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Quark fragmentation in the θ-vacuum

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The vacuum of Quantum Chromodynamics is a superposition of degenerate states with different topological numbers that are connected by tunneling (the θ-vacuum). The tunneling events are due to topologically non-trivial configurations of gauge fields (e.g. the instantons) that induce local $\mathcal{P}$-odd domains in Minkowski space-time. We study the quark fragmentation in this topologically non-trivial QCD background. We find that even though QCD globally conserves $\mathcal{P}$ and $\mathcal{CP}$ symmetries, two new kinds of $\mathcal{P}$-odd fragmentation functions emerge. They generate interesting dihadron correlations: one is the azimuthal angle correlation $\sim \cos(\phi_1 + \phi_2)$ usually referred to as the Collins effect, and the other is the $\mathcal{P}$-odd correlation $\sim \sin(\phi_1 + \phi_2)$ that vanishes in the cross section summed over many events, but survives on the event-by-event basis. Using the chiral quark model we estimate the magnitude of these new fragmentation functions. We study their experimental manifestations in dihadron production in e^+e^- collisions, and comment on the applicability of our approach in deep-inelastic scattering, proton-proton and heavy ion collisions.

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1. Introduction. QCD is at present firmly established as the theory of the strong interactions. Equations of motion in QCD possess topologically non-trivial solutions [1] signaling the presence of degenerate ground states differing by the value of topological charge [2]. The physical vacuum state of the theory is a superposition of these degenerate states, so-called θ-vacuum [3]. To reflect this vacuum structure one may equivalently introduce a θ-term in the QCD Lagrangian. Unless θ is identically equal to zero, this term explicitly breaks $\mathcal{P}$ and $\mathcal{CP}$ symmetries of QCD. However stringent limits on the value of $\theta < 3 \times 10^{-10}$ deduced from the experimental bounds on the electric dipole moment of the neutron [4] indicate the absence of *global* $\mathcal{P}$ and $\mathcal{CP}$ violation in QCD.

Nevertheless it has been proposed that the *local* $\mathcal{P}$- and $\mathcal{CP}$-odd effects due to the topological fluctuations characterized by an effective $\theta = \theta(\vec{x}, t)$ varying in space and time could be directly observed through multi-particle correlations [5]. In heavy ion collisions, the existence of magnetic field (and/or the angular momentum) in the presence of topological fluctuations can induce the separation of electric charge with respect to the reaction plane, so-called Chiral Magnetic Effect [6–9]. There is a recent experimental evidence for this effect from STAR Collaboration at RHIC [10]. The interpretation of STAR result in terms of the local parity violation is under intense scrutiny at present, see e.g. [11–13].

In this paper, we study the role of QCD topology in hard processes using the formalism based on factorization theorems [14]. From the QCD factorization point of view, the cross section in high energy collision can be factorized into a convolution of perturbatively calculable partonic cross section and the non-perturbative but universal parton distribution and fragmentation functions (FFs). In the conventional formalism, these distribution and FFs are required to be $\mathcal{P}$-even because of the parity-conserving nature of the strong interaction. However, in the presence of local (in space and time) $\mathcal{P}$-odd domains $\mathcal{P}$-odd FFs can emerge [15]; note that only the cross section of the entire process has to be $\mathcal{P}$-even, not the FFs.

In this letter, we derive the most general form of the quark FF for a quark fragmenting into a pseudoscalar meson which is consistent with the Lorentz invariance. Abandoning the parity constraint, we obtain two $\mathcal{P}$-odd FFs besides the well-known $\mathcal{P}$-even spin-averaged FF [16] and Collins function [17]. We obtain the exact operator definitions and estimate the size of these new $\mathcal{P}$-odd FFs using the chiral quark model [18]. As a first step, we present their observable effect in the back-to-back dihadron production in e^+e^- collisions. We encourage the experimentalists to carry out the related analyses at RHIC and elsewhere.

2. Quark FFs in locally $\mathcal{P}$-odd background. The quark FFs are defined through the following matrix [19]:

$$\Delta\left(z, p_\perp\right) = \frac{1}{z}\int \frac{dy^- d^2y_\perp}{(2\pi)^3} e^{ik\cdot y}\langle 0|\mathcal{L}_y\psi(y)|PX\rangle \langle PX|\bar{\psi}(0)\mathcal{L}_0^\dagger|0\rangle|_{y^+=0}, \tag{1}$$

where p is the momentum of the final state hadron with a transverse component $p_\perp$ relative to the fragmenting quark k. We choose the hadron moving along $+\hat{z}$ direction, and define the light-cone momentum $p^\pm = (p^0 \pm p^z)/\sqrt{2}$. For convenience, we define two light-like vectors: $\bar{n}^\mu = \delta^{\mu+}$ and $n^\mu = \delta^{\mu-}$. The momentum fraction $z = p^+/k^+$, and $\vec{k}_\perp = -\vec{p}_\perp/z$. $\mathcal{L}_y = \mathcal{P}\exp\left(ig\int_0^\infty d\lambda n\cdot A(y+\lambda n)\right)$ is the gauge link needed to make $\Delta\left(z, p_\perp\right)$ gauge invariant.

Since QCD is a theory conserving $\mathcal{C}$, $\mathcal{P}$, and $\mathcal{T}$ globally, one usually expands the above matrix using the following constraints [20]:

$$\begin{aligned}
\text{Hermiticity :} \qquad & \Delta^\dagger(p,k) = \gamma^0 \Delta(p,k)\gamma^0 & (2)\\
\text{Parity :} \qquad & \Delta(p,k) = \gamma^0 \Delta(\bar{p},\bar{k})\gamma^0 & (3)\\
\text{Time}-\text{reversal :} \qquad & \Delta^*(p,k) = V_T \Delta(\bar{p},\bar{k})V_T^{-1} & (4)
\end{aligned}$$

where $V_T = i\gamma^1\gamma^3$ and $\bar{p}^\mu = p_\mu = (p^0, -\vec{p})$. Using the basis of gamma matrices $\Gamma = \{1, \gamma^\mu, \gamma^\mu\gamma^5, \sigma^{\mu\nu}, i\gamma^5\}$, and the available momenta p and k, one can expand $\Delta(p,k)$ in the most general form:

$$\begin{aligned}
\Delta(p,k) \;=\; & \left[MA_1 1 + A_2 \not{p} + A_3 \not{k} + A_4 \sigma^{\mu\nu}\frac{k_\mu p_\nu}{M}\right] \\
& + \left[A_5 \not{p}\gamma^5 + A_6 \not{k}\gamma^5 + MA_7 i\gamma^5 + A_8 \sigma^{\mu\nu} i\gamma^5 \frac{k_\mu p_\nu}{M}\right], \qquad (5)
\end{aligned}$$

where M is the hadron mass used to make all A_i's have the same dimension. Since the time-reversal changes out-state to in-state, it does not really give any constraint on the coefficients A_i [20]. One the other hand, if one applies the Hermiticity constraint, all of the A_i's have to be real. If one further applies Parity constraint, one finds $A_5 = A_6 = A_7 = A_8 = 0$. However, as we stated in the Introduction, we are interested in the situation in which a local $\mathcal{P}$-odd domain develops in space-time, and the quark fragmentation happens inside such a $\mathcal{P}$-odd domain (or in other words, the quark scatters off the non-trivial gauge field configuration prior to transforming into a pseudoscalar meson). In this case, the $\mathcal{P}$-odd modes in the quark fragmentation could be populated [15] and one has to release the parity constraint in Eq. (3). Note that even though the FF is not a local quantity (the gauge link in Eq. (1) extends to infinity along the light-cone), a $\mathcal{P}$-odd domain in Minkowski space is elongated along the light-cone [15], and thus the $\mathcal{P}$-odd terms do not average to zero. Without parity constraint, we thus need to keep all 8 terms A_1 through A_8 in Eq. (5). Applying the twist-expansion by parametrizing the momenta as $p^\mu \approx p^+\bar{n}^\mu$ and $k^\mu \approx (p^+\bar{n}^\mu - p_\perp^\mu)/z$ and keeping the leading terms,

$$\begin{aligned}
\Delta(z,p_\perp) \;=\; & \frac{1}{2}\left[D(z,p_\perp^2)\not{\bar{n}} + H_1^\perp(z,p_\perp^2)\sigma^{\mu\nu}\frac{p_{\perp\mu}\bar{n}_\nu}{M}\right] \\
& + \frac{1}{2}\left[\widetilde{D}(z,p_\perp^2)\not{\bar{n}}\gamma^5 + \widetilde{H}_1^\perp(z,p_\perp^2)\sigma^{\mu\nu}i\gamma^5\frac{p_{\perp\mu}\bar{n}_\nu}{M}\right] \qquad (6)
\end{aligned}$$

where $D(z,p_\perp^2)$ and $H_1^\perp(z,p_\perp^2)$ are the usual $\mathcal{P}$-even FFs: $D(z,p_\perp^2)$ is the transverse momentum dependent spin-averaged FF [16], and $H_1^\perp(z,p_\perp^2)$ is the Collins function describing a transversely polarized quark fragmenting into an unpolarized hadron [17]. Now besides the two conventional $\mathcal{P}$-even FFs, we also obtain two new $\mathcal{P}$-odd FFs: $\widetilde{D}(z,p_\perp^2)$ and $\widetilde{H}_1^\perp(z,p_\perp^2)$. As we will show below, $\widetilde{H}_1^\perp(z,p_\perp^2)$ generates a new kind of azimuthal correlation. Its role is similar to $H_1^\perp(z,p_\perp^2)$: $H_1^\perp(z,p_\perp^2)$ represents an asymmetric distribution $\propto (\hat{p}\times p_\perp)\cdot\vec{s}_q$, while $\widetilde{H}_1^\perp(z,p_\perp^2)$ represents an asymmetric distribution $\propto p_\perp \cdot \vec{s}_q$ for a transversely polarized quark with spin vector $\vec{s}_q$ to fragment into a pseudoscalar meson. The newly derived $\mathcal{P}$-odd FFs will lead to interesting $\mathcal{P}$-odd effects in experiment as we will show in the next section.

In order to study the experimental effects generated by these $\mathcal{P}$-odd FFs, we need to estimate their magnitude. For this purpose we use the effective chiral quark model developed by Manohar and Georgi [18], which is an effective theory of QCD at low energy scale. This model has also been adopted for an estimate of the Collins functions in [21, 22]. The effective Lagrangian describing the interaction between the quarks and the pion in the leading order is given by

$$L_{qq\Pi} = -\frac{g_A}{2f_\pi}\bar{\psi}_q\gamma^\mu\gamma^5\vec{\tau}\cdot\partial_\mu\vec{\pi}\psi_q \qquad (7)$$

where $f_\pi \approx 93$ MeV is the pseudoscalar decay constant.

At tree level, the fragmentation of a quark is modeled through the process $q^* \to \pi q$, see Fig. 1. One can obtain the unpolarized quark FF $D(z,p_\perp^2)$ from the definition in Eq. (6) which has been done in [21]. The Collins function can be calculated similarly, though one needs to go beyond the tree diagram and consider the π-loop to obtain the final result, see Ref. [21].

Since the chiral quark model Lagarangian in Eq. (7) conserves parity, it does not generate $\mathcal{P}$-odd FFs $\widetilde{D}(z,p_\perp^2)$ and $\widetilde{H}_1^\perp(z,p_\perp^2)$. As we stated in the Introduction, QCD contains topological gauge field configurations, and their effect can be mimicked by an effective space-time dependent θ field [5, 6]. One can thus add to the Lagrangian of QCD the

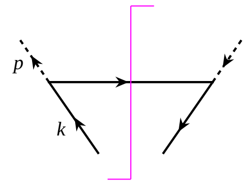

FIG. 1: Lowest-order Feynman diagram for a quark with momentum k fragmenting into a π meson with momentum p.

term $(g^2/32\pi^2)\theta(x,t)F_a^{\mu\nu}\tilde{F}^a_{\mu\nu}$; performing an axial $U(1)$ rotation this term can be transformed into $\frac{1}{2N_f}\partial_\mu\theta\bar{\psi}_q\gamma^\mu\gamma^5\psi_q$ [9]. Let us define an effective $\bar{\theta}_\mu \equiv \partial_\mu\theta/2N_f$, whose zero (time) component is the chiral chemical potential μ_5 [9]. This new term leads to the following equation of motion for the quark field $\psi(x)$:

$$\left(i\partial\!\!\!/ - m + \bar{\theta}\!\!\!/\gamma^5\right)\psi(x) = 0, \tag{8}$$

which is not $\mathcal{P}$-invariant any more, and yields a modified quark propagator $i\widetilde{S}(p,\bar{\theta}) = i/(p\!\!\!/ - m + \bar{\theta}\!\!\!/\gamma^5)$ given by

$$\begin{aligned} i\tilde{S}(p,\bar{\theta}) &= i\left[\mathcal{P}_{\mathcal{R}}S(p+\bar{\theta}) + \mathcal{P}_{\mathcal{L}}S(p-\bar{\theta})\right] \\ &\quad \times \left[1 + m\gamma^5\left(S(p+\bar{\theta}) - S(p-\bar{\theta})\right)\right] \\ &\quad \times \left[1 + \frac{4m^2\bar{\theta}^2}{\left((p+\bar{\theta})^2 - m^2\right)\left((p-\bar{\theta})^2 - m^2\right)}\right]^{-1} \end{aligned} \tag{9}$$

where $\mathcal{P}_{\mathcal{L},\mathcal{R}}$ are the left (right) projection operators $\mathcal{P}_{\mathcal{L},\mathcal{R}} = (1 \pm \gamma^5)/2$, and $iS(p) = i(p\!\!\!/ + m)/(p^2 - m^2)$ is the conventional quark propagator. Note that we have treated $\bar{\theta}$ as a constant in deriving Eq. (9). This is because the time scale associated with the soft non-perturbative dynamics in the FFs due to the non-trivial gauge field configurations $t_{soft} \sim 1/\Lambda_{QCD}$ is much longer than the time scale for the hard collision $t_{hard} \sim 1/Q$, so the soft gauge fields are effectively frozen during hard scattering in each event, and $\bar{\theta}$ can be considered constant. With the $\mathcal{P}$-odd terms contained in the quark field $\psi(x)$, we can now derive the $\mathcal{P}$-odd FFs directly from Eqs. (1) and (6). To the first non-trivial order, $\langle 0|\psi(y)|PX\rangle$ is proportional to

$$\begin{aligned} &\propto \langle 0|\,\psi(y)\bar{\psi}(x)\gamma^\mu\gamma^5\partial_\mu\,\pi(x)\,\psi(x)|q(k-p),\pi(p)\rangle \\ &\propto i\tilde{S}(k,\bar{\theta})p\!\!\!/\gamma^5 u(k-p), \end{aligned} \tag{10}$$

where $u(k-p)$ is the wave function for the final-state quark with momentum $k-p$. Using a similar expression for $\langle PX|\bar{\psi}(0)|0\rangle$, we immediately obtain the results for the $\mathcal{P}$-odd FFs. In terms of Feynman diagrams, they are represented by Fig. 1.

Our calculations further imply that these new FFs are suppressed by a factor $\bar{\theta}^{0,3}/p^+$ when $\bar{\theta}^\mu$ is along time or z-direction. Since p^+ is a large component in our twist expansion, we thus neglect the contribution of the $0,3$ components of $\bar{\theta}$ to be self-consistent. On the other hand, if the $\bar{\theta}$ is along the transverse direction that is perpendicular to $p_\perp$, we find that the $\mathcal{P}$-odd FFs vanish. We thus only consider the situation when $\bar{\theta}$ is along the $p_\perp$ direction: $\bar{\theta}^\mu = \bar{\theta}_\perp\hat{p}^\mu_\perp$, in which case we find

$$\begin{aligned} \widetilde{D}(z,p_\perp^2) &= \frac{g_A^2}{64f_\pi^2\pi^3 z}\frac{4\bar{\theta}_\perp p_\perp}{p_\perp^2 + z^2m_q^2 + (1-z)m_\pi^2}\left[1 - \frac{z}{2}\right. \\ &\quad \left. - \frac{4(1-z)^2z^2m_q^2\,m_\pi^2}{\left(p_\perp^2 + z^2m_q^2 + (1-z)m_\pi^2\right)^2}\right], \end{aligned} \tag{11}$$

$$\begin{aligned} \widetilde{H}_1^\perp(z,p_\perp^2) &= \frac{g_A^2}{4f_\pi^2}\frac{m_qm_\pi}{8\pi^3}\frac{\bar{\theta}_\perp}{p_\perp}\frac{1}{\left(p_\perp^2 + z^2m_q^2 + (1-z)m_\pi^2\right)^3} \\ &\quad \times\left[\left(p_\perp^2 + z^2m_q^2\right)^2(z-2) + (1-z)^2m_\pi^2\right. \\ &\quad \left. \times\left[(3z-2)m_\pi^2 - 4(p_\perp^2 - z^2m_q^2)\right]\right], \end{aligned} \tag{12}$$

which is valid when $\bar{\theta}_\perp \ll p_\perp, p^+$, since we do an expansion and neglect terms $\sim \mathcal{O}(\bar{\theta}^2)$. From the above equations we see that both of the $\mathcal{P}$-odd FFs are proportional to $\bar{\theta}_\perp/p_\perp$. Since $p_\perp$ is a small component, the effect is not suppressed. We will now estimate the size of the observable effect generated by the $\mathcal{P}$-odd FFs within the same model.

3. Observable effect of parity-odd FFs. Let us now discuss the experimental consequences of the $\mathcal{P}$-odd FFs. As a first step, we study a relatively simple process, the back-to-back dihadron production in e^+e^- collisions $e^+e^- \to h_1 h_2 + X$. The method we presented here can be generalized to study $\mathcal{P}$-odd effects in heavy ion collisions.

At leading order in QCD coupling, the two hadrons h_1 and h_2 in e^+e^- collisions are the fragmentation products of a quark and an antiquark originating from $e^+e^- \to q\bar{q}$ annihilation. Following Ref. [23], we choose a reference frame such that the $e^+e^- \to q\bar{q}$ annihilation occurs in the x-z plane, with the back-to-back quark and antiquark moving along the z-axis. The final hadrons h_1 and h_2 carry light-cone momentum fractions z_1 and z_2 and have intrinsic transverse momenta $p_{1\perp}$ and $p_{2\perp}$ with respect to the directions of the fragmenting quarks. Using the fragmentation parameterization in Eq. (6), one can derive the differential cross section as

$$\begin{aligned}
\frac{d\sigma}{d\mathcal{PS}} \;=\; & \sigma_0 \sum_q e_q^2 \Big\{ (1+\cos^2\theta) \\
& \times \Big[D_q(z_1) D_{\bar{q}}(z_2) - \widetilde{D}_q(z_1)\widetilde{D}_{\bar{q}}(z_2) \Big] \\
& + \sin^2\theta \cos(\phi_1+\phi_2) \\
& \times \Big[H_q^\perp(z_1) H_{\bar{q}}^\perp(z_2) + \widetilde{H}_q^\perp(z_1)\widetilde{H}_{\bar{q}}^\perp(z_2) \Big] \\
& + \sin^2\theta \sin(\phi_1+\phi_2) \\
& \times \Big[H_q^\perp(z_1) \widetilde{H}_{\bar{q}}^\perp(z_2) - \widetilde{H}_q^\perp(z_1) H_{\bar{q}}^\perp(z_2) \Big] \Big\},
\end{aligned} \tag{13}$$

where the phase space $d\mathcal{PS} = dz_1 dz_2 d\cos\theta d(\phi_1+\phi_2)$, $\sigma_0 = N_c \alpha_{em}^2/4Q^2$, and θ is the angle between the initial beam direction and the z-axis, not to be confused with the $\theta(x)$ field. In Eq. (13), we have integrated over the moduli of the intrinsic momenta, $p_{1\perp}$ and $p_{2\perp}$, and over the azimuthal angle ϕ_1. The $p_\perp$-integrated functions $D_q(z)$ and $H_q^\perp(z)$ are defined as

$$D_q(z) \;=\; \int d^2p_\perp D_q(z, p_\perp^2), \tag{14}$$

$$H_q^\perp(z) \;=\; \int d^2p_\perp \frac{|\vec{p}_\perp|}{M} H_1^{\perp q}(z, p_\perp^2). \tag{15}$$

The definition of $\widetilde{D}_q(z)$ (or $\widetilde{H}_{\bar{q}}^\perp(z)$) is similar to $D_q(z)$ (or $H_q^\perp(z)$).

The $\cos(\phi_1+\phi_2)$ correlation is usually referred to as the Collins effect, analyzed recently by BELLE Collaboration [24, 25]. However, we find that the product of two $\mathcal{P}$-odd FFs $\widetilde{H}_q^\perp(z)$ leads to the same azimuthal correlation. We emphasize that such products of two $\mathcal{P}$-odd functions in general do not average to zero. They are proportional to $\langle \partial_\mu\theta(x)\partial^\mu\theta(x')\rangle$, and thus are connected to the correlator of pseudoscalar gluon field operators $\langle F\tilde{F}(x)F\tilde{F}(x')\rangle$ that does not vanish [26]. The existence of this term complicates the extraction of the Collins function, but may in effect provide an alternative view of the origin of the Collins effect and puts an experimental constraint of the $\mathcal{P}$-odd FF $\widetilde{H}_q^\perp(z)$. It is interesting that a new azimuthal correlation also emerges: the $\sin(\phi_1+\phi_2)$ term, which is explicitly $\mathcal{P}$-odd. Note that for the $\sin(\phi_1+\phi_2)$ contribution, the first term corresponds to the situation when the antiquark fragments inside the $\mathcal{P}$-odd bubble, whereas the second term corresponds to the situation when the quark fragments inside the $\mathcal{P}$-odd bubble. They have the opposite sign, and thus when averaged over many events, the effect will vanish. Thus a $\mathcal{P}$-odd effect happens only on the event-by-event basis [8].

To estimate the effect, let us assume that the antiquark fragments inside the $\mathcal{P}$-odd bubble; the relative magnitude of the correlation will depend on the following factor $I(\bar{\theta}, z_1, z_2)$, besides the kinematic factor $\sin^2\theta/(1+\cos^2\theta)$,

$$I(\bar{\theta}, z_1, z_2) = \frac{H_q^\perp(z_1)\widetilde{H}_{\bar{q}}^\perp(z_2)}{D_q(z_1)D_{\bar{q}}(z_2) - \widetilde{D}_q(z_1)\widetilde{D}_{\bar{q}}(z_2)}. \tag{16}$$

Certainly $I(\bar{\theta}, z_1, z_2)$ depends on the size of $\bar{\theta}_\perp$.

To estimate $\bar{\theta}_\perp$, we resort to the instanton vacuum model (for a review, see [27]). According to [27], the two most important parameters are the mean size of the instanton $\rho \sim 1/3$ fm and the typical separation R between instantons, with $\rho/R \sim 1/3$. The spatial gradient of the effective field $\theta(x)$ within the instanton vacuum model is proportional to the inverse instanton size ρ^{-1} that is the only dimensionful parameter characterizing the solution. The probability for a quark moving along the light cone to interact with the instanton is $\sim \rho^2/R^2$; note that in Minkowski space-time the instanton event is elongated along the light cone [15]. We thus estimate the average value of $\bar{\theta}_\perp$ sampled by the quark in a given event as

$$\langle \bar{\theta}_\perp \rangle \equiv \frac{1}{2N_f} \langle \partial_\perp \theta(\vec{x}, t) \rangle \sim \frac{1}{2N_f} \cdot \frac{1}{\rho} \cdot \frac{\rho^2}{R^2} \sim 10 \text{ MeV}, \tag{17}$$

where we have used $N_f = 3$, $\partial_\perp \theta \approx \Delta\theta / \Delta x_\perp$ with $\Delta\theta \sim \mathcal{O}(1)$ and $\Delta x_\perp \sim \rho$. With $\bar{\theta}_\perp = 10$ MeV, and other standard parameters of the chiral quark model [18], and using the calculation of the FFs taken from Ref. [21], we find $I(\bar{\theta}, z_1, z_2) \sim 1.5\%$ for a typical $z_1 = z_2 = 0.5$ at BELLE experiment, with the final two hadrons as π^+ and π^-. We urge the experimentalists at BELLE, RHIC and elsewhere to carry out an analysis to constrain the $\mathcal{P}$-odd FFs. Because of the universality of the FFs, we expect that the formalism developed here could be generalized to other processes.

4. Conclusion. In this letter we have studied the quark fragmentation in the topologically non-trivial QCD background. We have found two new FFs besides the well-known spin-averaged FF and the Collins function. Both of the new FFs are $\mathcal{P}$-odd. We have related the magnitude of these functions to the typical size of the topological fluctuations (described by the effective $\theta(x)$ field). We have studied the observable effects of the $\mathcal{P}$-odd FFs in back-to-back dihadron production in e^+e^- collisions, and have found that a new azimuthal correlation $\propto \sin(\phi_1 + \phi_2)$ appears. Since the new azimuthal correlation is explicitly $\mathcal{P}$-odd, it can be observed only on an event-by-event basis. Our results also offer a new interpretation of the Collins correlation. We encourage the experimentalists to carry out an analysis to constrain the $\mathcal{P}$-odd FFs, and anticipate new applications.

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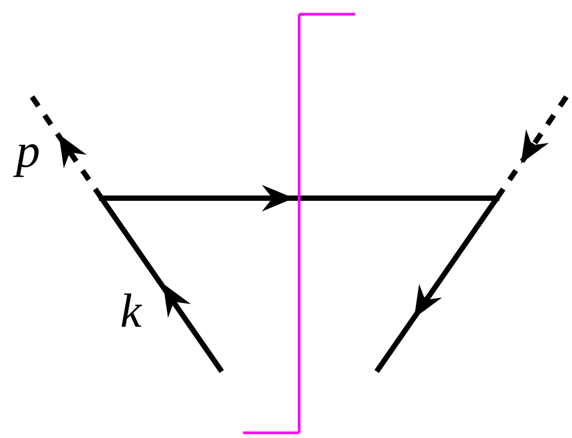
[1] A. A. Belavin *et al.*, Phys. Lett. B **59** (1975) 85.
[2] S. S. Chern and J. Simons, Annals Math. **99** (1974) 48.
[3] G. 't Hooft, Phys. Rev. Lett. **37**, 8 (1976); Phys. Rev. D **14**, 3432 (1976) [Erratum-ibid. D **18**, 2199 (1978)]; R. Jackiw and C. Rebbi, Phys. Rev. Lett. **37**, 172 (1976); C. G. Callan, R. F. Dashen and D. J. Gross, Phys. Lett. B **63**, 334 (1976).
[4] C. A. Baker *et al.*, Phys. Rev. Lett. **97**, 131801 (2006).
[5] D. Kharzeev, R. D. Pisarski and M. H. G. Tytgat, Phys. Rev. Lett. **81**, 512 (1998); arXiv:hep-ph/9808366; arXiv:hep-ph/0012012.
[6] D. Kharzeev, Phys. Lett. B **633**, 260 (2006); Annals Phys. **325**, 205 (2010).
[7] D. Kharzeev and A. Zhitnitsky, Nucl. Phys. A **797**, 67 (2007).
[8] D. E. Kharzeev, L. D. McLerran and H. J. Warringa, Nucl. Phys. A **803**, 227 (2008).
[9] K. Fukushima, D. E. Kharzeev and H. J. Warringa, Phys. Rev. D **78**, 074033 (2008).
[10] B. I. Abelev *et al.* [STAR Collaboration], Phys. Rev. Lett. **103**, 251601 (2009); Phys. Rev. C **81**, 054908 (2010).
[11] A. Bzdak, V. Koch and J. Liao, Phys. Rev. C **81**, 031901 (2010); Phys. Rev. C **82**, 054902 (2010).
[12] F. Wang, Phys. Rev. C **81**, 064902 (2010).
[13] M. Asakawa, A. Majumder and B. Muller, Phys. Rev. C **81**, 064912 (2010).
[14] J. C. Collins, D. E. Soper and G. Sterman, Adv. Ser. Direct. High Energy Phys. **5**, 1 (1988).
[15] A. Efremov and D. Kharzeev, Phys. Lett. B **366**, 311 (1996).
[16] J. C. Collins and D. E. Soper, Nucl. Phys. B **194**, 445 (1982).
[17] J. C. Collins, Nucl. Phys. B **396**, 161 (1993).
[18] A. Manohar and H. Georgi, Nucl. Phys. B **234**, 189 (1984).
[19] See e.g. A. Bacchetta *et al.*, JHEP **0702**, 093 (2007).
[20] See e.g. D. Boer, P. J. Mulders and F. Pijlman, Nucl. Phys. B **667**, 201 (2003).
[21] A. Bacchetta *et al.*, Phys. Rev. D **65**, 094021 (2002).
[22] A. Bacchetta, A. Metz and J. J. Yang, Phys. Lett. B **574**, 225 (2003); D. Amrath *et al.*, Phys. Rev. D **71**, 114018 (2005).
[23] M. Anselmino *et al.*, Phys. Rev. D **75**, 054032 (2007).
[24] D. Boer, R. Jakob and P. J. Mulders, Nucl. Phys. B **504**, 345 (1997); D. Boer, Nucl. Phys. B **806**, 23 (2009).
[25] R. Seidl *et al.* [Belle Collaboration], Phys. Rev. Lett. **96**, 232002 (2006); Phys. Rev. D **78**, 032011 (2008).
[26] E. Witten, Nucl. Phys. B **156**, 269 (1979).
[27] T. Schafer and E. V. Shuryak, Rev. Mod. Phys. **70**, 323 (1998).



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