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Evolution of the Normal State of a Strongly Interacting Fermi Gas from a Pseudogap Phase to a Molecular Bose Gas

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Abstract

Wave-vector resolved radio frequency (rf) spectroscopy data for an ultracold trapped Fermi gas are reported for several couplings at T_c, and extensively analyzed in terms of a pairing-fluctuation theory. We map the evolution of a strongly interacting Fermi gas from the pseudogap phase into a fully gapped molecular Bose gas as a function of the interaction strength, which is marked by a rapid disappearance of a remnant Fermi surface in the single-particle dispersion. We also show that our theory of a pseudogap phase is consistent with a recent experimental observation as well as with Quantum Monte Carlo data of thermodynamic quantities of a unitary Fermi gas above T_c.

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While the existence of a high-temperature superfluid phase in the BCS-BEC crossover of a strongly interacting Fermi gas is experimentally well established, important questions remain as to the nature of the gas above the superfluid transition temperature T_c. In particular, the question of whether or not a pseudogap state exists and how to identify it is of importance [1]. This is a question that may have relevance to the controversy surrounding the pseudogap state in the high-T_c cuprates. While the origin of this state in the cuprates is a hotly debated topic, with atomic Fermi gases we can answer the simpler question of whether or not strong interactions and pairing fluctuations alone can lead to a pseudogap phase. This, in turn, tells us whether using such an approach to explain the pseudogap phase in the cuprates is a viable option or if other mechanisms are required.

As a function of increasingly strong attractive interactions, a Fermi gas exhibits a smooth crossover (called the BCS-BEC crossover), from a weakly attractive Fermi gas with a superfluid transition explained by conventional BCS theory, to a Fermi gas where interparticle attractions are so strong that the fermion pairs form molecules and the gas is well described as a molecular Bose gas with a Bose-Einstein condensation transition. In the BCS limit the phenomena of Cooper pairing and superfluidity occur simultaneously at the phase transition, while in the BEC limit pairing and Bose condensation are decoupled with pairing of fermionic atoms into molecules occurring well above the condensation temperature. The *pseudogap phase* refers to the normal state of a strongly interacting Fermi gas in the center of this crossover, where it is proposed that pairs exist above the superfluid transition in analogy with the normal state of the gas in the BEC limit. However, unlike the pairs in the BEC limit, the pairs in the pseudogap state have many-body character with *the underlying Fermi statistics playing a crucial role*, in analogy with the Cooper pairs of the BCS limit. A key prediction of theories of the pseudogap phase is that there should be a smooth evolution from the many-body pairs in the center of the crossover to the molecular pairs in the BEC limit [1, 2] and accordingly, in order to verify the existence of a pseudogap phase, it is critical to examine the evolution of the spectral function from the center of the crossover to the molecular limit [3].

Based on two recent experiments, conflicting conclusions have been reached about the existence of a pseudogap state in the strongly interacting Fermi gas. On the one hand, thermodynamic measurements [4] have been interpreted as well described by Fermi liquid theory, without the need for a pseudogap state. On the other hand, momentum-resolved rf spectroscopy [5], which measures the single-particle spectral function, has been interpreted as evidence for a pseudogap state above T_c.

In this work, we present a theoretical investigation of the pseudogap regime based on the t-matrix pairing-fluctuation approach of Ref.[3], addressing both the single-particle spectral function *and* the thermodynamics of the gas, as a function of interaction strength in the BCS-BEC crossover. We find that, in the pseudogap regime, the single-particle dispersion back-bends at a wave vector k_L near the Fermi wave vector k_F, indicating the existence of *a remnant Fermi surface* in this strongly interacting gas and the importance of Fermi statistics to the pairing. As interactions are increased towards the BEC limit, k_L disappears rapidly when entering the regime of molecular pairing. This picture is supported by a comparison of our theoretical results, where we include the effects of the trapping potential, with new experimental data using momentum resolved rf spectroscopy to probe the gas for different interaction strengths. In addition, we show that the theory also reproduces the observed linear behavior in the thermodynamics.

By the experimental technique introduced in Ref.[6], excitations of the trapped gas produced by an rf pulse are analyzed by time-of-flight imaging to determine the wave vector of the excited atoms once the trap has been switched off. The new data are presented with an improved signal-to-noise ratio at the critical temperature T_c, which is accurately determined as the temperature where the condensate fraction disappears. We concentrate in the coupling range $0.0 \lesssim (k_F a_F)^{-1} \lesssim 1.0$, because the evolution of interest from the pseudogap state to the molecular Bose gas occurs on the positive side of the resonance. Here, a_F is the scattering length associated with the Fano-Feshbach resonance and k_F is given by $\hbar^2 k_F^2/(2m) = E_F = \hbar\omega_0(3N)^{1/3}$, where $\hbar$ is Planck constant, m the atom mass, N the total number of atoms, and ω_0 the average trap frequency (we set $\hbar = 1$).

Ultracold Fermi gases are peculiar systems, in that their interparticle coupling can be increased to the point when a description in terms of a gas of molecular bosons holds, for which a real gap exists in the single-particle spectra. This molecular (two-body) physics is of no interest in the context of the pseudogap, in a similar fashion of molecular binding in vacuum being distinct from Cooper pairing at finite density in the presence of a Fermi surface (cf. footnote 18 of Ref.[7]). The question then arises about what fermionic feature distinguishes the pseudogap from the molecular phase. We shall find that the back-bending of the dispersion curves obtained from the single-particle spectral function $A(k, \omega)$ (with

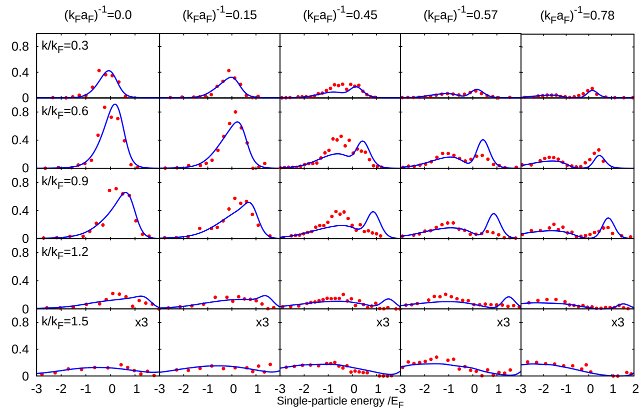

FIG. 1. Experimental (circles) and theoretical (full lines) EDC for the trap at T_c, for several couplings and wave vectors.

wave vector k and frequency ω) occurs at a wave vector k_L which remains close to k_F over a wide coupling range even when approaching the molecular limit. We refer to this special wave vector as k_L because it is reminiscent of the Luttinger theorem [8], according to which in a normal Fermi liquid the radius k_F of the Fermi sphere is unaffected by the interaction.

Figure 1 compares the experimental and theoretical energy distribution curves (EDC) at T_c for five different couplings in the window of interest (see Ref.[9] for details). We emphasize that the experimental data bear on *an absolute normalization*, in that only the integral over wave vector and energy of the EDC curves (and not the separate spectra) has been normalized to unity [9]. For this reason, there is no independent normalization in the various panels at different k. This renders quite stringent the comparison with the corresponding theoretical calculations, which in turn contain no adjustable parameters. Good agreement results from this comparison. In particular, the theoretical calculations well reproduce the asymmetry of the experimental curves between positive and negative energies, in addition to the peak positions, widths and heights (note how the latter change by about one order of magnitude from small to large k). Note further the excellent agreement between the theoretical and experimental negative energy tails, and the gradual flattening of the EDC curves for increasing coupling due to the increase of intrapair correlations.

In Fig. 2 the dispersion and full width at half maximum of the peak at lower energies are reported over a dense set of k values for the same couplings of Fig. 1, and compared with our theoretical calculations. Note that a characteristic *back-bending* is revealed from

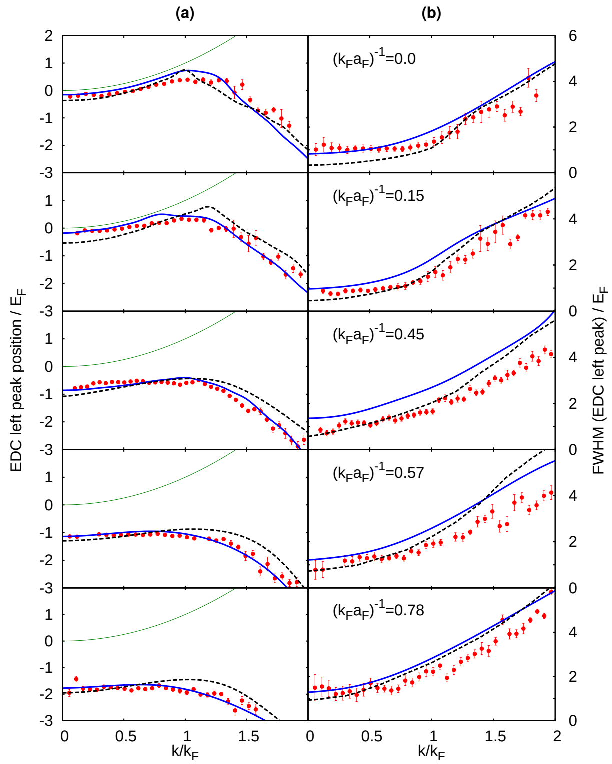

FIG. 2. (a) Dispersions and (b) widths of the low-energy EDC peak. Experimental data (circles) and theoretical calculations for the trap (full lines) are shown for the same couplings of Fig. 1, and compared with the contribution from the radial shell with the largest particle number (dashed lines). In the left panels the free-particle dispersion $k^2/(2m)$ is also reported for comparison (thin full lines).

these dispersions [10]. This kind of back-bending is typical of a BCS-like dispersion, and is associated with the presence of a pseudogap in a strongly interacting Fermi system [3, 5, 12–14]. In addition, the large values of the widths (which are at least of the order of E_F) and their asymmetric behavior between $k < k_F$ and $k > k_F$ are associated with strong deviations from the expected behavior of a normal Fermi liquid (which requires instead the quasi-particle widths to be vanishingly small at k_F [15]), and confirm the fact that single-particle states in this region constitute poor quasi-particles. Large values of the widths are not surprising in the context of the pseudogap physics that results from pairing fluctuations [3]. Large widths were also obtained by the self-consistent t-matrix approach of Ref. [16], which however masked the occurrence of a pseudogap near k_F.

It is relevant to discuss how trap averaging affects the above results, because different

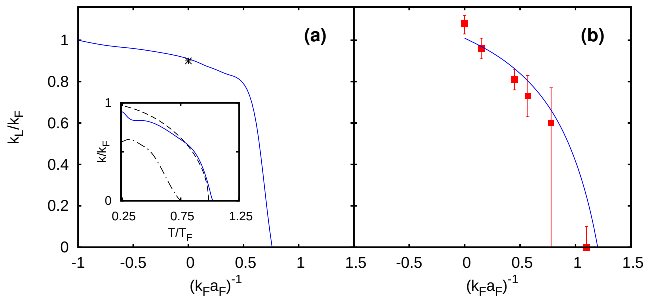

FIG. 3. (a) Coupling dependence of the Luttinger wave vector k_L for a homogeneous system at T_c, according to the theory of Ref.[3] (full line) [the value at unitarity from the QMC calculation of Ref.[17] is also reported (star)]. The inset shows the temperature dependence of k_L at unitarity (full line), and compares it with those obtained from the temperature dependence of the chemical potential of the non-interacting (dashed line) and interacting (dashed-dotted line) systems. (b) Theoretical (full line) and experimental (squares) coupling dependence of k_L for the trap system at T_c.

radial shells in the trap correspond to different locations in the coupling-vs-temperature phase diagram of the homogeneous system. A reasonable hypothesis is that the radial shell with the largest particle number (whose radius $r_{\max}$ is estimated to be $(0.5-0.6)R_F$ where $R_F = [2E_F/(m\omega_0^2)]^{1/2}$ is the Thomas-Fermi radius) contributes most to the total signal. The dispersions and widths contributed by this shell at $r_{\max}$ are represented by dashed lines in Fig. 2, which show good agreement with the complete calculation. This indicates that both the back-bending of the dispersion relations and the associated large widths are not an artifact of trap averaging.

Despite these deviations from the behavior of a normal Fermi liquid, in the experimental data and theoretical calculations there yet appears a feature which is preserved from the physics of a Fermi liquid. That is the Luttinger wave vector k_L where the back-bending occurs, which is plotted at T_c vs $(k_F a_F)^{-1}$ in Fig. 3, for a homogeneous [panel (a)] and trapped [panel (b)] system.

Figure 3(a) shows that for a homogeneous system k_L drops rapidly to zero when $(k_F a_F)^{-1} \simeq 0.75$, where the pseudogap in $A(k,\omega)$ turns into a real gap and the molecular limit is reached. Accordingly, we identify the boundary between the pseudogap and molecular phases where this drop occurs. Along this evolution into the molecular regime, the disappearance of the underlying Fermi surface about occurs when the molecular size becomes smaller than the interparticle spacing. The existence of a remnant Fermi surface with an enclosed volume consistent with Luttinger theorem was already pointed out by ARPES experiments for the pseudogap phase of high-T_c superconductors [18], but its importance for delimiting the pseudogap region was not appreciated in that context [19] because the interparticle interaction could not be controlled. The inset of Fig. 3(a) shows the temperature dependence of k_L calculated for a homogeneous system at unitarity (full line). At high temperatures when the pseudogap closes up, we have identified k_L as the value where the dispersion of the peak at lower energy in $A(k, \omega)$ crosses the chemical potential [9]. This does not contradict our argument that at low temperatures the presence of a pseudogap requires an underlying Fermi surface, since at high temperatures the underlying Fermi surface of a Fermi liquid is not related to a pseudogap. The plot also shows the temperature dependence of $k_{\mu^0} = \sqrt{2m\mu^0(T)}$ (dashed line) and $k_\mu = \sqrt{2m\mu(T)}$ (dashed-dotted line), where $\mu^0(T)$ and $\mu(T)$ are the chemical potentials of the non-interacting and interacting Fermi systems, in the order, at the temperature T. Note that k_L about coincides with k_{μ^0}, while k_μ is not related with k_L.

Figure 3(b) shows the coupling dependence of k_L at T_c for the trapped system, for which the theoretical predictions can be directly compared with the experimental data (the latter are obtained by a BCS-like fit to the dispersions of Fig. 2(a), as explained in Ref.[9]). The good comparison that results between theory and experiment confirms our identification of k_L as the relevant quantity for identifying the remnant Fermi characteristics of the system in the pseudogap phase.

However, the occurrence of a pseudogap for a unitary Fermi gas above T_c has recently been questioned, following a result reported in Ref.[4] where a linear dependence of the equation of state as a function of $[k_B T/\mu(T)]^2$ (k_B being Boltzmann constant) was fitted by the Fermi-liquid equation of state and then interpreted [20] as evidence that the Fermi-liquid theory with no pseudogap can describe a unitary Fermi gas above T_c. To compare with the data of Ref.[4] and resolve this controversy, we have used the theoretical approach of Ref.[3], which contains a robust pseudogap associated with a non-Fermi-liquid behavior consistent with the data obtained by momentum resolved rf spectroscopy, also to calculate the thermodynamic properties of a homogeneous system above T_c. Figure 4(a) reports the

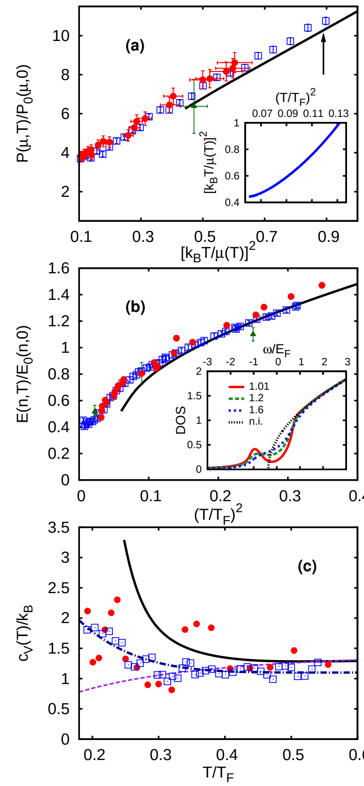

FIG. 4. Thermodynamics of a homogeneous Fermi gas at unitarity. (a) Pressure vs $[k_BT/\mu(T)]^2$: Experimental data from Ref.[4] (circles) are compared with QMC data from Refs.[21] (squares) and [22] (triangles), and with the t-matrix (full line). In the inset, the variable $[k_BT/\mu(T)]^2$ is transformed to $(T/T_F)^2$ according to the t-matrix. (b) Energy vs $(T/T_F)^2$ at fixed density: Experimental data from Ref.[23] (circles) are compared with QMC data from Refs.[21] (squares) and [22] (triangles), and with the t-matrix (full line). The inset shows the density of states per spin (in units of $mk_F/(2\pi)^2$) for several temperatures in units of T_c according to the t-matrix, and contrasts it with the non-interacting (n.i.) result. (c) Specific heat per particle vs T/T_F obtained from the t-matrix (full line), the experimental data of Ref.[23] (circles), and the QMC data of Ref.[21] (squares) - the dotted line is a guide to the eye for the QMC data. The behavior of the non-interacting Fermi gas (broken line) is reported for reference [9].

pressure in the grand-canonical ensemble vs $[k_BT/\mu(T)]^2$ as in Ref.[4], and shows that the *linear behavior* seen in the experimental data and QMC calculations also results from our t-matrix approach, both above and below the temperature at which the pseudogap appears (indicated by the vertical arrow). The inset of Fig. 4(a) shows that this linear behavior can be ascribed to the pronounced temperature dependence of the chemical potential, because a non-linear behavior results when transforming $[k_BT/\mu(T)]^2$ to $(T/T_F)^2$ over the relevant range. The same change of variables can be performed in the experimental [23] and QMC [21, 22] data, to obtain the total energy in the canonical ensemble as a function of $(T/T_F)^2$ reported in Fig. 4(b). This shows that in the new variable the linear behavior is lost.

Yet, it remains difficult to appreciate directly from this thermodynamic quantity the presence of a pseudogap in a unitary Fermi gas above T_c even by the t-matrix calculation, despite the fact that a pseudogap is clearly present in the single-particle density of states obtained by the t-matrix as shown in the inset of Fig. 4(b) where deviations from the non-interacting behavior $\sqrt{(\omega + \mu(T_c))/E_F}$ are evident. Accordingly, by suitable numerical differentiation of the energy data we have obtained in Fig. 4(c) the *specific heat* vs T/T_F. A sharp upturn of this thermodynamic quantity, beginning at a temperature T^* well above T_c where the pseudogap sets in, results clearly from the t-matrix calculation, and it is also visible from the QMC data at the corresponding value of T_c.

The experimental data in Fig. 4(c) appear too scattered to draw definite conclusions about the presence of the upturn and thus of a pseudogap above T_c. It should be mentioned, however, that a similar upturn of the specific heat at a temperature T^* above T_c was measured in underdoped high-T_c cuprates and interpreted as revealing the onset of the pseudogap regime, whereby a "residual superconductivity" remains far above T_c [24].

In conclusion, we have provided clear experimental and theoretical evidence for non-Fermi-liquid behavior in the normal phase of a strongly interacting Fermi gas, which we have qualified in terms of a pseudogap picture. We have further shown that this picture, that appears evident in the single-particle dynamics, is also consistent with the thermodynamic behavior of the system.

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