\pi^2}{16} [2g_d^2(\max(\Omega - V/2, \Gamma))] \quad (15)$$

Expanding Eq. 15 in powers of the interaction g , we have checked that this is consistent with Eqn. (4).

For $V < \Omega < T_K$ the T -matrix is described by a cusp with power Δ . Apparently, for $\Gamma < |\Omega - V/2| \ll V$ this exponent changes to $\Delta/2$ and one may want to interpret this as a new nonequilibrium scaling regime governed by a $2K$ channel, instead of K channel behavior. However, decoherence is so strong that this regime is ‘unmeasurable’: the extra factor in Eq. (14) $1 > (D/V)^{\Delta/2} \gtrsim (\Gamma/V)^{\Delta/2} \approx e^{-(\ln K)/K}$ always remains close to 1. Moreover, the peak at $\Omega=V/2$ is not much higher than the minimum at $\Omega=0$, $\text{Im}[T(V/2) - T(0)]/\text{Im}[T(0)] \sim \left(\frac{V}{T_K}\right)^\Delta \frac{\ln K}{K} \ll 1$. These results are shown schematically in Fig. 1 where we have used $\max(a, b) = \sqrt{a^2 + b^2}$.

Above, we have shown that a $2K$ channel fixed point cannot be stabilized by a finite voltage in an extended regime. However, if one considers a model where initially $g_{LR} \ll g_d$ such a regime becomes accessible. Remarkably, one can even calculate the nonequilibrium conductance exactly in this regime for $K=1$ [16]. It has also been suggested by one of us [9] that a finite voltage stabilizes the $2K$ channel regime efficiently for $K=1$ for sufficiently

small g_{LR} . It is therefore interesting to study this case also for $K \gg 1$.

For $g_{LR}=0$ the system is described by a $2K$ channel fixed point with a Kondo temperature $T_K = D_0(Kg_{d0})^K e^{-1/g_{d0}}$, and voltage has no effect as no currents can flow. However g_{LR} is a relevant variable for $V=0$. While at the scale T_K it is small, $g_{LR}(T_K) \approx \frac{g_{LR0}}{g_{d0}^2 K^2}$, it grows below T_K with scaling dimension $-1/K$ (compared to the exact result [4] $-1/(K+1)$), thus inducing a flow back to the K channel fixed point which is reached at the scale $T_K^* \approx T_K \left(\frac{g_{LR0}}{K g_{d0}^2} \right)^K$.

New physics can arise in the regime $T_K^* < V < T_K$ governed by the $2K$ -channel fixed point. For example, the conductance in this regime is given by $G_m \sim \frac{e^2}{h} g_{LR}(V)^2 \sim \frac{e^2}{h K^2} \left(\frac{T_K^*}{V} \right)^{2/K}$ which crosses smoothly over to Eqn. (11) for $V \sim T_K^*$. Similarly, the decoherence rate in this regime is given by $\Gamma \sim K V g_{LR}(V)^2 \sim \frac{V}{K} \left(\frac{T_K^*}{V} \right)^{2/K}$. To decide whether the range of validity of the $2K$ channel regime is enhanced compared to the equilibrium case we compare Γ with T_K^* and V for $T_K > V > T_K^*$. We find

$$\frac{\Gamma}{T_K^*} \sim \frac{1}{K} \left(\frac{V}{T_K^*} \right)^{1-2/K}, \quad \frac{\Gamma}{V} \sim \frac{1}{K} \left(\frac{V}{T_K^*} \right)^{-2/K} \quad (16)$$

While the ratio Γ/V is reduced in this regime, the decoherence rate is always *larger* than T_K^* for $V/T_K^* \gtrsim K$ and approaches $\Gamma/V \approx 1/K$ as before for smaller voltages (up to small corrections of $O(\ln K/K)$). This implies that voltage bias does *not* enhance the regime where $2K$ channel physics is observable in contrast to the suggestion of Ref. [9].

Leads with different voltage differences: We now discuss the case where (for $g_{LR}=g_d$) a fraction p and hence Kp leads are at voltage $V \ll T_K$, while $K(1-p)$ are at $V=0$. (In [6] one has $p=1/2$ and $K=2$). The question arises whether the decoherence Γ again prohibits the flow to new fixed points for finite V . Analyzing as above the logarithmic corrections to the T -matrix in the regime $D < V$ we obtain

$$\begin{aligned} \frac{dg_0}{d \ln D} &= - \left[g_0^2 - \frac{K}{2} g_0 \{ (1-p)g_0^2 + p(2g_{dV}^2) \} \right] \\ \frac{dg_{dV}}{d \ln D} &= - \left[g_{dV}^2 - \frac{K}{2} g_{dV} \{ (1-p)g_0^2 + p(2g_{dV}^2) \} \right] \end{aligned} \quad (17)$$

where g_{dV} and $g_0 = 2g_d$ are the coupling constants for the leads with and without an applied bias voltage respectively. The initial values $g_{dV}(V)=g_0(V)/2=g(V)/2$ are obtained from Eq. (9) or (10). For our initial condition $g_{dV} < g_0$, the solution apparently flows towards the fixed point $g_{dV}^*=0, g_0^*=2/(K(1-p))$ where only the channels without bias voltage contribute to screening and powerlaws are governed by the exponent $\Delta_p=2/(K(1-p))$.

This is however misleading because the decoherence stops the flow towards this fixed point for *all* p . To see this note that close to the K channel fixed point $g_{dV}=g_0/2=1/K$ the RG equations are $\frac{dg_0}{d \ln D} \approx -2p/K^2$ and $\frac{dg_{dV}}{d \ln D} \approx (1-p)/K^2$. Therefore all changes arising from the flow from $D=V$ to $D=\Gamma \simeq \pi p V/K$ remain for arbitrary p smaller than $\frac{2}{K^2} \ln(V/\Gamma) < \frac{2}{K^2} \ln(K^2) \ll 1/K$. Therefore the new fixed points can never be approached.

In summary we have studied the nonequilibrium over-screened Kondo problem in the perturbatively solvable limit of a large number of leads $K \gg 1$. For this model, current induced decoherence is very small ($\Gamma/V = O(1/K) \ll 1$). However, in this limit the renormalization group flows also become very slow. Our calculations show that the net result is that the flow to any new voltage induced fixed points is stopped very effectively by the decoherence. We have made predictions for the splitting of the T -matrix for $V \ll T_K$ which can be observed experimentally. An important open question is to study this nonequilibrium problem in the presence of an external magnetic field.

Acknowledgements: We thank E. Sela and D. Schuricht for useful comments. This work was supported by NSF-DMR (Award No. 0705584 and 1004589) and by the DFG within SFB 608 and FOR 960.

-
- [1] A. C. Hewson, *The Kondo Problem to Heavy Fermions* (Cambridge University Press, Cambridge, England, 1993).
 - [2] P. Nozières and A. Blandin, *J. Phys. (France)* **41**, 193 (1980).
 - [3] N. Andrei and C. Destri, *Phys. Rev. Lett* **52**, 364 (1984); A. M. Tsvelick and P. B. Wiegmann, *Z. Phys. B* **54**, 201 (1984).
 - [4] I. Affleck and A. W. W. Ludwig, *Phys. Rev. B* **48**, 7297 (1993).
 - [5] J. Gan, *J. Phys.: Condensed Matter* **6**, 4547 (1994).
 - [6] R. M. Potok, I. G. Rau, H. Shtrikman, Y. Oreg, and D. Goldhaber-Gordon, *Nature (London)* **446**, 167 (2007).
 - [7] X. G. Wen, cond-mat/9812431, P. Coleman, C. Hooley and O. Parcollet, *Phys. Rev. Lett.* **86**, 4088 (2001).
 - [8] A. Kaminski, Yu. V. Nazarov, and L. I. Glazman, *Phys. Rev. B* **62**, 8154 (2000).
 - [9] A. Rosch, J. Kroha and P. Wölfle, *Phys. Rev. Lett.* **87**, 156802 (2001).
 - [10] S. Kehrein, *Phys. Rev. Lett.* **95**, 056602 (2005).
 - [11] Aditi Mitra and A. J. Millis, *Phys. Rev. B* **76**, 085342 (2007).
 - [12] H. Schoeller and F. Reininghaus, *Phys. Rev. B* **80**, 045117 (2009).
 - [13] A. Kamenev, *Nanophysics: Coherence and Transport*, Les Houches 2004 session No. LXXX1 (Elsevier, Amsterdam, 2005).
 - [14] S. G. Jakobs, V. Meden and H. Schoeller, *Phys. Rev. Lett.* **99**, 150603 (2007).
 - [15] J. Paaske, A. Rosch, J. Kroha, and P. Wölfle, *Phys. Rev. B* **70**, 155301 (2004).

- [16] E. Sela and I. Affleck, *Phys. Rev. Lett.* **102**, 047201 (2009).