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# The dynamo effect in decaying helical turbulence

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We show that in decaying hydromagnetic turbulence with initial kinetic helicity, a weak magnetic field eventually becomes fully helical. The sign of magnetic helicity is opposite to that of the kinetic helicity—regardless of whether or not the initial magnetic field was helical. The magnetic field undergoes inverse cascading with the magnetic energy decaying approximately like  $t^{-1/2}$ . This is even slower than in the fully helical case, where it decays like  $t^{-2/3}$ . In this parameter range, the product of magnetic energy and correlation length raised to a certain power slightly larger than unity, is approximately constant. This scaling of magnetic energy persists over long time scales. At very late times and for domain sizes large enough to accommodate the growing spatial scales, we expect a cross-over to the  $t^{-2/3}$  decay law that is commonly observed for fully helical magnetic fields. Regardless of the presence or absence of initial kinetic helicity, the magnetic field experiences exponential growth during the first few turnover times, which is suggestive of small-scale dynamo action. Our results have applications to a wide range of experimental dynamos and astrophysical time-dependent plasmas, including primordial turbulence in the early universe.

PACS numbers: Magnetohydrodynamics – Dynamo theory, Turbulence, Astrophysical fluid dynamics

## I. INTRODUCTION

In electrically conducting fluids such as plasmas and liquid metals, steady helical turbulence is known to lead to an efficient conversion of kinetic energy into magnetic energy—a process referred to as a dynamo. Dynamos with swirling (helical) motions can be excited at relatively small magnetic Reynolds numbers, i.e., at moderate turbulent velocities and length scales, as well as moderate electric conductivities [1, 2]. This is why many dynamo experiments have employed helical flows both in the constrained and basically nonturbulent flows of the experiments performed in Riga [3, 4] and Karlsruhe [5, 6], as well as the unconstrained (turbulent) von Kármán flows in the experiments in Cadarache [7, 8]. Many other experiments are currently being worked upon [9–12]. Their success is limited by the power that can be delivered by the propellers or pumps. A more economic type of dynamo experiment is driven by the flow that results inside a spinning torus of liquid sodium after abruptly breaking it. This leads to turbulence from the screw-like diverters inside the torus [13–15]. Theoretical studies of laminar screw dynamos have been performed [16], but the evolution of hydromagnetic turbulence is usually parameterized in ways that ignore the effects of kinetic and magnetic helicity.

The problem of magnetic field evolution in decaying helical turbulence in conducting media is far more general. Neutron stars, for example, have convective turbulence during the first minute after their formation [17, 18]. The early universe could be another example of turbulence driven by expanding bubbles after a first-order phase transition [19, 20]. Turbulence can also be driven by magnetic fields generated at earlier times during inflation [21, 22]. Transient turbulence is also being generated as a consequence of merging galaxy clusters [23, 24]. Even accretion discs may provide an example of decaying turbulence when the magnetorotational instability is not excited during certain phases

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[25]. A related example is that of tidal disruption events, where a star has a close encounter with a supermassive black hole and gets disrupted. During this process, tremendous shearing motion is being dissipated. A fraction of it can be dissipated magnetically via strongly time-dependent dynamo action and Joules dissipation [26]. Dynamo effects are also suspected to occur over durations of microseconds in inertial fusion confinement plasmas [27–29]. In all these cases, one deals with decaying turbulence. This is what makes the interpretation in terms of a dynamo effect complicated. Here we focus on general aspects of the dynamo mechanism rather than trying to model specific laboratory or astrophysics conditions.

In this paper, we demonstrate for the first time that in decaying helical turbulence, an initially nonhelical seed magnetic field undergoes a quasi-exponential increase. In the presence of initial kinetic helicity, this growth is followed by a long ( $\sim 50,000$  turnover times) transient decay where the magnetic energy decays like  $t^{-1/2}$ . This is slower than in the case of an initially fully helical magnetic field, which decays like  $t^{-2/3}$ . It develops inverse cascade-type behavior already well before the magnetic field becomes fully helical. To what extent the transient decay owing to initial kinetic helicity can be modeled in terms of advanced mean-field dynamo theory remains open, although potentially suitable tools such as two- and three-scale dynamo theories have been developed [30]. Previous decay simulations were always performed with strong initial magnetic fields. Only recently, the need for studying the evolution of hydromagnetic turbulence in *kinetically dominated* systems has been emphasized [31]. However, no detailed study has been presented as yet, except for our own work [32], which focused on the case without kinetic helicity.

## II. HELICAL DYNAMOS WITH TIME-DEPENDENT COEFFICIENTS

A simple example of a dynamo is one that works owing to the presence of kinetic helicity,  $\langle \boldsymbol{\omega} \cdot \mathbf{u} \rangle$ , where  $\boldsymbol{\omega} = \nabla \times \mathbf{u}$  is the vorticity and  $\mathbf{u}$  is the turbulent velocity. In stationary isotropic turbulence a statistically averaged mean magnetic field  $\overline{\mathbf{B}}$  obeys [1, 2]

$$\partial \overline{\mathbf{B}} / \partial t = \nabla \times [\alpha_{\text{dyn}} \overline{\mathbf{B}} - (\eta_t + \eta) \mu_0 \overline{\mathbf{J}}], \quad (1)$$

where  $\alpha_{\text{dyn}} \approx -\tau \langle \boldsymbol{\omega} \cdot \mathbf{u} \rangle / 3$  is the  $\alpha$  effect,  $\eta_t \approx \tau \langle \mathbf{u}^2 \rangle / 3$  is the turbulent magnetic diffusivity,  $\eta$  is the microphysical magnetic diffusivity, and  $\overline{\mathbf{J}} = \nabla \times \overline{\mathbf{B}} / \mu_0$  is the mean current density with  $\mu_0$  being the vacuum permeability. If the coefficients are spatially constant and the domain is periodic, the solutions are eigenfunctions of the curl operator with eigenvalue  $k$ . If the coefficients were also constant in time,  $|\overline{\mathbf{B}}|$  would be proportional to  $\exp(i\mathbf{k} \cdot \mathbf{x} + \gamma t)$ . There would then be a growing solution that obeys  $\gamma = |\alpha_{\text{dyn}} k| - \eta_T k^2$  with  $\eta_T = \eta_t + \eta$  if  $C \equiv |\alpha_{\text{dyn}}| / \eta_T k_1 > 1$ , where  $k_1 = 2\pi/L$  is the smallest wave number that fits into the cubic domain of size  $L^3$ .

We define the fractional helicity  $\epsilon_f$  such that  $\langle \boldsymbol{\omega} \cdot \mathbf{u} \rangle = \epsilon_f \langle \mathbf{u}^2 \rangle / \xi_K$ , where  $\xi_K$  is the scale of the energy-carrying eddies, which we will later identify with the integral scale that is formally defined in terms of energy spectra. Thus,  $C = \epsilon_f / (\iota k_1 \xi_K)$ , where  $\iota = 1 + 3 \text{Re}_M^{-1}$  with

$$\text{Re}_M = u_{\text{rms}} \xi_K / \eta \quad (2)$$

being the magnetic Reynolds number,  $\tau = \xi_K / u_{\text{rms}}$  is the turnover time, and  $u_{\text{rms}} = \langle \mathbf{u}^2 \rangle^{1/2}$  is the rms velocity [33]. The effective wave number of the large-scale field,  $k_m$ , is not normally at the minimal wave number  $k = k_1$ , but at a larger value, so  $k_1 \leq k_m \leq (2\xi_M)^{-1}$ ; see, e.g., Fig. 17 of Ref. [34].

In decaying hydrodynamic turbulence, we have  $u_{\text{rms}}^2 \propto t^{-p}$  with exponent  $p = 10/7$  if the Loitsiansky integral [35] is conserved, or  $p = 6/5$  if the Saffman integral [36] is conserved. In these cases, we have  $|\overline{\mathbf{B}}| = B_0 \exp[\int_0^t \gamma(t') dt']$ , where

$$\gamma(t) = (\epsilon_f - \iota k_m \xi_K) u_{\text{rms}} k_m / 3 \quad (3)$$

with  $\epsilon_f = \epsilon_f(t)$ ,  $\xi_K = \xi_K(t)$ ,  $k_m(t) \geq k_1$ , and  $\iota = \iota(t)$  now all being time-dependent functions. Thus, we expect a time-dependent (instantaneous) growth rate that is, to leading order, given by  $u_{\text{rms}}(t) k_m(t) / 3$ . With these preliminary expectations in mind, let us now turn to three-dimensional turbulence simulations.

## III. DYNAMOS IN DECAYING TURBULENCE

We are primarily interested in subsonic turbulence with initial Mach numbers of the order of 0.1. At those low Mach numbers, the equation of state no longer affects the flow (see Fig. 2 of the supplemental material to Ref. [37]) and compressibility effects are unimportant. We choose to solve for an isothermal gas where the pressure  $p$  is proportional to the local density  $\rho$  with  $p = \rho c_s^2$ . This equation of state applies to the early universe where  $c_s^2 = c^2/3$  with  $c$

being the speed of light. Solving for a weakly compressible gas is computationally more efficient than solving for an incompressible fluid where the pressure is a nonlocal function of the velocity.

We neglect kinetic and two-fluid effects in our present work, which is appropriate for many astrophysical plasmas, including the early universe [38]. We thus solve the three-dimensional hydromagnetic equations

$$\frac{\partial \mathbf{u}}{\partial t} = -\mathbf{u} \cdot \nabla \mathbf{u} - c_s^2 \nabla \ln \rho + \frac{1}{\rho} (\mathbf{J} \times \mathbf{B} + \nabla \cdot 2\rho\nu\mathbf{S}), \quad (4)$$

$$\frac{\partial \ln \rho}{\partial t} = -\mathbf{u} \cdot \nabla \ln \rho - \nabla \cdot \mathbf{u}, \quad (5)$$

$$\frac{\partial \mathbf{A}}{\partial t} = \mathbf{u} \times \mathbf{B} - \eta\mu_0\mathbf{J}, \quad (6)$$

where  $\mathbf{S}_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) - \frac{1}{3}\delta_{ij}\nabla \cdot \mathbf{u}$  is the traceless rate of strain tensor,  $\nu$  is the kinematic viscosity,  $\mathbf{B} = \nabla \times \mathbf{A}$  is the magnetic field,  $\mathbf{J} = \nabla \times \mathbf{B}/\mu_0$  is the current density, and  $\mu_0$  is the vacuum permeability. We consider a triply periodic domain of size  $L^3$ , so the smallest wave number in the domain is  $k_1 = 2\pi/L$ .

We take the initial velocity to be solenoidal and define it in Fourier space as

$$u_i(\mathbf{k}) = \left[ \mathbf{P}_{ij}(\mathbf{k}) + i\sigma_K \epsilon_{ijl} \frac{k_l}{k} \right] \frac{u_0 k_0^{-3/2} g_j(\mathbf{k}) (k/k_0)^{\alpha/2-1}}{[1 + (k/k_0)^{2(\alpha+5/3)}]^{1/4}}, \quad (7)$$

where  $\mathbf{P}_{ij} = \delta_{ij} - k_i k_j / k^2$  is the projection operator,  $\mathbf{g}(\mathbf{k})$  is the Fourier transform of a spatially  $\delta$ -correlated vector field in three dimensions with Gaussian distributed fluctuations, and  $k_0$  is wave number of the peak of the initial spectrum. It corresponds to the initial wave number of the energy-carrying eddies. We choose  $k_0/k_1 = 60$ . The exponent  $\alpha$  (not to be confused with the mean-field dynamo coefficient  $\alpha_{\text{dyn}}$ ) denotes the slope of the spectrum at low wave numbers. We choose  $\alpha = 4$  for a causally generated solenoidal field [39, 40]. The fractional initial helicity is controlled by the parameter  $\sigma_K$  and given by  $\epsilon_f = 2\sigma_K/(1 + \sigma_K^2)$ . For the initial magnetic field, we take the same spectrum, but with  $\sigma_M$  instead of  $\sigma_K$ , and amplitude  $B_0$  instead of  $u_0$ . The velocity is initially fully helical ( $\sigma_K = 1$ ) and solenoidal. We consider an initial  $\mathbf{B}(\mathbf{k})$  with  $\sigma_M = 0, 1$ , and  $-1$ . The initial density is constant and given by  $\rho_0$ .

Viscosity  $\nu$  and magnetic diffusivity  $\eta$  are usually very small in physical systems of interest. This is generally difficult to simulate, especially at early times if we fix  $\nu$  and  $\eta$  to be that small. However, a self-similar evolution is made possible by allowing  $\nu$  and  $\eta$  to be time-dependent (after some time  $t > t_0$ ; see below) with

$$\nu(t) = \nu_0 \max(t, t_0)^r, \quad (8)$$

where  $r = (1 - \alpha)/(3 + \alpha)$  [41], which gives  $r = -3/7$  for  $\alpha = 4$ . The time  $t_0$  is chosen to be short ( $t_0 u_{\text{rms}}/\xi_M \approx 1$ ), but non-vanishing to prevent  $\nu$  and  $\eta$  from becoming singular for  $r < 0$ . In most of the cases reported below, we assume  $\eta(t) = \nu(t)/\text{Pr}_M$ , where  $\text{Pr}_M = 1$  is chosen for the magnetic Prandtl number. In some cases, we also compare with cases where  $\text{Pr}_M \neq 1$  and with cases where  $\nu \equiv \nu_0$  and  $\eta \equiv \eta_0$  are constant in time.

We define kinetic and magnetic energy spectra,  $E_K(k, t)$  and  $E_M(k, t)$ , respectively. They are normalized such that  $\int E_i(k, t) dk = \mathcal{E}_i$  for  $i = K$  or  $M$ , where  $\mathcal{E}_K = \rho_0 u_{\text{rms}}^2/2$  and  $\mathcal{E}_M = B_{\text{rms}}^2/2\mu_0$  are the kinetic and magnetic mean energy densities, and  $B_{\text{rms}}$  is the rms magnetic field. Time is given in units of the initial turnover time,  $\tau_0 = \tau(0)$ , and

$$\xi_i(t) = \int_0^\infty k^{-1} E_i(k, t) dk / \mathcal{E}_i(t) \quad (9)$$

is the integral scale. We have chosen  $t_0/\tau_0 = 0.1$  for the time when viscosity and magnetic diffusivity become time-dependent. Our runs are given in Table I, where the initial Alfvén speed  $v_{A0} = B_0/\sqrt{\mu_0\rho_0}$  has been introduced and the end time of the run  $t_e$  is given.

In the following, we characterize the values of  $\nu_0$  and  $\eta_0$  by the time-dependent magnetic Reynolds and Lundqvist numbers,

$$\text{Re}_M = u_{\text{rms}}\xi_M/\eta \quad \text{and} \quad \text{Lu} = B_{\text{rms}}\xi_M/\eta, \quad (10)$$

respectively. Their initial and final values are indicated in Table I, respectively. Note also that we have now chosen to define  $\text{Re}$  and  $\text{Re}_M$  in terms of  $\xi_M$  instead of  $\xi_K$ . We do this because the magnetic energy spectrum has a more

clearly defined peak, while that of the kinetic energy spectrum is less clear and, at least after some time, its evolution is enslaved by the magnetic field. Furthermore, we define instantaneous scaling exponents of  $\mathcal{E}_i(t)$  and  $\xi_i(t)$  as

$$p_i(t) = d \ln \mathcal{E}_i / d \ln t, \quad q_i(t) = d \ln \xi_i / d \ln t, \quad (11)$$

and plot  $p_i(t)$  versus  $q_i(t)$  for  $i = M$  and  $K$  and discuss the evolution of the point

$$\mathbf{P}_i = (p_i, q_i) \quad (12)$$

in the  $pq$  diagram. Solutions that obey invariance under rescaling [37, 41, 42],

$$k \rightarrow k' \ell^{-1} \quad \text{and} \quad t \rightarrow t' \ell^{1/q_i}, \quad (13)$$

all lie on the line  $p_i = 2(1 - q_i)$  in this diagram.

In the case of a self-similar evolution [41, 42], the magnetic energy spectra can be described by a single function  $\phi(k\xi_M)$  of the product  $k\xi_M$  such that [37]

$$E_M(k\xi_M(t), t) \approx \xi_M^{-\beta_M} \phi(k\xi_M), \quad (14)$$

where  $\phi(k\xi_M)$  is a function of magnetic Reynolds and Prandtl numbers, but not of time. Note that  $\xi_M(t)$  varies such that the peak of the spectrum is always at  $k\xi_M \approx 1$ . If the solutions are invariant under rescaling, they must obey  $q_i = 2/(\beta_i + 3)$  [41].

By integrating  $\mathcal{E}_M(t) = \int E_M(k, t) dk$ , one can see that

$$\mathcal{E}_M(t) \propto \xi_M^{-(\beta_M+1)} \propto t^{-q_M(\beta_M+1)}, \quad (15)$$

and therefore we have  $1 + \beta_M = p_M/q_M$  [37]. On dimensional and physical grounds [43], one expects the rate of change to obey

$$\frac{d\mathcal{E}_M}{dt} \propto \xi_M^{-1} \mathcal{E}_M^{3/2}. \quad (16)$$

Further details regarding the relation between  $\xi_M$  and  $\mathcal{E}_M$  depend of the conservation laws that are being obeyed. For example, when magnetic helicity is conserved, we have  $\langle \mathbf{A} \cdot \mathbf{B} \rangle \propto \mathcal{E}_M \xi_M = \text{const}$ , so  $\xi_M \propto \mathcal{E}_M^{-1}$  and therefore  $d\mathcal{E}_M/dt \propto \mathcal{E}_M^{5/2}$ , which yields [43]

$$p_M = q_M = 2/3. \quad (17)$$

This, in turn, implies  $\beta_M = 0$ , so the height of the peak of  $E_M(k, t)$  stays unchanged; see Eq. (14).

For our numerical simulations we use the **PENCIL CODE** (<https://github.com/pencil-code>), a public MHD code that is particularly well suited for simulating turbulence. In all cases we use  $1152^3$  meshpoints, which is large enough to ensure that the inverse-cascade effects are well reproduced; see Ref. [44] for earlier work highlighting the importance of high resolution in connection with the inverse cascade in nonhelical hydromagnetic turbulence.

TABLE I: Summary of the runs discussed in this paper.

Run	$\sigma_K$	$\sigma_M$	$v_{A0}/u_0$	$\text{Re}_M$	Lu	$t_e/\tau_0$	$q_M(t_e)$	$p_M(t_e)$
A	1	0	0.1	38–323	17–830	49,000	0.55	0.58
B	1	1	0.1	35–120	14–182	23,000	0.46	0.59
C	1	−1	0.1	37–326	29–1090	15,000	0.53	0.57
D	1	0	0.01	34–37	3–21	2500	0.38	1.10
E	1	0	0.001	30–22	0.5–4.5	400	0.35	0.70
E'	0	0	0.001	30–22	0.5–2.5	1300	0.29	0.44
F	1	0	0.1	27–21	9–17	63	0.47	1.31
G	1	0	0.1	14	3–5	26	0.29	0.50
H	1	0	0.1	8	1–6	3400	0.29	0.50

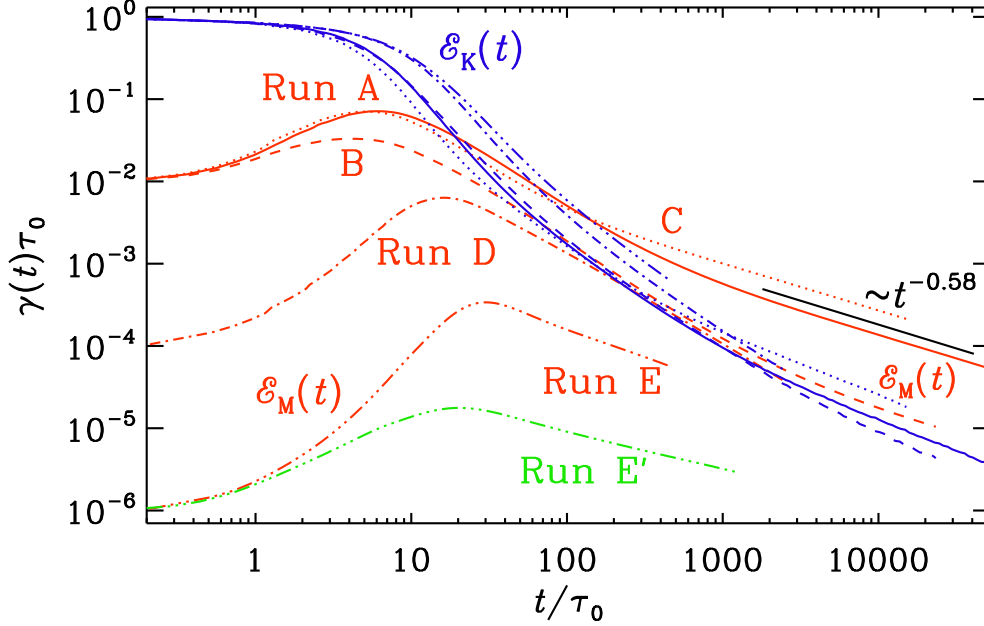


FIG. 1: Evolution of  $\mathcal{E}_K$  (blue) and  $\mathcal{E}_M$  (red) for  $\sigma_M = 0$  (solid),  $\sigma_M = 1$  (dashed), and  $\sigma_M = -1$  (dotted) for  $v_{A0}/u_0 = 0.1$  (Runs A–C), as well as 0.01 (dot-dashed, Run D) and 0.001 (triple dot-dashed, Run E) for  $\sigma_M = 0$ . The green triple dot-dashed line denotes Run E', which has zero initial kinetic helicity.

## IV. RESULTS

### A. Kinetic and magnetic energy evolution

In Fig. 1, we plot  $\mathcal{E}_K(t)$  and  $\mathcal{E}_M(t)$  for Runs A–E.  $\mathcal{E}_M$  is found to increase at first, reaches a maximum at  $t/\tau_0 \approx 10$ , and then approaches a late-time magnetic decay law approximately proportional to  $t^{-p}$  with  $p \gtrsim 0.5$ . We see that kinetic energy is transferred to magnetic energy, whose value eventually exceeds  $\mathcal{E}_K$ . The time when this happens depends on the initial magnetic energy. For Run A with  $v_{A0}/u_0 = 0.1$ , this time is  $t/\tau_0 \approx 20$ ; see Fig. 1, and for Run D it is  $t/\tau_0 = 200$ .

Although the turbulence is decaying, it is still possible to define a meaningful growth rate of the magnetic field and to estimate a critical value of the magnetic Reynolds number above which dynamo action is possible. We do this by plotting the instantaneous growth rate,

$$\gamma(t) = d \ln B_{\text{rms}} / dt, \quad (18)$$

of the rms magnetic field  $B_{\text{rms}}$ . The result is shown in Fig. 2, where we plot  $\gamma(t)\tau_0$  versus  $t/\tau_0$ . We see that the values with the weakest initial field (i.e., in the kinematic limit) are  $\gamma(t)\tau_0 \lesssim 0.5$  at early times. At later times, however,  $\gamma(t)$  decreases. This is roughly consistent with Eq. (3). Furthermore, the decay is faster if  $\eta$  is larger, i.e.,  $\text{Re}_M$  smaller.

In the case with zero kinetic helicity, the initial growth rate is nearly the same as with kinetic helicity; see Run E' in Fig. 2. This suggests that there is also small-scale dynamo action. Owing to the absence of kinetic helicity,  $\gamma(t)$  follows a slightly steeper power law of the approximate form  $t^{-0.9}$ . At later times, however, the magnetic field of Run E' decays in the same way as that of Run E.

To understand what has happened, we look at the spectra  $E_K(k, t)$  and  $E_M(k, t)$  in Fig. 3. We see that, during the late evolution ( $t/\tau_0 > 1000$ ), the magnetic energy spectra are shape-invariant and just translate toward smaller  $k$ . This is suggestive of an inverse cascade, where  $E_M(k\xi_M(t), t)$  collapses onto the same curve  $\phi(k\xi_M)$  with  $1 + \beta_M = p_M/q_M$  [37]; see Eq. (14). Here the correlation length  $\xi_M$  increases like  $t_M^q$  such that  $\langle B^2 \rangle \xi_M^{1+\beta_M}$  stays constant; see Eq. (15). The value of this constant depends on the total amount of magnetic helicity that is produced in the system. To compensate for the decay in magnetic energy, we multiply  $E_M$  by  $\xi_M^{\beta_M}$  with an exponent  $\beta_M$  such that the compensated spectra collapse onto a single function

A closer inspection of the magnetic decay gives  $q_M \approx 0.55$  and  $p_M \approx 0.58$  at the end time for Run A, so that  $\beta_M \approx 0.05$ ; see the  $pq$  diagram in Fig. 5. In Fig. 6 we show a similar plot for Run C. Since the magnetic decay is not

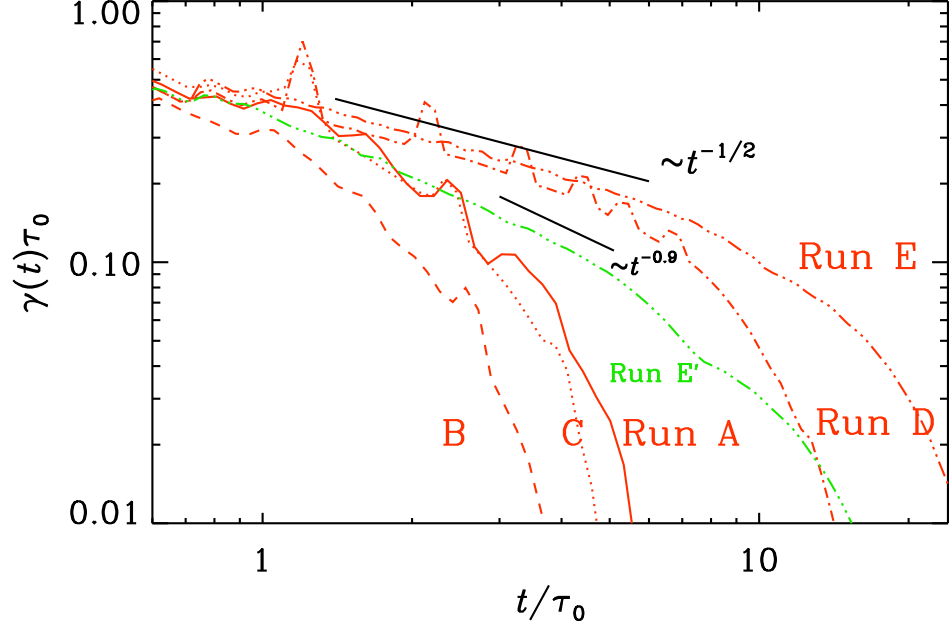


FIG. 2: Instantaneous growth rates  $\gamma(t)\tau_0$  of  $B_{\text{rms}}$  for  $\sigma_M = 0$  (solid),  $\sigma_M = 1$  (dashed), and  $\sigma_M = -1$  (dotted) for  $v_{A0}/u_0 = 0.1$  (Runs A–C), as well as 0.01 (dot-dashed, Run D) and 0.001 (triple dot-dashed, Run E). The green triple dot-dashed line denotes Run E' with zero initial kinetic helicity.

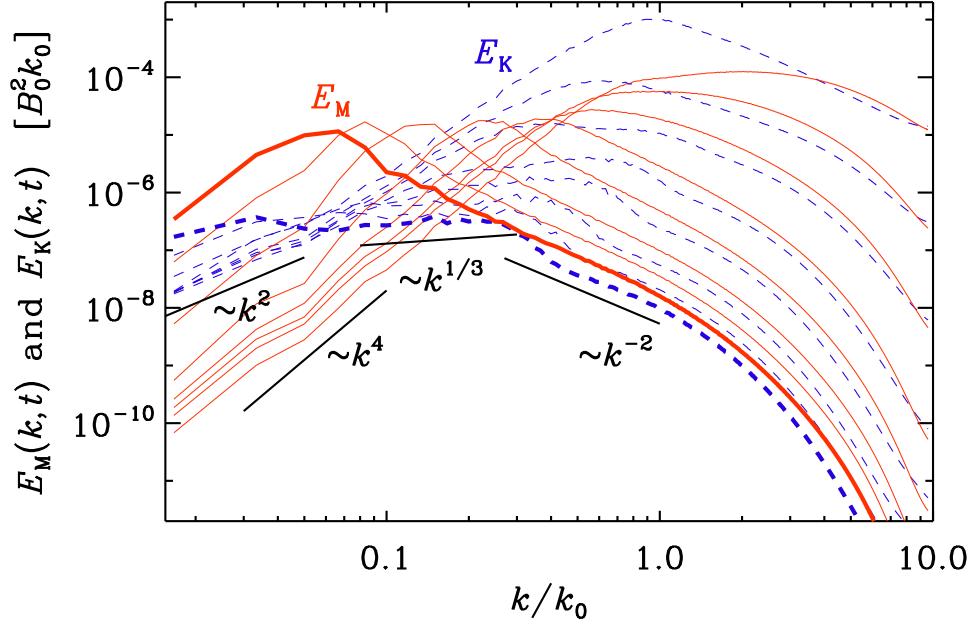


FIG. 3:  $E_K(k, t)$  and  $E_M(k, t)$  for  $t/\tau = 16, 60, 200, 800, 2000, 8000$ , and  $17,000$ , for Run A. Time is decreasing downward and the last time is shown as fat lines.

truly self-similar, it does not obey the scaling relation  $\beta_M = 2/q_M - 3$  [41] and does not fall on the line  $p_M = 2(1 - q_M)$ , which is indicated in Fig. 5 by a solid line.

At late times, although  $p_M$  and  $q_M$  are still different from the expected law with  $p_M = q_M = 2/3$ , there are several other similarities to earlier calculations of magnetically dominated hydromagnetic turbulence. In particular, we see a change of the low wave number slope of  $E_K$  from  $k^4$  to  $k^2$  at later times and at small  $k$ . This is a consequence

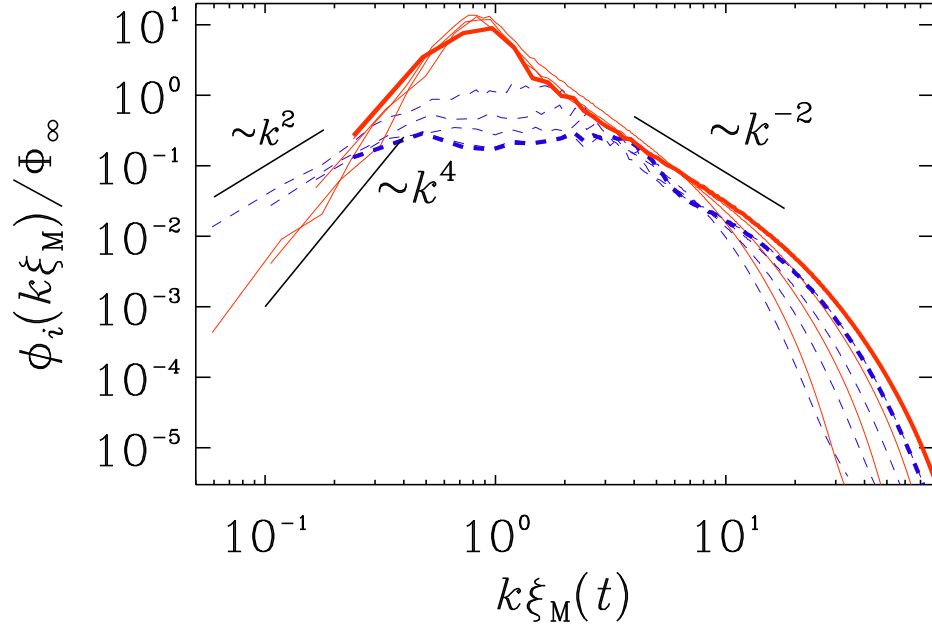


FIG. 4:  $E_K(k, t)$  and  $E_M(k, t)$  at  $t/\tau = 2600, 9400, 24,000$ , and  $45,000$ , collapsed spectra using  $\beta_M = 0$  for Run A.

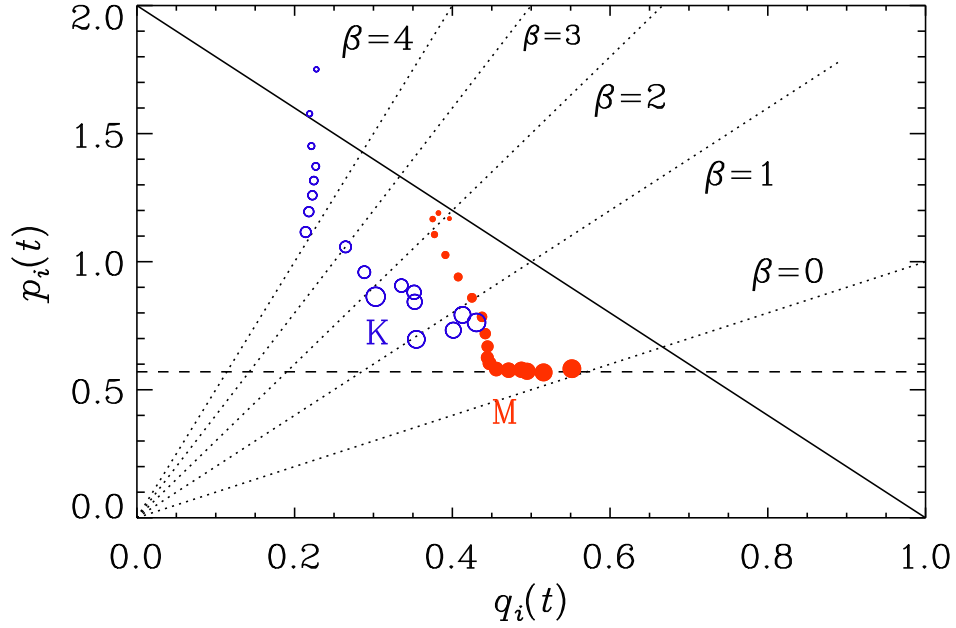


FIG. 5:  $pq$  diagram for Run A for kinetic (blue open symbols) and magnetic (red filled symbols) energy spectra. Near the end of the run (larger symbols), the solution evolves along the  $p_M = 0.58$  line (dashed) and  $q_M \approx 0.55$  with  $\beta_M = p_M/q_M - 1 \approx 0.05$  is found at the end of the run. Smaller (larger) symbols denote earlier (later) times.

of compressibility [44, 45] and is not seen in the incompressible case; see the supplemental material of Ref. [46]. At larger wave numbers, near the point where  $E_M$  peaks, the kinetic energy is proportional to  $k^{1/3}$ ; see Figs. 5 and 6 and Ref. [44]. The  $k^2$  law for the kinetic energy  $\mathcal{E}_K$  is likely a consequence of turbulent interactions over the scale of the domain since the initial time.

To inspect the slow changes of  $\beta_M(t)$ ,  $p_M(t)$ , and  $q_M(t)$  in more detail, we show in Fig. 7 their evolution for Run A using again a logarithmic time axis. We see that there is an intermediate plateau when their values are indeed



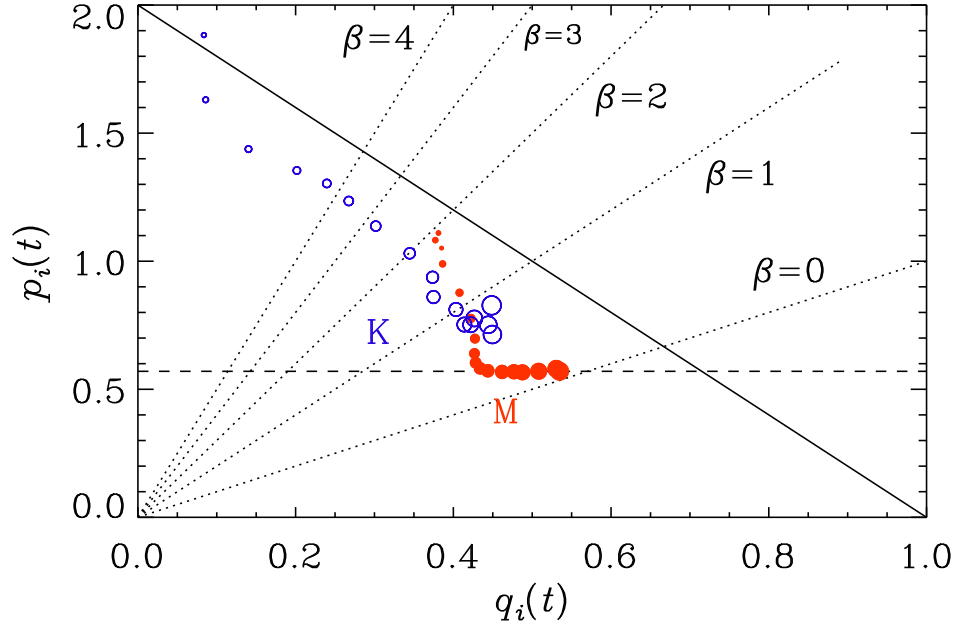


FIG. 6: Same as Fig. 5, but for Run C, where the solution evolves along the  $p_M = 0.57$  line (dashed) and  $q_M \approx 0.53$  with  $\beta_M = p_M/q_M - 1 \approx 0.08$  is found at the end of the run.

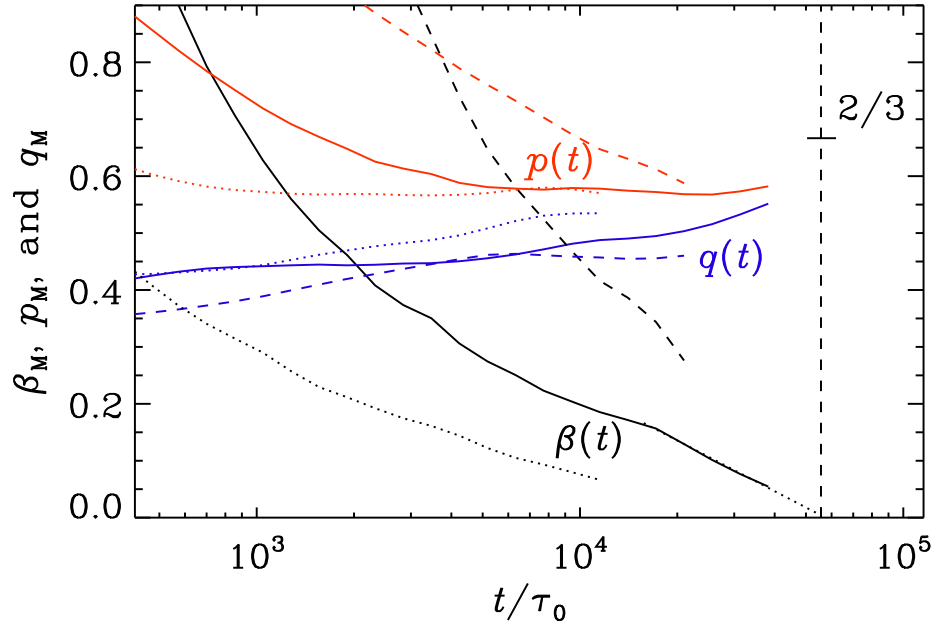


FIG. 7: Evolution of  $\beta_M(t)$  (black),  $p_M(t)$  (red), and  $q_M(t)$  (blue) for Run A (solid lines), Run B (dotted lines), and Run C (dashed line). Note that  $\beta_M(t)$  would reach zero at an extrapolated time of  $t_* \approx 5300$ .

approximately constant. At late times, however, we see that  $\beta_M(t)$  is well described by an expression of the form  $\beta_M(t) = \beta_{M0} - \beta_{M1} \ln(t/\tau_0)$ . This implies that  $\exp(\beta_M - \beta_{M0}) = (t/\tau_0)^{-\beta_{M1}}$ . We also see that the extrapolated time  $t_*$  when  $\beta_M(t_*) = 0$  is given by  $t_*/\tau_0 = \exp(\beta_{M0}/\beta_{M1})$ . Looking at Fig. 7 suggests that the exponents  $p_M$  and  $q_M$  both increase, although it is not obvious that they attain the value  $2/3$  by the extrapolated time  $t_* \approx 5300$ .

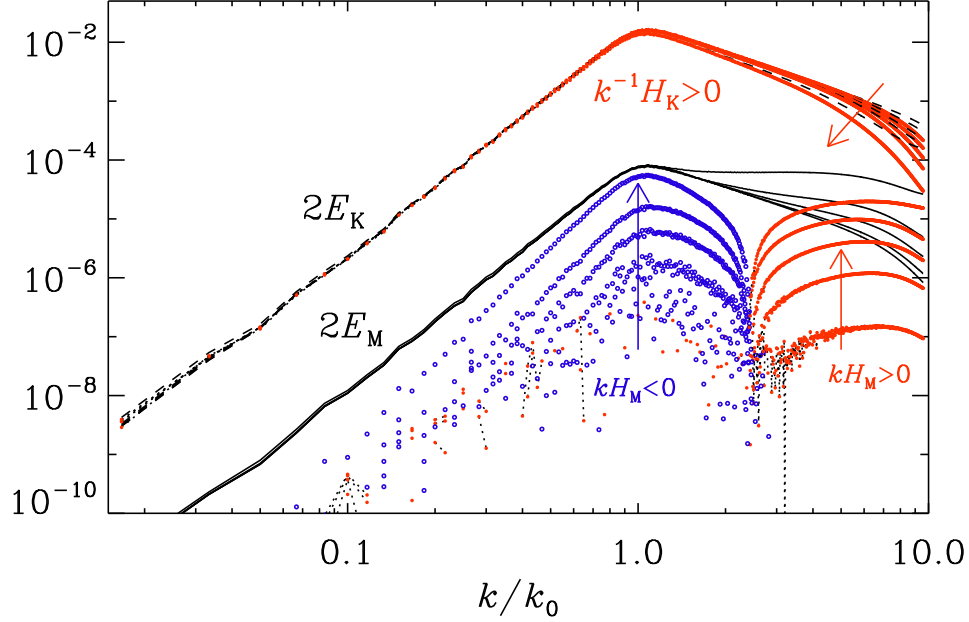


FIG. 8:  $k^{-1}H_K(k, t)$  (red) and  $kH_M(k, t)$  along with  $2E_K(k, t)$  and  $2E_M(k, t)$  (black lines) at  $t/\tau = 0.05, 0.16, 0.3, 0.6$ , and  $1.7$  for Run A (red for positive values and blue for negative values). The blue and red arrows indicate the change of  $kH_M(k, t)$  with time.

### B. Effect of finite initial magnetic helicity

We recall that, except for Runs B and C, no magnetic helicity was present initially, i.e.,  $\sigma_M = 0$ ; see Table I. Magnetic helicity is a conserved quantity and it can only change through resistive effects and at small scales. To understand how magnetic helicity gets produced, we show in Fig. 8 magnetic and kinetic helicity spectra,  $H_K(k, t)$  and  $H_M(k, t)$ , respectively. They obey the realizability conditions,  $k^{-1}|H_K(k, t)| \leq 2E_K(k, t)$  and  $k|H_M(k, t)| \leq 2E_M(k, t)$ , respectively, and are normalized such that  $\int H_K dk = \langle \boldsymbol{\omega} \cdot \mathbf{u} \rangle$  and  $\int H_M dk = \langle \mathbf{A} \cdot \mathbf{B} \rangle$ .

We see that, at early times, a bihelical magnetic helicity spectrum is produced, where positive and negative contributions are present simultaneously, though separated in  $k$  space, just like in driven turbulence [30, 34]. Thus, there remains a near-cancellation of the net magnetic helicity until the magnetic helicity spectrum saturates at  $k = O(k_0)$ . When that happens, magnetic helicity at large scales continues to increase only slowly such that at small scales magnetic helicity continues to dissipate resistively. Eventually, at late times, the positive magnetic helicity at small scales has disappeared and it has at all wave numbers a negative sign; see Fig. 9. Since Run C starts with  $\sigma_M = -1$ , we do not need to wait until the field with positive magnetic helicity gets dissipated. This leads to a more efficient transfer of kinetic energy to magnetic energy, which is why we see a stronger growth in Fig. 1. The opposite happens in the case with  $\sigma_M = 1$ , where the entire spectrum has initially the ‘wrong’ (positive) sign, making it even harder to establish a negative magnetic helicity at all wave numbers. The total magnetic energy decays then subject to resistive decay in the presence of magnetic helicity.

### C. Interpretation

In Fig. 10 we plot the evolution of  $\langle \mathbf{B}^2 \rangle \xi_M$  for different Reynolds numbers (Runs A, F, and G). We see that the magnetic helicity produced depends on the magnetic Reynolds number. For comparison, we also plot  $\langle \mathbf{B}^2 \rangle \xi_M^{1.05}$  for Run A. This is indicated by the dotted line, which has a flat tangent at the last time, and corresponds to  $\beta_M = 0.05$ .

To make contact with the mean-field interpretation developed in Sect. II, we show in Fig. 11 that  $\langle \boldsymbol{\omega} \cdot \mathbf{u} \rangle$  dies out while  $-\langle \mathbf{J} \cdot \mathbf{B} \rangle$  increases such that  $\langle \boldsymbol{\omega} \cdot \mathbf{u} \rangle - \langle \mathbf{J} \cdot \mathbf{B} \rangle / \rho_0 \approx \text{const}$  during the first 10,000 turnover times. It is this combination of kinetic and current helicity densities that replaces the otherwise kinematic  $\alpha$  effect in the nonlinear regime [30, 47]. At  $t/\tau_0 \approx 200$ , the sign of  $\langle \boldsymbol{\omega} \cdot \mathbf{u} \rangle$  changes and has now the same sign as  $\langle \mathbf{J} \cdot \mathbf{B} \rangle$ . This can be explained by the strong dominance of the magnetic field over the velocity field, which begins already at  $t/\tau_0 \approx 20$ ; see Fig. 1.

In Fig. 11, we also plot  $\langle \mathbf{A} \cdot \mathbf{B} \rangle$  and see that it never reaches a constant—not even until the end of the run. This

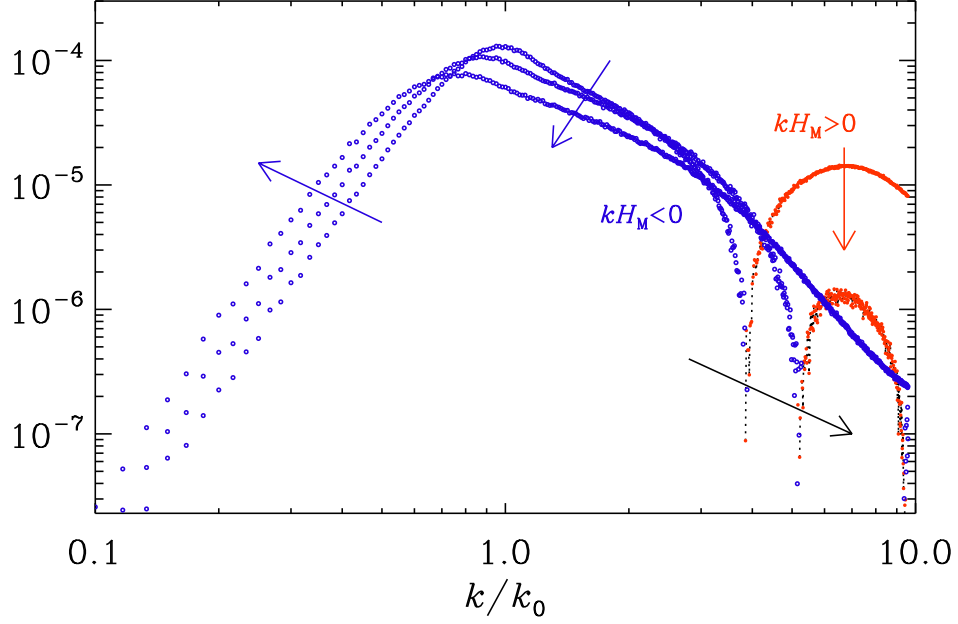


FIG. 9:  $kH_M(k, t)$  (red for positive values and blue for negative values) at later times:  $t/\tau = 5, 10$ , and  $25$  for Run A. The arrows indicate the temporal change of  $kH_M(k, t)$ .

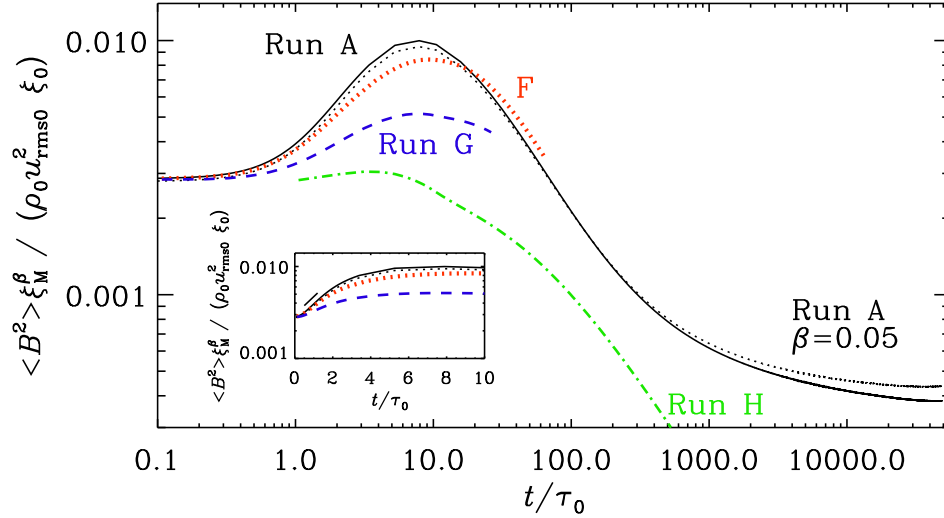


FIG. 10: Evolution of  $\langle B^2 \rangle_{\xi_M^\beta}$  with  $\beta = 0$  for Run A (black solid), F (red dotted), G (blue dashed), and H (green dash-dotted). In fully helical turbulence, we expect  $\langle B^2 \rangle_{\xi_M} \rightarrow \text{const}$ , but here  $\langle B^2 \rangle_{\xi_M^{1+\beta_M}} \approx \text{const}$  with  $\beta_M = 0.05$  at the end of the run.

explains why  $p_M$  and  $q_M$  are still different from  $2/3$ . By comparison with the other helicities, the magnetic helicity appears to rise sharply at  $t/\tau_0 \gtrsim 3 \times 10^4$  in this double-logarithmic plot. This signals the end of the  $p_M \approx 1/2$  scaling of magnetic energy and the beginning of a  $t^{-2/3}$  scaling after even later times.

We emphasize that in Fig. 11 we have plotted the time axis logarithmically and have normalized by the time-varying rms velocity. In this way we were able to display the various sign changes of kinetic and current helicities, but it also distorted the view. For this reason we now show in Fig. 12 in separate panels the kinetic, current, and magnetic helicities with a constant normalization using the initial velocity and the initial peak wavenumber in panels (a)–(c) along with time-dependent normalizations using the wavenumber of the domain  $k_1$  in panels (d)–(f), with a linear time axis for the magnetic helicity in panel (f). We see that magnetic helicity is always negative, reaches a peak

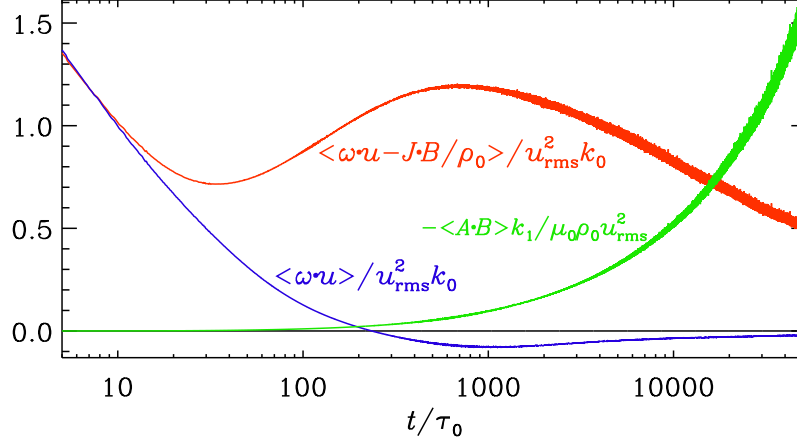


FIG. 11: Evolution of  $\langle \omega \cdot \mathbf{u} \rangle$  (blue),  $\langle \omega \cdot \mathbf{u} - \mathbf{J} \cdot \mathbf{B} / \rho_0 \rangle$  (red), and  $\langle \mathbf{A} \cdot \mathbf{B} \rangle$  (green) for  $\text{Re} = 160$  (Run A).

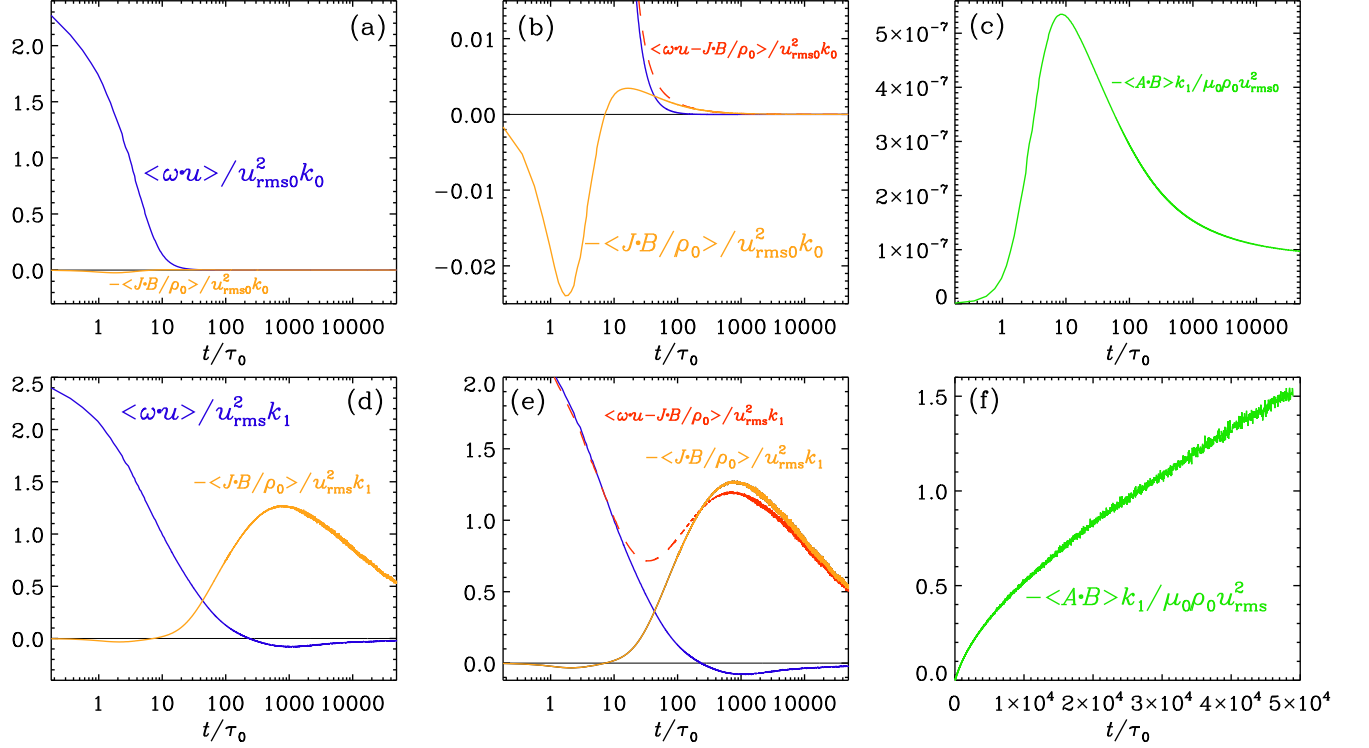


FIG. 12: Similar to Fig. 11, but with different normalizations, using in (a)–(c) the initial velocity and the initial peak wavenumber, in (d)–(f) the time-dependent velocity using  $k_1$ , with a linear time axis for magnetic helicity in (f) in separate panels focusing on  $\langle \omega \cdot \mathbf{u} \rangle$  (blue),  $\langle \omega \cdot \mathbf{u} - \mathbf{J} \cdot \mathbf{B} / \rho_0 \rangle$  (red),  $-\langle \mathbf{J} \cdot \mathbf{B} / \rho_0 \rangle$  (orange), and  $\langle \mathbf{A} \cdot \mathbf{B} \rangle$  (green) for  $\text{Re} = 160$  (Run A).

at  $t/\tau_0 = 10$  and then decays, before asymptoting to a finite value. When normalized by  $u_{\text{rms}}^2$ , the modulus of the magnetic energy increases approximately linearly; see Fig. 12(f).

The current helicity normalized by  $u_{\text{rms}}^2$  shows a negative peak at  $t/\tau_0 \approx 1000$ . This is when  $\langle \omega \cdot \mathbf{u} \rangle$  reached a negative peak, confirming again that the reason for its sign change is indeed related to the current helicity, which is then also negative and much stronger than the kinetic helicity.

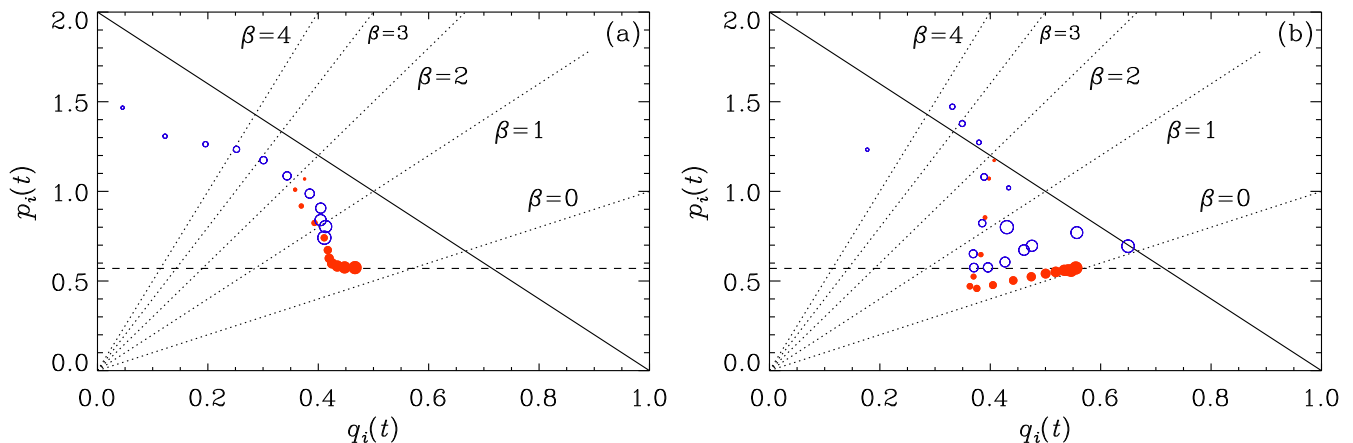


FIG. 13:  $pq$  diagram for  $\text{Pr}_M = 0.1$  (a) and 10 (b). Smaller (larger) symbols denote earlier (later) times. The  $p_M = 0.58$  line (dashed) is shown for comparison.

#### D. Robustness of the $t^{1/2}$ scaling

It is here for the first time that the  $t^{1/2}$  scaling has been observed. Several potentially important assumptions have been made and it needs to be seen to what extent they might affect our findings. Here we examine both the assumption of using  $\text{Pr}_M = 1$  and the assumption of using a time-dependent viscosity and a time-dependent magnetic diffusivity. In Figs. 13(a) and (b), we plot the results for  $\text{Pr}_M = 0.1$  and 10, respectively. In both cases, a similar evolution of  $(p_M, q_M)$  along the line  $p_M \approx 1/2$  is seen while  $q_M$  increases and approaches the  $p_M = 2(1 - q_M)$  line. Reaching this point would require a larger dynamical range and thus much larger domains and computation times than what has been possible so far. This is because, in the present runs,  $k_1 \xi_M$  becomes rather small ( $\lesssim 3$ ) toward the end of the run, so inverse transfer is no longer independent of the system size.

In Fig. 13(b), we also see that the trajectory overshoots the  $p_M \approx 1/2$  line when  $\text{Pr}_M = 10$ . This overshooting indicates that  $\eta$  is still too small for our numerical resolution of  $1152^3$  meshpoints. We have seen a similar behavior also when using a time-independent, but with a very small value of  $\nu = \nu_0$ ; see Fig. 14 for such an example.

Thus, the principal finding of an evolution along the  $p_M \approx 1/2$  line with increasing  $q_M$  toward the  $\beta = 0$  line, which is shortly before it reaches the  $p_M = 2(1 - q_M)$  equilibrium line, is recovered over a range of different circumstances, but the quality of convergence depends on how well we can approach the high magnetic Reynolds number limit.

#### V. CONCLUSIONS

Our work has demonstrated for the first time that the decay of turbulence with kinetic helicity leads to a non-conventional intermediate magnetic decay law with  $p_M \approx 1/2$  and  $q_M$  slowly increasing from about 0.4 to 0.6 before both  $p_M$  and  $q_M$  are expected to approach  $2/3$ . Qualitatively, our results are easily explained. At early times, a bihelical magnetic helicity spectrum develops and it grows until it reaches equipartition at the wave number where the magnetic spectrum peaks. At small scales, the sign of the magnetic helicity agrees with that of the kinetic helicity. At early times, however, no net magnetic helicity can be produced. Therefore, magnetic field with negative helicity is generated simultaneously at larger scales. This can also be understood as a result of mean-field dynamo theory, where the sign of magnetic helicity at large scales agrees with the sign of  $\alpha_{\text{dyn}}$  which, in turn, is a negative multiple of the kinetic helicity [1, 2].

At later times, the magnetic helicity at small scales gets dissipated resistivity, so that part of the magnetic helicity spectrum is gradually lost until the entire magnetic helicity spectrum has the same sign (negative) at all  $k$ . The kinetic helicity has then also reversed sign, but it is very small and sustained only by the current helicity. After that time, the magnetic energy spectrum shows an inverse cascade during which  $\langle B^2 \rangle_{\xi_M}^{1+\beta_M} \approx \text{const}$  with  $\beta_M \rightarrow 0$ ; see Eq. (15).

These new insights affect our understanding of all cases of decaying turbulence with initial kinetic helicity in electrically conducting media, such as plasma and liquid metal experiments, specifically the braked torus experiment, neutron stars, galaxy clusters, inertial fusion confinement plasmas, and the early universe. Thus, we predict that experiments should approach an evolutionary track in the  $pq$  diagram close to the  $p_M \approx 1/2$  line for many tens of

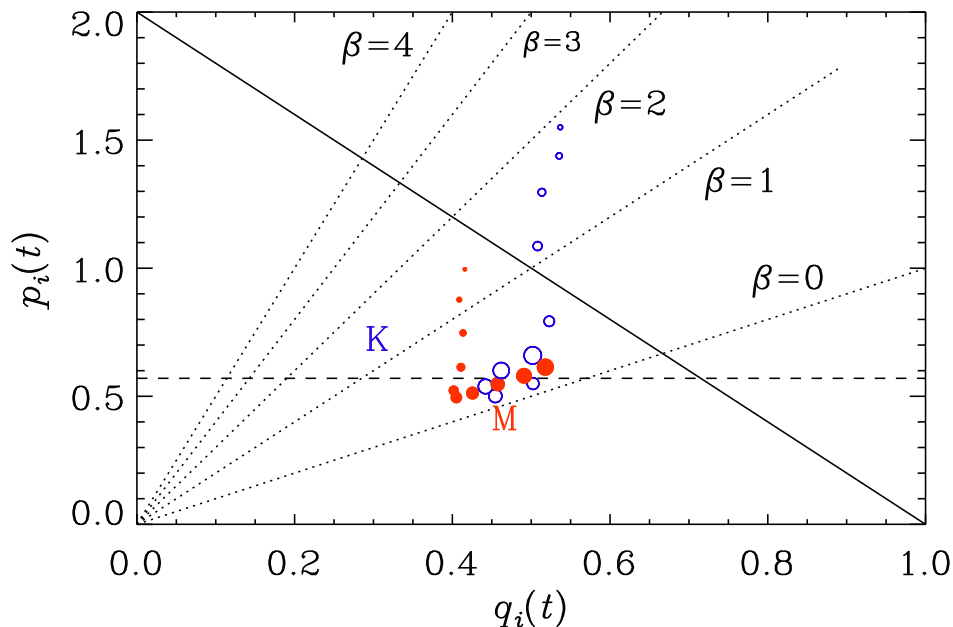


FIG. 14:  $pq$  diagram with constant  $\nu = \eta = 10^{-6}$  for  $\text{Pr}_M = 1$ . Again, smaller (larger) symbols denote earlier (later) times and the  $p_M = 0.58$  line (dashed) is shown for comparison.

thousands of turnover times if  $\text{Re}_M$  is large enough. Regarding applications to the early universe, the evolution in the  $\langle \mathbf{B}^2 \rangle$ – $\xi_M$  diagram (see Fig. 11 of Ref. [32]) will be slightly steeper than for an initially fully helical magnetic field. This is because  $\beta_M$  is already close to zero.

We recall that kinetic and two-fluid effects have been neglected in the present work. While this should be appropriate for liquid metal experiments and the early universe, it may not be accurate for plasma experiments and galaxy clusters. Even in the dense neutron stars, Hall drift may play a role [48]. At present, not much is known about the importance of kinetic and two-fluid effects for plasma decay in the presence of helicity, so there is still a lot of room for basic studies in the field.

The new evolutionary phase of decaying magnetic fields with initial kinetic helicity follows after an early phase of an exponential increase of the magnetic energy by dynamo action. It is here for the first time that such a process has been simulated. To be able to achieve this, it was necessary to reach rather large values of  $\text{Re}_M$ . The subsequent decay phase with  $p_M \approx 1/2$  has so far only been seen in the decaying phase of such a dynamo process. During that time, the magnetic energy is already decaying, but the system clearly captures signatures of the initial kinetic helicity in the system, which then, at later times, disappears in favor of producing first current helicity and later magnetic helicity.

The phenomenon of magnetic field amplification at intermediate times is a new phenomenon specific to high magnetic Reynolds numbers, which are only now becoming accessible to simulations. At present, no detailed comparison with dynamo decay experiments is possible yet, because no time-dependence of the magnetic field has been obtained. The best such experiment is that of two laser beams producing colliding plasma jets directed toward each other, leading to magnetic field generation that can be monitored through Faraday rotation measurements [29]. The situation is complicated further by the fact that in the experiments performed so far, the build-up phase of the turbulence constitutes a significant fraction of the total time available. One might therefore want to consider a model for the build-up of the turbulence as well, which has not yet been attempted.

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